ELEMENTARY MATHEMATICS

Prepared in accordance with the Course of Study prescribed for The Secondary School-Leaving Certificate

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WITH A PREFACE

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PART I.

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PREFACE.

AN accession to the already copious stream of modern text-books on the subjects of elementary mathematics for school use requires, perhaps, some excuse, but I think that the present book will have no difficulty in justifying its existence.

The central idea of the book is to develop concurrently the subjects of arithmetic, mensuration, algebra and geometry, and to ignore the artificial and mischievous distinction which in the older text-books separated these subjects into non-communicating compartments; it attempts to give prominence to principles and to present them in a form which the youthful intellect can readily assimilate; attention is paid to shortened methods of computation and to approximation; the examples are mostly of a practical kind; the importance of checking results is insisted upon. These and other good features are of course to be found in some of the more modern English text-books, but the present work has the additional merit for Indian school use that it has been written with special reference to Indian conditions.

It contains the Indian weights, measures and coins besides the English ones in current use; a large proportion of the examples are questions which occur in ordinary Indian life, and the statistical examples, being drawn from Indian statistical returns, railway time-tables, etc., have not that air of unreality which similar questions in English publications present to Indian pupils.

Part I is intended for the middle forms of Secondary schools, but pupils of the higher forms would be all the better for a thorough knowledge of its contents.

THE PRESIDENCY COLLEGE,
MADRAS,
4th December, 1911.

E. W. MIDDLEMAST.

CONTENTS.

[*N.B.*—Articles and examples marked with an asterisk may be omitted on a first reading.

It is not intended that the several chapters and sections should be taken in the order in which they are given here].

ELEMENTARY MATHEMATICS.

CHAPTER I.

NOTATION AND NUMERATION.

§ **1. Counting.** Suppose you are given one book and then another book; then you have *one* book and *one* other book, or *two* books in all. Now if you are given yet another book, you have *two* books and *one* other book, or *three* books in all. This process of adding on things of the same kind one by one is called **counting.**

Number. Thus if we are given a group of things, by counting them one by one we find how *many* there are in that group; and to express the result of counting we use **numbers** one, two, three, four, etc. Every time we count one more of the same kind, we use a new word to denote the number of things counted. In this way, we proceed up to *ten*, and when we count *one* more, we do not use altogether a new word to denote the number counted, but a derived word *eleven* (in Tamil பதினொன்று; in Telugu పదునొకండు) meaning ten and one. When we count one more, we have another derived word to denote the number, *viz.*, *twelve* (in Tamil பனிரண்டு and in Telugu పండ్రెండు) meaning ten and two, and so on, until we come to ten and ten, or *twenty*, meaning two tens (in Tamil இருபது; in Telugu ఇరువది). Similarly we have *thirty*, *forty*, (three tens, four tens, etc.) up to ten tens for which we use a new word, *viz.*, a *hundred*. A similar process is continued in counting above a hundred.

In this process of counting, it should be noted that **the order in which the things are counted does not in any way affect the result.**

§ 2. **Figures and Notation.** The numbers one, two, three......nine are respectively denoted by the symbols 1, 2, 3, 4, 5, 6, 7, 8, 9 which are called **figures.**

We have seen that, in counting after ten, no altogether new words are introduced, but only derived words, such as twelve (ten and two), thirteen (ten and three), twenty (two tens), thirty (three tens), etc., thus indicating that things are, as it were, collected and arranged in bundles or groups of ten, the remaining things under ten being counted separately and left loose. Similarly in expressing numbers by figures, no new figures are employed to denote numbers above ten ; but the groups of ten and the number of loose things left over under ten are separately indicated. Thus thirteen and twenty-five may be represented as shown below.

Groups of ten	Loose things left over
1	3
2	5

N.B.—A single object or a thing is called a **unit.** 5 such loose things will thus mean 5 units, and hereafter we shall call the column headed 'loose things' the 'units' column.

Now to represent twenty (two tens and no loose things or units), we write 2 in the column 'groups of ten' and 0 in the column 'units.' Thus 0 (read '**zero**') is a symbol used to denote nothing ; similarly to represent ten, we write 1 in the column 'groups of ten' and 0 in the column 'units.' We have thus seen how numbers up to ninety-nine may be represented. For numbers above this, we open

another column on the left, for 'groups of hundred,' and other columns for thousands, etc., as shewn in the following tabular form :—

Crores.	Tens of lakhs.	Lakhs.	Tens of thousands.	Thousands.	Hundreds.	Tens.	Units.	Number expressed in words.	Remarks.
							1	One	Least number of 1 figure.
							9	Nine	Greatest ,, ,,
						1	0	Ten	Least ,, 2 figures.
						9	9	Ninety-nine	Greatest ,, ,, ,,
					1	0	0	One hundred	Least ,, 3 ,,
					9	9	9	Nine hundred and ninety-nine	Greatest ,, ,, ,,
				1	0	0	0	One thousand	Least ,, 4 ,,
				9	9	9	9	Nine thousand nine hundred and ninety-nine	Greatest ,, ,, ,,
			1	0	0	0	0	Ten thousand	Least ,, 5 ,,
			9	9	9	9	9	Ninety-nine thousand, nine hundred and ninety-nine	Greatest ,, ,, ,,
		1	0	0	0	0	0	One lakh	Least ,, 6 ,,
		9	9	9	9	9	9	Nine lakhs, ninety-nine thousand, nine hundred and ninety-nine	Greatest ,, ,, ,,
	1	0	0	0	0	0	0	Ten lakhs	Least ,, 7 ,,
	9	9	9	9	9	9	9	Ninety-nine lakhs, ninety-nine thousand, nine hundred and ninety-nine	Greatest ,, ,, ,,
1	0	0	0	0	0	0	0	One crore	Least ,, 8 ,,

Exercise—Oral.

1. Read the numbers in the tabular form.

Tens of Thousands.	Thousands.	Hundreds.	Tens.	Units.
		3	5	1
2	3	4	5	6
	9	0	7	8
1	2	3	4	6
	8	9	3	4

2. Put into figures in the proper column :—
   (*a*) Three hundred and fifty-seven.
   (*b*) Eight thousand and forty.
   (*c*) Seventy-eight thousand four hundred and twenty-seven.
   (*d*) Ninety thousand and nine.
   (*e*) Ten thousand and one.

3. Mention (*a*) the least number of two digits beginning with 3 ;
   (*b*) the greatest number of three digits beginning with 7.

4. Find the greatest and the least number that can be formed with the digits, without repeating them : (*a*) 3, 5 ; (*b*) 4, 7 ; (*c*) 0, 8 ; (*d*) 5, 7 and 8.

Thus, in this system of **notation** (*i.e.*, the method of denoting by symbols numbers expressed in words) we use only ten symbols called figures, *viz.*, **1, 2, 3 ......9** and **0.**

It is called the **Decimal System of Notation*** because ten is taken as the basis for making our groups.†

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* Also called the **Arabic System** because it was first introduced into Europe by the Arabs who are believed to have obtained it from the **Hindus.**

† There are traces of other systems of notation having been in use, *e.g.*, the *Duodecimal System*, *i.e.*, one with twelve as the basis. We have the words *dozen* and *gross*.

Ten has been made the basis probably because man first counted on his fingers which are ten in number; hence also the name **digits** (meaning fingers) applied to the figures 1, 2, 3, etc.

§ 3. Intrinsic and local values of digits. Thus it will be seen that, in this decimal system, any number however large can be expressed by means of the nine digits and zero. This is made possible because digits are given, in addition to the values they have in the scale 1, 2, 3......9 called their **intrinsic** values, values corresponding to the columns they occupy which are called their **place** or **local** values. *E.g.*, the number three thousand, three hundred and thirty-three is represented in figures by 3333. Here the intrinsic value of each of the digits in the same, *viz.*, three, whereas the local values are different; the 3 on the extreme left standing for three thousands; the 3 coming next for three hundreds, the next 3 for three tens and the next 3 for three units.

The symbol zero has no intrinsic value and consequently has no local value; it is brought in to fix the local values of other digits.

Exercise—Oral.

What is the local value of each of the figures dotted in :—

1. $\dot{8}378$. **2.** $46\dot{7}89$. **3.** $\dot{9}0\dot{9}8\dot{9}$.

4. $\dot{6}2\dot{8}3\dot{6}6$. **5.** $\dot{8}008\dot{8}5$. **6.** $8\dot{8}\dot{5}376$?

§ 4. Grouping of figures in Notation. When the student becomes familiar with the notation, the use of columns for writing the figures may be given up, and they may be written simply side by side, *e.g.*, 25 to represent two tens and five units.

When figures are thus written side by side, they are marked off into groups by commas, so as to enable us to easily ascertain the local values of figures, thus facilitating the reading of the numbers. The first 3 digits (from the right) are marked off into one group; and the rest of the digits into groups of two. Thus twenty-three crores, forty-seven lakhs, thirty-five thousand, five hundred and ninety-six is written—23,47,35,596. This is the system prevailing in India.

In England, the digits are marked off into groups of three up to millions, and after millions into groups of six. The Indian lakhs is read in England as a hundred thousand. A million means a thousand thousands, or ten lakhs. A million millions is called a billion; a million billions is called a trillion; similarly we have quadrillion, quintillion, &c.

Exercise 1 (a).

1. Construct a table like that on page (3) and place in it the numbers—

(1) One lakh, two thousand and thirty-three.

(2) Five lakhs, sixty-five thousand and forty-four.

(3) Sixteen lakhs, twenty thousand, three hundred and forty-five.

(4) Two crores, seven lakhs, fifteen thousand, three hundred and ninety.

2. Express in figures the following numbers marking them off with commas into proper groups:—

(*a*) (1) Two lakhs, thirty-five thousand, four hundred and ninety-five.

(2) Fifteen lakhs, six thousand and twenty-four.

(3) Two crores, seventy-eight thousand and forty-five.

(4) Fifteen crores, seven lakhs, four hundred and ninety-seven.

(*b*) (1) One hundred and fifty thousand, three hundred and forty-nine.

(2) Two millions, forty-five thousand, seven hundred and three.

(3) One thousand and forty-three millions, two hundred and seven thousand, five hundred and sixty-four.

3. Express in words the following numbers :—

(*a*) (1) 20.687.
(2) 4,32,099.
(3) 8,05,807.
(4) 21,36,639.
(5) 2,06,89,703.

(*b*) (1) 206,598.
(2) 809,777.
(3) 5.603,708.
(4) 25.009,078.
(5) 1047,687,343.

4. Distinguish between the intrinsic and local values of digits and point out the distinction with reference to the number 5555.

5. What is meant by saying that we are using a decimal system of notation ? Give an example of any other system in use.

6. What is the use of zero in the notation of numbers ?

* ROMAN SYSTEM.

§ **5.** Besides the decimal system explained in the previous articles, there is also another system in use for expressing numbers in figures, called the **Roman System.** The figures used in this system, are given in two columns below, and their values are noted against them (in brackets):

I (one)
X (ten)
C (hundred)
M (thousand)

V (five)
L (fifty)
D (five hundred)

Some of these figures you might have seen on the dial of a clock where you have the first twelve numbers expressed in Roman Notation.

In this system, the figures given in the left-hand column may be repeated twice or thrice, each figure has only its intrinsic value wherever it may be, and the figures added together give the number expressed; *e.g.*, XV means 10

and 5 or 15; XXI means 10 and 10 and 1 or 21; CLXV means 100 and 50 and 10 and 5 or 165. But when I is placed before V or X and similarly X before L or C, and C before D or M, it indicates that the number denoted by the former figure is to be taken off from the number denoted by the latter: thus IV means 5 minus 1 or 4, CIX means 100 and 10 minus 1 or 109. In Sanskrit, we have phrases expressing the idea of subtraction as indicated above, *viz.*, Ekónavimsati, Ekónashashti meaning (20—1, or) 19 and (60—1, or) 59 respectively.

This system is, however, not suitable for purposes of calculation. At present its use is mainly confined to indicate serial number by way of distinction from, and to avoid confusion with, the ordinary symbols used.

Exercise 1 (b).

1. Express, in figures of the Roman system, the following numbers :—

(1) 27; 39; 48; 153; 297; 2306.

(2) 9; 35; 41; 129; 305; 5768.

2. Express in Arabic notation, the following numbers :—

(1) XIV; LX; LXV; LXXIX; CXXXIV; CCXXIX; DCCLXXV; MCCXXXVIII; MCMX.

CHAPTER II.

LINES, POINTS AND MEASUREMENT OF LENGTH.

§ 6. Here you see three wooden models of different sizes and shapes. Fig. (1) is a picture of what is called a cube; Fig. (2) is that of a pyramid; and Fig. (3) is that of

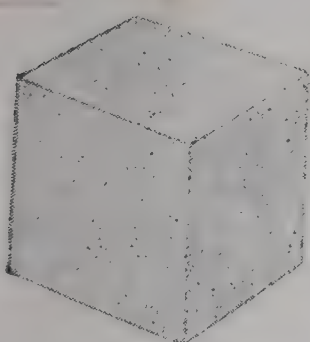


FIG. 1.

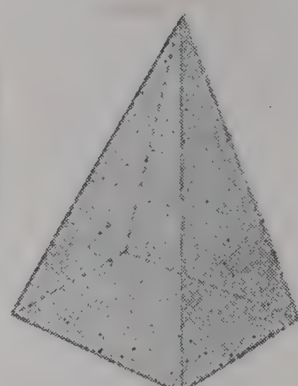


FIG. 2.

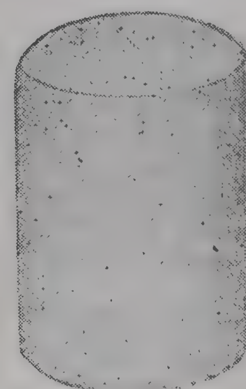


FIG. 3.

a cylinder. These are called **Solids**. If you examine the outside of each of these solids, you will find that the cube has six sides, or boundaries, which separate it from the outside space; that the pyramid has five and the cylinder three. The boundaries of these solids are their **surfaces**. Observe where any two such surfaces meet: they meet in **edges or lines**. These edges thus form the boundaries between two surfaces. Again observe where the edges or lines meet in the 1st two models; they meet in **points**.

Surfaces. You may have seen a tumbler containing some water and oil. There the water stands up to a certain level and above it the oil. You can see the boundary or the surface separating the two liquids. Has this surface any thickness? If it had, it must be either water or oil, in which case it cannot be the common boundary. Thus it will be seen that a **surface has no thickness**. Similarly the

sides of the cube, the pyramid, etc., have no thickness; but they have only *length* and *breadth.*

Lines. The walls of a school-room are generally painted blue up to a certain height and white above it. What is it that separates the blue from the white? It is a line Has it any breadth? No, it has not; for, if it had, it must be either blue or white, but it is neither. **So a line has no breadth**; but only length. Similarly the edges of the cube, the pyramid, etc., have no breadth

Points. Take a piece of paper, fold it once. What do you get? A line. Now keeping the original fold, fold it once again as in figure and notice that you get a sharp **point** at A in the figure. If you unfold the paper completely how many lines do you see? Two lines What is there where the two lines meet? A point. Has the point any length? No. Has it any breadth? No

FIG. 4.

Here is a pin. What difference do you notice between its two ends? One end is sharp and the other end has a nob. What do you mean by 'sharp'? It is well-pointed or ends in a sharp point; can you show any such sharp point in any object around you? The sharp end of the pencil is such a point.

Thus **the ends of lines are points.** Where two lines meet you have a point and **a point has no length and no breadth**; but it merely marks **position**.

Draw the sharp point of a pencil across a paper You get a line; this shows that a *line is traced by a moving*

point If the point of your pencil is blunt, you will get a thick line but if it is very sharp, you will get a very thin line.

In any case the line you draw has necessarily some breadth, but the finer the line and the less breadth it has, the more it approaches a true line.

Draw two clear lines on paper so as to cut each other. You will notice that there is some small space common to them. The thinner your lines the smaller is this space, and the nearer it approaches a true point.

Exercise—Oral.

1. (*a*) How many faces has a cube?
   (*b*) How many edges has each face?
   (*c*) How many edges on the whole has a cube?
2. Similar questions for the pyramid.
3. What is the difference in shape between the faces of a cube and those of a pyramid?
4. What is the difference in form between the surface of a cube and the surface of a cylinder?

§ 7. Kinds of lines and tests of straight lines. Here are drawn a number of lines.

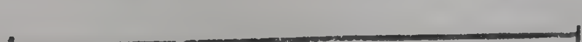


FIG. 5.

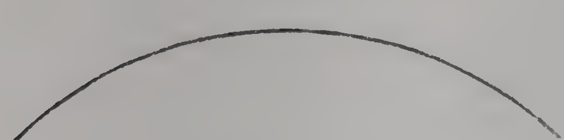


FIG. 6.

FIG. 7.

Do you find any difference in their form? Yes: The line shewn in Fig. (5) is **straight** whereas the lines shewn in Figs. (6) and (7) are not straight. What is the difference between (6) and (7)? (6) is a continuous line, whereas (7)

is a broken line. Continuous lines which are not straight are called **curved lines.**

Hold each end of a piece of string between a finger and thumb in each hand and keep it tight. What kind of a line do you then get? **A straight line.** Keep the string slack. What kind of a line is it now? **A curved line.** Twist the ends of the string between your fingers (1) when it is tightly held, (2) when it is slack, and watch how the string behaves in each case.

Draw a line AB as in the two figures shown below. Put a piece of tracing paper over the line AB in Fig (9) and make a neat tracing of it Mark A and B and turn the tracing paper over, face downwards and put it again over the line so that the reversed A falls on A, and the reversed B on B, and observe whether the traced line falls over AB throughout its length. Observe that it does not fall over AB. Do the same with the line AB in Fig. (8). Here it will be found that the traced line when reversed falls exactly over AB. Apply this test to the lines ruled with your ruler to see if they are straight.

FIG. 8.

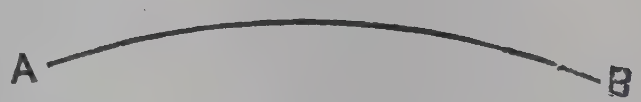


FIG. 9.

A second test will be to take a fine piece of thread and keep it tightly stretched along the line AB (Fig. 9). If the thread lies along the line all the way, the line AB is straight. You know what the drill-master does. when he forms the line before beginning to teach exercises, to test if the line is straight. He stands near the first boy and looks along the line with his eye placed just over his head and directed

towards the head of the last boy. He then sees that the heads of all the boys are along the line of his vision. You may test the straightness of a line by looking along it with the eye placed at one end.

§ 8 Marking points and drawing lines.

Rule two straight lines so as to give as exact a point as possible; thus : ×

This is the best way of marking a point and not by a dot thus :

Exercise 1.—Mark a point on your paper and draw a straight line with a straight edge or ruler so as to pass through it. This must be done carefully; otherwise the line may not pass through the point. How many straight lines can be drawn passing through this given point? Any number, is it not? Draw half a dozen such straight lines.

Exercise 2.—Now mark two points A and B and draw a straight line so as to pass through them both. Repeat this exercise a number of times by taking A and B at different distances and in different positions. Again mark two points A and B and draw a straight line (1) beginning exactly at A and passing through B, (2) beginning exactly at A and ending exactly at B. Do this neatly and note that the second line falls exactly upon the first.

Thus you see that only one straight line can be drawn between two points. You have already seen that any number of lines can be drawn through a point and now you see that only one straight line can be drawn through two points; Thus **a straight line is determined by two points and two straight lines cannot enclose a space.** How many curved or broken lines can be drawn between two points? Any number. If you take a thread and measure these lines and also the straight line between them you will find that the straight line is the shortest.

of them. You also know that if you have 2 ways to go from a place A to a place B: (1) along a straight road, (2) along a winding lane, the straight way is the shortest. **Thus the straight line is the shortest distance between two points.**

Exercise 3.—Draw any straight line AB, and produce it to any length. Produce it again to a different length and note that the produced portions fall upon each other. **Thus two straight lines which have a common part coincide throughout, however far they may be produced.**

N. B.—Every boy should have two pencils—(1) a pencil with a point to mark points, (2) a pencil with a chisel edge to draw lines.

Measurement of length. When you go to buy cloth for a coat, how does the shopman measure the cloth? With a yard measure. Why does he not allow you to use your arm for measuring? Because different persons have arms of different lengths, whereas the yard measure used is of the same length everywhere.

To secure uniformity and to avoid deceit, the British Parliament has fixed a standard of measure called **a yard** which is defined as the distance betweeen two marks upon a bronze bar kept in the Exchequer Chambers, London. To measure small lengths, a yard is divided into three parts, each part being called a *foot*; and a foot is divided into twelve parts, each part being called an *inch*.

On the next page is given the figure of a 4-inch rule one edge of which is divided into inches and each inch subdivided into ten equal parts. To measure the length of a line in inches, place the scale so that the zero mark of the scale is at one end of the line (say A) and the scale is close to the line. If the other end (say B) then falls at one of the

main division marks 1, 2, 3, &c., you can read off the length at once, and you will get an exact number of inches. In Fig. (10) the length of AB is 2 inches, since B falls at

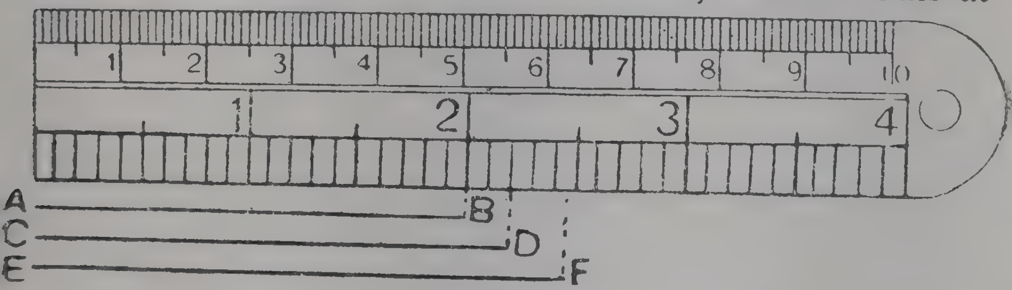


FIG. 10.

the main division 2. If the other end does not fall exactly at one of the main division marks as in the case of the line CD in the figure, it must fall between two such divisions; say, at the sub-division 2, as shown in Fig. (10); then the length is 2 inches and 2 sub-divisions If the other end does not fall exactly at one of the sub-division marks, it must fall between two sub divisions as in the case of EF in Fig. (10). If F is nearer the sub-division 4 than the sub-division 5 the length of EF may be taken to be 2 inches and 4 sub-divisions. If F is nearer the sub-division 5 than 4, the length of EF may be taken to be 2 inches and 5 sub-divisions. In other words, when the last remaining portion of the line to be measured is less than half a sub-division we neglect it and do not count it; when it is greater than half a sub-division we regard it to be nearly equal to a sub-division and count it as one sub-division. This is what is called measuring a line *correct* to a sub-division. The same principle is observed when examination marks are entered in registers. A boy getting $13\frac{1}{4}$ marks will find his marks entered in the register as 13; but a boy getting $13\frac{3}{4}$ marks will find it entered as 14.

§ **10**. The length of the line CD in Fig. (10) is 2 inches and 2 sub-divisions. Since ten such sub-divisions make one

inch, each sub-division is a tenth of an inch. The length of CD is 2 inches and two-tenths of an inch which is usually written 2·2 inches, just as 2 rupees 4 annas is written Rs. 2-4. Similarly if a line is 3 inches and seven-tenths long, its length is written 3·7 inches.

The abbreviations "*in.*" and "*ft.*" are generally used for *inch* and *foot* respectively. A foot is also denoted by the mark (′) and an inch by the mark (″). Thus '2 *feet* 3 *inches*' may be written either as '2 *ft.* 3 in.' or as 2′ 3″. Again '3·7 *inches*' is generally written 3·7 *in.* or 3·7″.

It is important that the scale should be quite close to the line measured ; if it is not so, there would be difference in the reading for slight alterations in the position of your eye. *E.g.*,

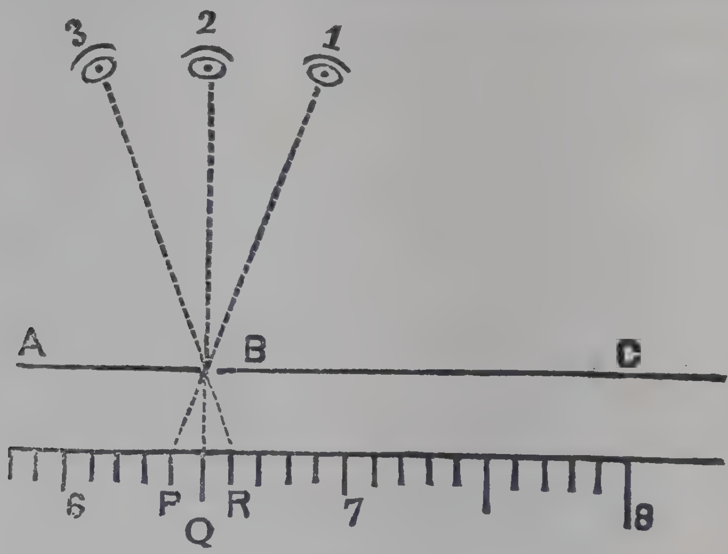


FIG. 11.

ABC is a straight line. See Fig. (11). The point B appears to be at the sub-division R when seen with the eye in position 3 ; it appears to be at Q when seen from the position 2 and at P when seen from the position 1. Hence the necessity to have the foot rule as close to the lines as possible.

§ **11. Use of dividers.** AB is a given line (Fig. 12). Suppose you want to draw another line of the

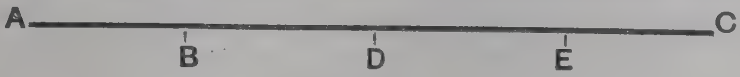


FIG. 12.

FIG. 13.

same length as. AB. Apply your dividers to AB so that the pin points are at A and B. Then take out the dividers and keeping the same distance between their legs slightly press the pin points elsewhere on the same page. Join the pin marks with your ruler. Thus you get AB of Fig. (13) equal in length to AB (Fig. 12.)

Produce the line AB to C ; suppose you want to mark off distances BD, DE, &c., each equal to AB, along BC. As before, open your dividers and place them so that the pin points are at A and B ; without altering the distance between them, keep the pin point at B fixed and turn the dividers on the leg resting at B until the pin point of the other leg occupies the position D on BC. Similarly, keep the leg at D fixed and turn the dividers on this leg until the pin point of the other leg comes to the position E, and so on. In this way you can mark off on a line any number of lengths each equal to a given line.

You can also measure lines by means of the dividers. Thus, to measure AB of Figure (13) open the dividers and apply them to AB so that the pin points are at A and B. Keeping the same distance between the legs, put one pin point on the zero reading of the scale and see where the other pin point falls on the scale ; then you can read off the length readily.

N.B.—In measuring lengths, it is advisable not to use always the end markings as the scale might then be worn out at the ends by frequent use.

§ **12. The Metric System.** If you look carefully at the 4 inch rule (Fig. 10) you will notice that the other edge of your scale is graduated in centimetres and millimetres. You learn from the scale that 10 millimetres are equal to 1 centimetre. 100 such centimetres make what is called a **metre,** which is the standard unit of length in the French system of measurement corresponding to the British yard. (This standard was intended to be one ten-millionth part of a quadrant of the meridian of the earth, *i.e.*, of the distance from the pole to the equator). It is a little more than a yard, being slightly over 39 inches.

The abbreviations *cm* and *mm.* are generally used for centimetre and millimetre respectively. Thus '*3 centimetres 5 millimetres*' is generally written 3 cm. 5 mm.

Exercise II (a).—Practical.

1. Measure in thumb-breadths the lengths of your pencils and the edges of your books and note-books.

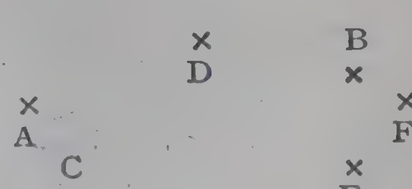
2. Measure the length of your table with your pencil and state the result.

3. (*a*) Mark three points A, B, C so that they do not lie all in one straight line; join the points in pairs by straight lines. How many such straight lines do you get?

(*b*) Repeat exercise (*a*) by taking 4 points no three of which lie in a straight line.

4. In your note book mark the points A, B, C, D, E, F, as shown here. Join

(1) A and C.
(2) D and E.
(3) E and F.
(4) A and B.
(5) D and B



(6) At what point does the line AB meet the line BF?

(7) At what point does the line AC meet the line CD?

(8) Produce the line CD to meet EF.

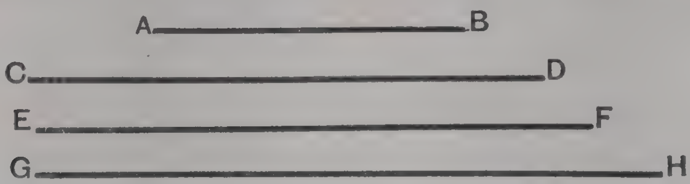
(9) Produce ED to meet AC.

(10) Produce AC and DB to meet each other.

5. From a point A draw a line 6·5 centimetres in length. Measure it also in inches.

6. Fill up the following table measuring the lines *A*B, CD, EF and GH given below :—

Name of the line measured.	Inches.	Tenths of an inch.	Result in Notation of Art. 10.
(1) AB			
(2) CD			
(3) EF			
(4) GH			

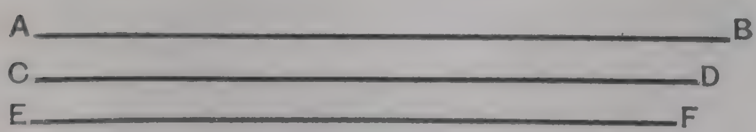


7. Fill up the above table using centimetres and millimetres.

8. Draw straight lines of the following lengths :—

(*a*) 2·3 in.	(*b*) 1·5 cm.
3·4 in.	3·6 cm.
4·9 in.	4·1 cm.
4·6 in.	7·6 cm.

9.



Fill up the following tabular form measuring the lines given above :—

Line measured.	Length guessed in cm. and mm.	Length actually measured correct to a mm.
AB		
CD		
EF		

10. Fill up the same table using inches and tenths of an inch for cm. and mm.

11. Draw lines of lengths given in the 8th question using your dividers.

12. Draw a straight line AB 5 in. long. In AB find points X, Y, and Z so that AX = XY = YZ = 1·5 inches. Use your dividers.

13. Draw a straight line AB, 4 in. long. Produce AB to C so that AC may be 4 times AB. Use your dividers.

14. Draw AB = 2·4″. Produce it to C so that AC = 5″. What is the length of the produced part?

15. Draw without using your scale the following lengths, and as nearly as you can guess the probable difference in each case:—
2″, 3″, 4″, 5″, 6″, 3·4″, 4·5″, 6·5,″ and verify by measurement.

§ 13. Representation of number by length. In Fig. 14 you have a line or a scale on which are taken equal lengths and their ends are marked 1, 2, 3, 4, 5,... 10,... 15, &c. The length OA drawn in the figure corresponds to the number 4, because it contains 4 of the equal lengths or its length **represents** the number 4. Similarly the length OB represents the number 7. We thus see that there can always be drawn a length representing or corresponding to a given number. Let

us take the number 16. This number is represented by the line OD.

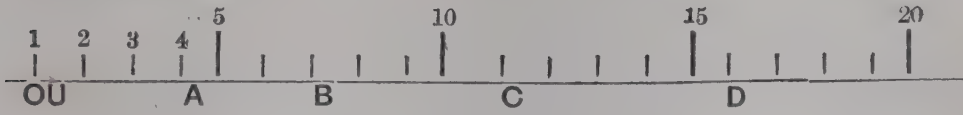


FIG. 14.

In the above representation of numbers by lengths, the lengths taken may be of any size: but, they must all be *equal*, so that when a certain length OU is taken to represent the unit, the length which represents 2 units is twice OU, that which is taken to represent 5 units is 5 times OU and so on.

Also concrete numbers such as 5 Rs. and 5 lbs. may be represented by lengths. Thus if you take a certain length to represent 1 Rupee or 1 lb., 5 Rs. or 5 lbs. will be represented by 5 times that length and so on.

N.B.—In all such representations, the length which stands for the unit must be clearly expressed.

Exercise II (b).—Practical.

1. Taking one sub-division of your squared paper to represent unity, draw lengths to represent 8 and 14.

2. Taking one mm. to represent unity, draw lengths to represent 25 and 37.

3. Measure the length and breadth of the class room with your foot-rule and represent these lengths on squared paper taking one small sub-division to represent a foot.

4. Measure the height and breadth of a window of your school room and represent these lengths on squared paper taking one main division to represent a foot.

5. Rama and Krishna have with them 15 Rs, and 25 Rs. respectively. Represent the money each has, taking a tenth of an inch to represent 1Rupee.

6. The weights of two cases of coffee are 30 lbs. and 40 lbs. respectively. Represent these weights by lengths, taking a mm. to stand for 1 lb.

7. The distance from Madras Fort to Saidapet is 5 miles. Represent that distance by a length taking half an inch to represent one mile.

8. The distances from Madras to Pallavaram and Chingleput are 12 and 36 miles respectively. Represent these distances in the same figure taking 1 mm. to represent a mile.

CHAPTER III.

ADDITION.

§ 14. Meaning and use of addition.—If you are given a group of things, you find by counting how many there are in that group. In actual life, however, you often have things in different groups and are required to find how many there are in all the groups taken together, *e.g.*, to find the total number of students in the different classes of a school. In such cases the groups may be all put together and the total number may be found by actual counting proceeding from one group to another and so on; but often it is not convenient or possible to put the groups together, and even if that be done, the counting becomes very laborious. Therefore in practice when the number in each group is known the total is found by a process known as **addition** (to be explained in this chapter). **Thus addition is the process of finding a number equal to a series of numbers taken together** and is used for finding the total number in several groups, the number in each group being known. **Addition is therefore a shortened form of counting.**

For this purpose, however, a table of addition of small numbers must be first prepared by actual counting and committed to memory.

The sign + (read 'plus') is the symbol for addition and denotes that the number following the symbol should be added to that preceding it.

The sign = stands for the words "is equal to" or "equals." Thus $8 + 5 = 13$ means that 5 added to 8 gives 13. When two or more numbers are added together the result is called the **sum**, *e.g.*, 13 is called the **sum** or **total** or amount of the two numbers 8 and 5. The numbers added are themselves called **addends.**

§ **15**. **Laws of addition.**—To find $5 + 3$, start with the number 5 and count 3 more numbers, *viz.*, 6, 7, 8. The last nnmber in this process, *viz.* 8 is the number equal to $5 + 3$. Similarly to find $3 + 5$, we start with 3 and count 5 more numbers, *viz.*, 4, 5, 6, 7, 8. The last number got in this process is the number denoted by $3 + 5$. The process may be represented thus :

1, 2, 3, 4, 5 ; 6, 7, 8.
1, 2, 3 ; 4, 5, 6, 7, 8.

The last number obtained is the same in both cases in whichever order we proceed. $\therefore\ 5 + 3 = 3 + 5$.

Similarly it may be proved that $5 + 4 = 4 + 5$ or in general that 5 + any number = that number + 5. Suppose then we use the letter a to represent any number, then $5 + a = a + 5$. It is usual to use letters to denote numbers whenever a general statement regarding numbers is to be made like the one given here. In the statement $5 + a = a + 5$, a may be put equal to any one of the numbers 1, 2, 3, 4... We may similarly prove that $6 + a = a + 6$, or any number + a = a + that number, or if b is used to denote any number, the relation becomes $a + b = b + a$. In this relation $a + b = b + a$, a and b stand for any two numbers. This law may be expressed in words thus :

The result of adding one number *b* to another number *a* is the same as the result of adding *a* to *b*, i.e., in whatever order the addends are taken the sum remains unaltered.

§ 16. Graphical representation of addition.—Suppose a person starts from O and walks 5 divisions to the right, reaching A. See Fig. (15). If he starts from A and walks again 3 divisions to the right, he will then reach the point B which is found to be 8 divisions to the right of O. Therefore $5 + 3 = 8$. Suppose the person starts from O and walks first only 3 divisions to the right reaching the point C. Then if he starts from C and walks 5 divisions to the right, he will reach the same point B, (eight divisions distant from O). Thus $3+5=8$. Whether he walks first 5 divisions, and then 3 divisions, or first 3 divisions and then 5 divisions the point reached by him is the same and distant 8 divisions from the starting point. The sum of two numbers is thus independent of the order in which they are added.

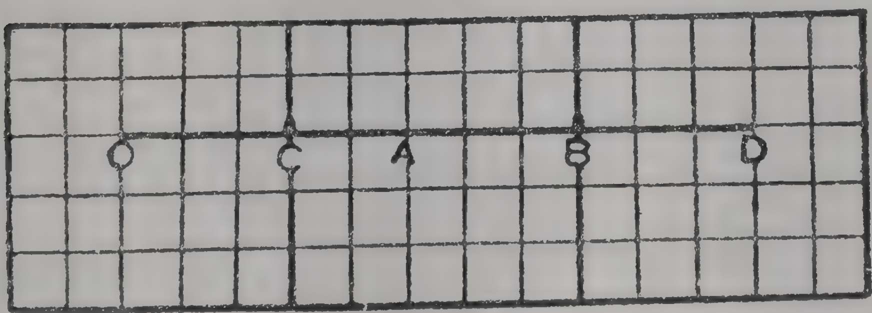


FIG. 15.

As before let the person start from O, walk 5 divisions to the right and reach the point A (Fig. 16); let him start from A, walk 4 divisions to the right, then he will reach the point B; again let him leave B and walk 3 divisions to the right. Then he reaches C which is 12 divisions from O. Thus in coming to C, he has taken in all $5+4+3$ steps. Suppose he starts from O, walks first 5 divisions, then 3 divisions and then 4 divisions, he will then reach the same point C; *i.e.*,

by taking $5+3+4$ steps he comes to the same point C. Or if he starts from O, walks first 3 divisions, then 4, and finally 5, he will even then reach the same point C. We thus have $5+4+3=5+3+4=3+4+5$. Or the sum of 5, 3 and 4 is the same in whatever order they may be added.

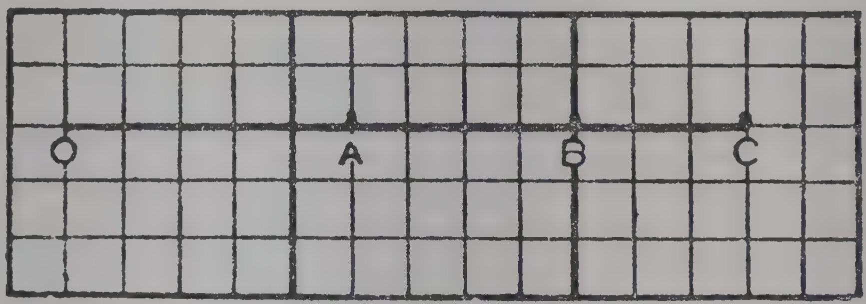


FIG. 16.

We may similarly show that

$$5+4+3=5+3+4=3+4+5=3+5+4=4+3+5$$
$$=4+5+3.$$

A similar result will be true if instead of 5, 4, 3 we have any three numbers a, b, c, and in general, we have

$$a+b+c=a+c+b=c+b+a=c+a+b=b+c+a$$
$$=b+a+c.$$

The law may be established by a similar mode of reasoning for four or five or any series of numbers. And this law for addition may be generalised thus: **The sum of any number of numbers is unaffected in whatever order the numbers may be added together.**

§ **17. Use of Brackets**—If in Fig. (16), the man, after walking first 5 divisions, be supposed to walk $4+3$ or 7 divisions, instead of 4 and 3 divisions separately, he would still reach the point C. This process of combining the two operations into one is denoted by placing $4+3$

within the symbols ( ) called **brackets**, thus: $(4+3)$. Then we get the result $5+(4+3)=12=5+4+3+\ldots$ Similarly we can show that $(5+4)+3=(5+3)+4$, etc.

The brackets in these examples denote that what is placed within them must be dealt with separately and the result used with the rest. Thus $5+(4+3)$ indicates the following order of operation:—first add 3 to 4 and add the result to 5. And $(5+3)+4$ denotes that 3 must be added to 5 and 4 must be added to the result.

And generally if a, b, c denote any numbers $a+(b+c)$ $=a+b+c=(a+c)+b=(a+b)+c$.

The statement $5+(4+3)=5+4+3$ or $a+(b+c)$ $=a+b+c$ may be graphically proved thus:—

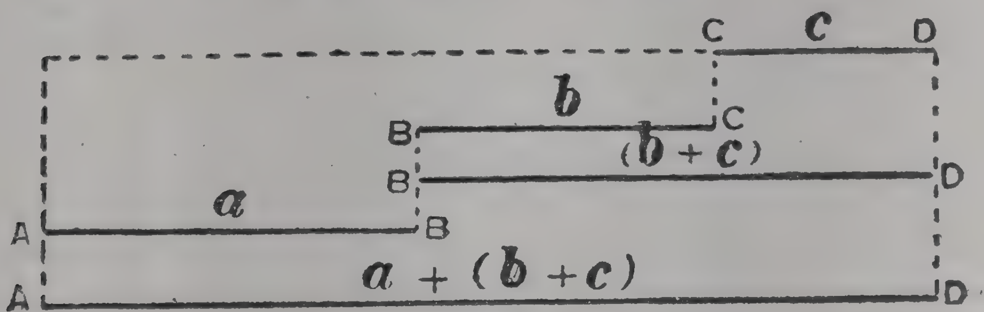


FIG. 17.

In Fig. (17) CD represents a length of c units
BC....................... b.....
∴ BD.................... $(b+c)$..
AB........................ a....
∴ AD............. $a+(b+c)$...

Again in Fig. (18) AB represents a length of a units
BC................... b......
∴ AC................. $a+b$....
∴ CD...................... c.....
∴ AD............. $a+b+c$....

Since AD is of the same length in both cases,

$\therefore \quad a + (b + c) = a + b + c.$

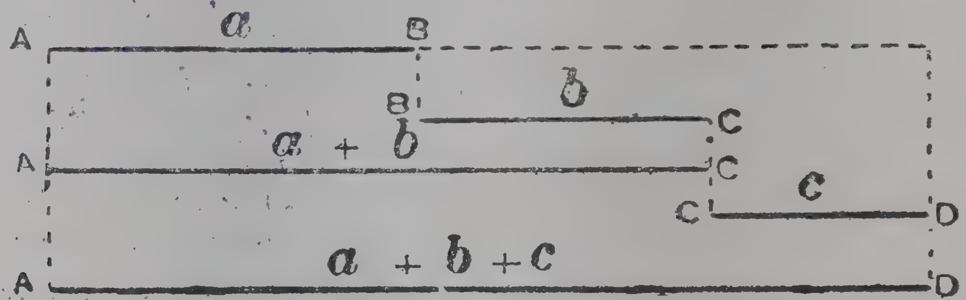


FIG. 18.

And more generally if a, b, c, d ... be a series of numbers, then $(a + b) + (c + d) + \ldots = a + b + c + d + \ldots = (a + c) + (b + d) \ldots$ &c.

This shows that **the sum of a series of numbers may be found by arranging the numbers in any groups and adding the totals of these groups.**

* You have learnt that, in counting, the order, in which the things counted are taken, does not in any way affect the result. Since addition is only a shortened form of counting, the laws mentioned above naturally follow.

Counting itself is primitive addition for, in counting you add things one by one.

Exercise—Oral.

1. Add together

(*a*) 9 and 8; (*b*) 7 and 5; (*c*) 9 and 4; (*d*) 7 and 8.

2. Find the value of

(*a*) $6+7$.	(*b*) $8+9$.	(*c*) $7+8$.
(*d*) $7+9$.	(*e*) $9+8$.	(*f*) $7+6$.
(*g*) $6+8+3$.	(*h*) $6+(9+4.)$	(*i*) $8+(9+9.)$

Exercise III—Symbolic.

1. If $5 + a = 8$, what is the value of a? 3

2. If $3 + b = 9$, what is b? 6

3. If $a + 7 = 16$, what is a? 9

4. If $b + 2 = 10$, what is b? 8

5. If $8 + 3 = a$, what is a?

6. If $9 + 4 = b$, what is b? 13

7. If $43 = a$ tens $+ 3$ units, what is a?

8. If $39 = 3$ tens $+ b$ units, what is b?

9. If $67 = a$ tens $+ b$ units, what is a and what is b?

10. If $783 = a$ hundreds $+ b$ tens $+ c$ units, what are the values of a, b and c?

11. What is the value of $a + 4$, if $a = 3; 6; 7; 8$?

12. What is the value of $a + b$, if
(1) $a = 3, b = 2$?
(2) $a = 8, b = 9$?
(3) $a = 4, b = 5$?
(4) $a = 6, b = 7$?

13. $a + b = 11$, what different values can be given to a and b?

14. If 3 and 5 are added, the process is denoted by $3 + 5$, similarly if a is added to 5, represent this process using the sign of addition.

15. If as in question 14, a is added to b, represent the process symbolically.

16. b is added to a and then c is added to the result, represent this symbolically.

Exercise III (a).—Practical and Graphical.

1. Show graphically the position of a man who walks 4 and 8 divisions to the right, and that of a man walking 8 and 4 divisions to the right respectively. Show that the distance walked in each case is the same. Hence deduce that $4 + 8 = 8 + 4$.

2. Show graphically that

(*a*) $8 + 3 = 6 + 5$

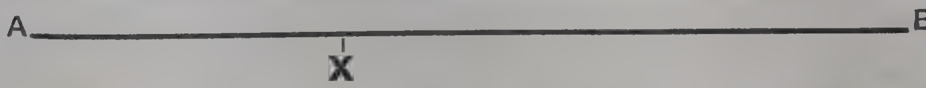
(*b*) $8 + 3 = 2 + 4 + 5$

(*c*) $3 + 6 + 4 = 5 + 1 + 7$.

(*d*) $6 + (3 + 2) = 6 + 3 + 2 = (6 + 2) + 3$.

3. Show that a man taking 2 steps of 4 divisions each will reach the same point as another person starting from the same point and taking 4 steps of 2 divisions each. Express the result symbolically.

4. Measure AX and XB in inches and tenths of an inch. Find



the sum of AX and XB and verify by measuring AB. Record your result thus

AX =
XB =
AB =

5. Repeat Ex. 4 using the metric scale. Also show that 5 + 3·2 = 8·2 with the help of your scale.

6. Find, by your scale, the value of

(1) 2·6 + 1·2 ; (2) 1·3 ÷ ·7 + ·6 ; (3) ·8 + 3·5 + ·1.

7. Shew, with the help of your scale, that

5 + 3·2 + 2·6 = 5 + 2·6 + 3·2 = 3·2 + 2·6 + 5......

§ 18. **Addition : General case.** In addition, as in counting, things added must be of the same kind or belong to the same class. It is meaningless to speak of adding 5 books and 3 hours. Consequently in the addition of numbers it is necessary that the numbers must be of the same class or 'order.' Thus 4 tens and 15 units cannot at once be added together but we must first convert 15 units into 1 ten and 5 units, and then add that one ten to 4 tens which gives 5 tens. And 5 tens with 5 units, make 55. Thus before adding together two or more numbers, we must first decompose them into hundreds, tens, units, &c., and then add the numbers of the same 'order.'

Example 1. Add together 125, 236, and 347.

Decompose the given numbers into hundreds, tens and units, and arrange them as shown below :—

Hundreds.	Tens.	Units.
1	2	5
2	3	6
3	4	7
7	0	8

Then 7 units and 6 units give 13 units ; 13 units and 5 units give 18 units which is equal to 1 ten and 8 units. Hence we put down 8 in the units column of the sum and carry this one ten to the column of tens for addition. 1 ten and 4 tens give 5 tens ; 5 tens and 3 tens, 8 tens ; 8 tens and 2 tens, 10 tens ; but 10 tens give us 1 hundred and no ten. We put down zero in the column of tens and carry 1 to the column of hundreds. 1 hundred and 3 hundreds give 4 hundreds ; 4 hundreds and 2 hundreds give 6 hundreds ; 6 hundreds and 1 hundred give 7 hundreds. So we put down 7 in the column of hundreds.

It is not necessary to say 7 units and 6 units, etc., in the course of addition. It is enough if we say 7 and 6, etc. Even this may be given up as the student advances. The mental calculation may then be as follows :—

The column of units : 7, 13, 18, set down 8 and carry 1.

The column of tens : 5, 8, 10, set down zero and carry 1 to the hundreds column.

The column of hundreds : 4, 6, 7, put down 7.

The process explained above may be symbolically expressed thus : (using the letters h, t and u for hundreds, tens and units respectively)

$$125 + 236 + 347 = (1h + 2t + 5u) + (2h + 3t + 6u) + (3h + 4t + 7u)$$

$$= 1h + 2t + 5u + 2h + 3t + 6u + 3h + 4t + 7u$$
$$= (1h + 2h + 3h) + (2t + 3t + 4t) + (5u + 6u + 7u)$$
$$= 6h + 9t + 18u = 6h + 9t + 1t + 8u$$
$$= 6h + 10t + 8u = 7h + 0t + 8n = 708$$

After sufficient practice, the columns may be dispensed with ; and the members to be added together, may be arranged one below another, units coming under units, tens under tens, &c.

Example 2. Add together 893, 1463, 987, 1293, 657, and 4837. Arrange them as shown below taking care to put units below units, tens below tens and so on.

$$\begin{array}{r} 893 \\ 1463 \\ 987 \\ 1293 \\ 657 \\ 4837 \\ \hline 10130 \\ \hline \end{array}$$

Explanation.

1st column. 14, 17, 24, 27, 30. Put 0 in the 1st column and carry 3 (to the next column).

2nd column. 6, 11, 20, 28, 34, 43. Put 3 in the 2nd column and carry 4 to the next column.

3rd column. 12, 18, 20, 29, 33, 41. Put 1 in the 3rd column and carry 4 to the next column.

4th column. 8, 9, 10. Put zero in this column, and carry 1 to the next column.

N.B.—Students are advised to see that the numbers are written neatly in vertical columns one below another. Squared paper may be used with advantage.

§ **19.** As De Morgan has said, 'Quick and Sure' must be the motto of a mathematician. In other words

speed and accuracy must be the two principal aims in the teaching of mathematics.

To acquire **speed,** a boy should have commited to memory the results of addition of numbers of 1 digit. Rapidity can also be promoted if a boy is trained to form a series mentally ; starting with a number, say 5, and going on adding 3 to it until a certain number is reached, thus :

5, 8, 11, 14, 17, etc.

The student is also advised to practice working simple additions mentally.

Example 1. Add together 37 and 45. The mental process must be 37 and 40 = 77 : and 5, 82.

Example 2. Add together 325 and 438. The mental process must be 325 and 400, 725 ; and 30, 755 ; and 8, 763.

Accuracy in addition is best attained by checking the results in various ways. If we have added from top to bottom, we may now add from bottom to top and see if the two results agree.

It is very useful to learn to add numbers horizontally taking care that units are added to units, etc. In this case if we have added from right to left we may add from left to right for the purpose of checking.

Exercise III (b).

Add together vertically 1 to 5 and from 18 to 23, and horizontally 6 to 11 and from 13 to 17 ; 12 is to be worked by adding totals of 1 to 5 and checking the result by adding the totals of 6 to 11.

1.	2.	3.	4.	5.		
6837	1345	7895	8637	2374	=	6.
943	6789	6435	407	1432	=	7.
123	1009	8920	9037	2065	=	8.
4567	2343	4865	5894	3904	=	9.
890	678	3478	1223	7867	=	10.
2375	1352	370	3641	9431	=	11.
					=	12.

13. $6\overset{*}{8}93 + \overset{*}{7}894 + 123\overset{*}{1} + 20\overset{*}{6}7 + 8\overset{*}{0}94 + 20\overset{*}{4}7.$

14. $70\overset{*}{6}3 + \overset{*}{4}057 + \overset{*}{9}38 + 100\overset{*}{2} + 325 + 6\overset{*}{2}067.$

15. $8\overset{*}{9}47 + 2\overset{*}{5}71 + \overset{*}{6}338 + 789\overset{*}{0} + 4001 + \overset{*}{3}4087.$

16. $\overset{*}{1}1035 + 1\overset{*}{2}037 + \overset{*}{4}338 + 690\overset{*}{8} + \overset{*}{3}0509 + 60\overset{*}{8}90.$

17. $120\overset{*}{3}6 + 20\overset{*}{4}67 + 308\overset{*}{9} + \overset{*}{7}308 + 41\overset{*}{3}9 + 503\overset{*}{7}.$

18. .... **19.** ...... **20.** ...... **21.** ...... **22.** ...... **23.** ......

24. Add together the totals of 13 to 17 and check the result by adding the totals of 18 to 23

25. Write down the sum of the numbers represented by the digits marked with an asterisk in each of the examples from (13) to (17).

26. Write the numbers to be added in questions 1, 2, 3 in any order different from that in which they occur and add them. Do you get the same answer? If so, how do you account for it?

27. Show that in each of the following magic squares the sum of the numbers in each row or column is the same :—

79	84	83
86	82	78
81	80	85

96	103	80	87	94
102	84	86	93	95
83	85	92	99	101
89	91	98	100	82
90	97	104	81	88

130	139	148	101	110	119	128
138	147	107	109	118	127	129
146	106	108	117	126	135	137
105	114	116	125	134	136	145
113	115	124	133	142	144	104
121	123	132	141	143	103	112
122	131	140	149	102	111	120

28. Write down the number 16,489. Add 21 to it and form another number, increase this number again by 21 and form a new number. Continue in this way until you get five numbers including the first. What is the sum of these 5 numbers?

29. Write down the number 21,853. Form 9 new numbers as in the example (28) by adding 100 successively. Find the sum of the ten numbers commencing with the given number.

SUBTRACTION.

§ **20.** If you have 10 marbles and give away 6 marbles to your brother how many marbles have you? Here you first give away 6 and count what is left and you find it to be 4. In this case you are said to **subtract** 6 from 10 and the result is given as 4. Similarly when we take away a number b from a number a we are said to subtract b from a. The process of taking away one number from another is called **subtraction.**

The 6 rupees given away and the 4 rupees left behind put together make 10 rupees. Thus 4 rupees is to be added to 6 rupees in order to make 10 rupees; *i.e*, the result of subtraction, *viz.*, 4 rupees, is also the answer to the question, "What must be added to 6 rupees to get 10 rupees?" And speaking generally, to subtract one number from another number is to find a number which, when added to the former, gives the latter. Thus **subtraction may be regarded as reversed addition.**

The result of subtraction is called the remainder or **difference**.

The number from which subtraction is made is called the **minuend**; whilst the number subtracted is called the **subtrahend.** In the example we have taken, 10 is the minuend, 6 the subtrahend, and 4 the remainder or difference.

The sign —, (read 'minus') is the symbol of subtraction and denotes that the number following it must be subtracted

from the number preceding it. The process of subtracting 6 from 10 and getting 4 may be symbolically represented thus: $10 - 6 = 4$; and $a - b$ denotes that b is to be subtracted from a.

As in the case of addition, it is desirable to form by counting and to commit to memory a table of subtraction containing the results of subtracting numbers of one digit from numbers below 20.

We have seen that $6 + 4 = 10$ ... ... (1)

and that $10 - 6 = 4$ ... ... (2)

and it can also be found by counting that

$10 - 4 = 6$ ... ... (3)

Similarly, whatever the minuend and subtrahend may be, we can prove that

Subtrahend + difference = minuend ... ... (4)

Minuend — subtrahend = difference ... ... (5)

Minuend — difference = subtrahend ... ... (6)

or $s + d = m$ ... ... (4)

$m - s = d$ ... ... (5)

$m - d = s$ ... ... (6)

where s, d and m are taken to stand for the subtrahend, difference and minuend respectively.

Exercise—Symbolic.

1. $8 + d = 10$, what is d?
2. $s + 4 = 12$, what is s?
3. $4 + 11 = m$, what is m?
4. $8 - a = 5$, what is a?
5. $b - 4 = 9$, what is b?
6. $16 - 9 = d$, what is d?

7. Given that $s+d=m$ (*a*) if $s=3$, $m=4$, what is d? (*b*) if $s=8$, $d=4$, what is m?

8. Given that $m-s=d$, if $s=5$ and $m=9$, what is d?

9. Given that $s=m-d$, if $s=13$, $m=18$, what is d?

10. Find $a-b$ when (1) $a=10$ and $b=3$.
(2) $a=10$ and $b=4$.

11. You had 15 peppermints of which you have eaten some, say a. Express using the symbol a the number of peppermints left. If the number left be 7, what number does a stand for?

12. If you had some marbles, say a, and gave 4 to your brother, how many marbles are left? If the number left be eight, how many marbles had you at first?

NOTE.—Let us find the value of $a-b$ when $a=8$ and $b=10$. Here we have to subtract 10 from 8, *i.e.*, a greater number has to be *subtracted or taken away* from a smaller number. This is not possible. Therefore subtracting or taking away 10 from 8 or the operation indicated by $8-10$ is at present meaningless to us.

§ **21. Graphic representation of subtraction.** In figure (19) the line AB contains 15 equal divisions,

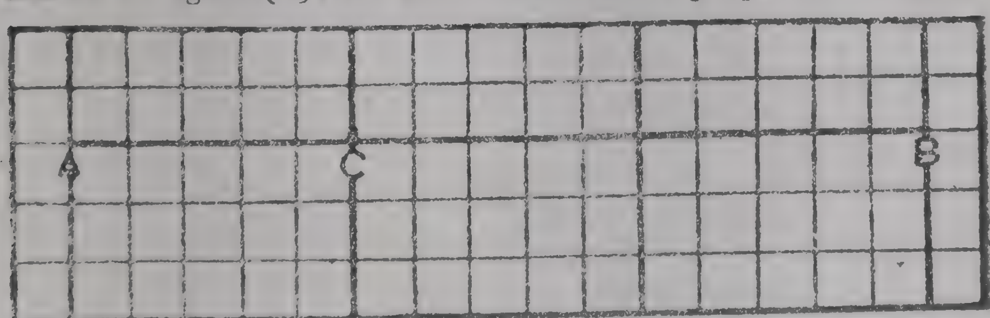
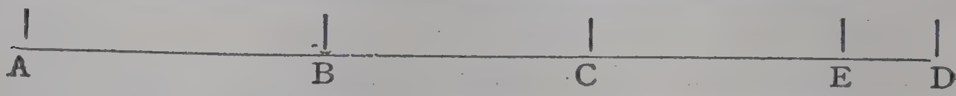


FIG. 19.

from which a bit AC containing 5 equal divisions is cut off. You will see that what is left, *viz.*, CB contains 10 divisions. Thus the figure shows that when 5 is taken from 15, 10 is left; or $15-5=10$. The student will notice that the figure also shows that $15-10=5$.

Exercise III (c).—Graphical and Practical.



1. Measure AD and AC in inches and tenths of an inch and arrange your results thus:

$$\begin{array}{l} AD = \quad \text{in.} \\ \underline{AC = \quad \text{in.}} \\ CD = \end{array}$$

Check by measuring CD.

2. In the same figure similarly find AD—BD. Check by measuring AB.

3. Find CD—AB. With your dividers mark off a point in CD so that CE = AB. Check the result you have got for CD—AB by measuring ED.

4. Find BD—AC. Show that it is equal to CD—AB.

5. Draw a line AB 5 inches long: and cut off from AB 4 lengths each equal to 1 inch. What is the length of the remainder? Verify by measurement.

6. Draw a straight line AB 9 cm. long, and starting from the ends A and B cut off lines equal to 2 cm. and 4 cm. respectively. What is the length of the remainder? Verify by measurement.

7. Draw a straight line AB 8 cm. long. Mark off a point P so that AP is 4 cm. Take any point Q in PB. Measure AQ, QB and PQ. Find AQ−QB and show that AQ−QB = twice PQ.

8. Find graphically the values of (*a*) $6{\cdot}3 - 4{\cdot}9$ (*b*) $3{\cdot}5 - 2{\cdot}8$.

9. With the help of your scale show that $4{\cdot}3 - 1{\cdot}2 = 5{\cdot}4 - 2{\cdot}3$.

10. Use your scale to find a number which when added to $3{\cdot}4$ will give $6{\cdot}2$.

§ 22. **Different methods of Subtraction.**—Just as in addition, addends ought to denote things of the same kind, so in subtraction, the minuend and the subtrahend must denote things of the same kind or belong to the same class. It is therefore necessary that, in writing the subtrahend below the minuend for subtraction, units must

come below units, tens below tens, hundreds below hundreds and so on.

Example. 1. Subtract 423 from 654

Hunds.	Tens.	Units.
6	5	4
4	2	3
2	3	1

Explanation.

Write 423 (the smaller number) below 654 (the larger) so that the units digit is below the units digit, the tens digit below the tens digit and so on.

3 units taken from 4 units leave 1 unit; so, we set down 1 in the units place of the remainder; 2 tens taken from 5 tens leave 3 tens; so, we set down 3 in the tens place; 4 hundreds taken from 6 hundreds leave 2 hundreds; so, we set down 2 in the hundreds place. Mentally the process is: 3 from 4 is **1**; 2 from 5 is **3**, 4 from 6 is **2**. As each of the figures in bold type is mentally thought of it is set down. When boys acquire considerable practice in subtraction, the mental process would be 1, 3, 2.

In this example each figure of the subtrahened is less than the corresponding figure of the minuend. Hence the subtraction is very simple. We shall now take up a more difficult example.

Example 2. Subtract 378 from 654.

$$\begin{array}{r} 654 \\ 378 \\ \hline 276 \end{array}$$

Write down 378 below 654 taking care to see that 8 comes below 4, &c. Now 8 cannot be subtracted from 4 (because 8 is greater than 4). If you have one ten-rupee note and 4 Rs. cash and want to pay 8 Rs. for some article purchased what will you do? Certainly you will get change for the ten-rupee note and then you will have 10+4 or 14 Rs. cash, and can easily give away 8 Rs. from 14 Rs. Similarly, out of 5 tens, take one ten and break it into ten units. You thus decompose 5 tens and 4 units into 4 tens and 14 units. Now taking

away 8 units from 14 units, you have 6 units which is set down in the units place in the difference. Again seven tens cannot be subtracted from 4 tens (1 ten has already been converted into units, so you have only 4 tens). As before out of the six hundreds, take one hundred and convert it into 10 tens. You thus decompose 6 hundreds and 4 tens into 5 hundreds and 14 tens. Taking away 7 tens from 14 tens you get 7 tens and 7 is set down in the tens place in the difference. Taking away 3 hundreds out of 5 hundreds (1 hundred has already been broken up into tens so you have only 5 hundreds) you get 2 hundreds and 2 is set down in the hundreds place in the difference. The process may be represented thus :—

$654 - 378 = 6$ hundreds $+ 5$ tens $+ 4$ units or
5 hundreds $+ 14$ tens $+ 14$ units $-$ (3 hundreds
$+ 7$ tens $+ 8$ units) $= 2$ hundreds $+ 7$ tens $+ 6$ units
$= 200 + 70 + 6 = 276.$

This is called the **decomposition method**.

* *Second Method.* This method depends on the fact that the difference between 2 numbers is unaltered by adding the same number to both, *e.g.*,

$$\begin{array}{r} 654 \\ 378 \\ \hline 276 \end{array}$$

$6 - 4 = (6 + 2) - (4 + 2)$, *i.e.*, $8 - 6$
or $(6 + 3) - (4 + 3)$. *i.e.*, $9 - 7$.

As before, 8 cannot be subtracted from 4. We add 10 as ten units to the minuend and also as 1 ten to the subtrahend. Then we have in the minuend 6 hundreds + 5 tens +14 units and in the subtrahend 3 hundreds + 8 tens + 8 units. Again we cannot take 8 tens from 5 tens so we add 1 hundred to the minuend as 10 tens and also as 1 hundred to the subtrahend. Then we have in the minuend 6 hundreds + 15 tens + 14 units and in the subtrahend 4 hundreds +8 tens +8 units. Now, the subtraction is very easy as in example (1). Therefore the difference is 2 hundreds + 7 tens + 6 units = 276.

This is called **the method of equal additions** (to minuend and subtrahend).

Third Method. This is the outcome of viewing subtraction as the process of finding a number which when added to the subtrahend will give the minuend. Here in subtracting 378 from 654 the mental process will be 8 and 6 make 14.

$$\begin{array}{r} 654 \\ 378 \\ \hline 276 \end{array}$$

Set down 6 and carry 1: *1 and 7, i.e.* 8 and **7** make 15. Set down 7 and carry 1; 1 and 3, *i e.* 4 and **2** make 6. Set down 2.

This method is known as the **complementary method.** It is also known as the **Austrian method.** It is the method used by our shopmen. Suppose you buy things in a shop for 7 annas. You give the shopman 1 rupee. Then he gives you back 9 annas saying "8, 10, 12, 16" giving you an anna first (saying 8), then 2 annas (saying 10), and then another 2 annas (saying 12) and 4 annas (saying 16 or 1 rupee).

To ensure accuracy in subtraction the best check is to add the difference and the subtrahend and to see if the minuend is got. The student should also practice subtracting the top line from the bottom line and also horizontally:

Example 3.

```
 18394
 56789
 -----
 38395
```

In this sum the top line has been subtracted from the bottom line The best check here is to add the difference and the top line and see if the bottom line is got.

Exercise III (d).

Subtract the bottom line from the top line in each of the following questions from 1 to 12:—

1.	**2.**	**3.**	**4.**	**5.**	**6.**
3867	6894	7546	6467	93763	43768
433	3908	6468	6353	28898	28934

7.	**8.**	**9.**	**10.**	**11.**	**12.**
387659	438716	110856	183488	3876548	1234567
26898	347865	98764	67406	999889	987654

Find the value of:—

13. 16038459 — 15938768.
14. 23679003 — 19876543
15. 33680009 — 32987654.
16. 48760099 — 7634489.
17. 10000000 — 8966554.
18. 1876594 — 444444.
19. 11111111 — 2222222.
20. 6350162099 — 55687699.

(From 13 to 20 one number should not be written below the other number for subtraction).

Fill up the dots in the addition sums from 21 to 24.

21.	**22.**	**23.**	**24.**
3809283	80304020	68473238	83847689
....	....	....	....
6847321	78643921	10008999	100000000

Fill up the dots in the subtraction sums from 25 to 28.

25.	**26.**	**27.**	**28.**
14648956	83893436	683564378	143287654
....	....	....	....
9876543	76000834	590000009	56399990

From 29 to 31 subtract the top line from the bottom line.

29.	**30.**	**31.**
863593	3283645	783645
1293986	92993876	1340000

32. Find the difference between 187654 and 32450.

33. Find the difference between 3248360 and 6 millions.

34. Which is the greater and by how much? 7847659 or 1006238.

35. Find 32486 $\backsim$ 783641. The sign $\backsim$ is used to denote the difference of two numbers.

36. (*a*) If the complement of a number of one digit is its defect from the least number of two digits, find the complements of 7; 8; 9.

(*b*) The complement of a number of two digits, three digits, or four digits is its defect from the least number of 3 digits, 4 digits or 5 digits respectively. Find the complements of 85; 346; 7899; 10,641.

LAWS OF ADDITION AND SUBTRACTION.

§ **23.** We have seen that + is the sign of addition and —, that of subtraction. Further + and — respectively indicate that the numbers following them should be added to and subtracted from the numbers preceding them. Also if a series of additions and subtractions is to be performed the order of operations is from left to right. $5 + 4 + 3$ means

that 4 is added to 5 and then 3 is added to the result; thus, $5 + 4 + 3 = 9 + 3$ or 12.

Again $5+4-3$ means 4 is added to 5 and then 3 is subtracted from the result, *i.e.*, $5 + 4 - 3 = 9 - 3$ or 6. Similarly $5 - 3 + 4$ means that 3 must be subtracted from 5 and to this result 4 must be added. $\therefore 5 - 3 + 4 = 2 + 4$ or 6.

Again $4 + 5 - 3$ means 5 is added to 4 and 3 is taken away from the result. $\therefore 4+5-3=9-3$ or 6. $4-3+5$ means that 3 is taken away from 4 and to this result 5 is added. $\therefore 4 - 3 + 5 = 1 + 5$ or 6.

Since the result obtained in each of these cases is 6, it may be concluded that $5 + 4 - 3 = 5 - 3+4=4+5-3 = 4 - 3 + 5$. As in the above instance it may be easily established that $6 + 5 - 2 = 6 - 2 + 5=5 + 6 - 2=5 - 2 + 6$. In both these instances the same set of three numbers occurring in different orders give but the same result, *the signs prefixed to each number of these sets remaining the same.* If the sign is changed the value will be affected, *e.g.*, $6 - 5 + 2$ is not the same as $6 + 5 - 2$ for the one is equal to 3 and the other is equal to 9. Similarly, in general, if instead of 5, 4 and 3 we have any three numbers a, b, c, then we get $a + b - c = a - c + b = b + a - c = b - c + a$ (provided that c is so taken as to be always less than a and b). Generally in a **series of additions and subtractions the final result will not be affected if the operations are performed in any order we please, provided there is no impossibility in the process of subtraction involved in the course of such operations.**

You have been told that in order to indicate that a certain operation must first be performed before another, *brackets* are used and that when numbers connected by the

signs + and — are placed within brackets, the meaning is that the operations indicated by the signs are to be performed first and the group of numbers within the brackets replaced by the result of such operations, and the brackets removed. Thus $5 + (4 - 3)$ means that 3 must be subtracted from 4 and the result added to 5.

Exercise—Oral.

1. Explain clearly the difference in the order of operations and compare the results, in (*a*) $5 + 4 - 3$ and $5 + (4 - 3)$
   (*b*) $9 - 8 - 6$ and $9 - (8 + 6)$.........
   (*c*) $9 - 8 + 3$ and $9 - (8 - 3)$.........
   (*d*) $7 - 6 + 5$ and $7 - (6 - 5)$.........

2. Simplify (*a*) $13 + 8 - 9 + 6 + 5$.
   (*b*) $16 - 9 + 7 - 3 - 2$.
   (*c*) $15 + 25 - 20 - 9 + 8$.
   (*d*) $17 - (2 + 3) - (4 + 6)$.
   (*e*) $23 - (5 + 4 - 6) + 2$.

§ **24. Graphical illustrations.** To show graphically that $12 - (5 + 4) = 12 - 5 - 4$ and more generally that $a - (b + c) = a - b - c$.

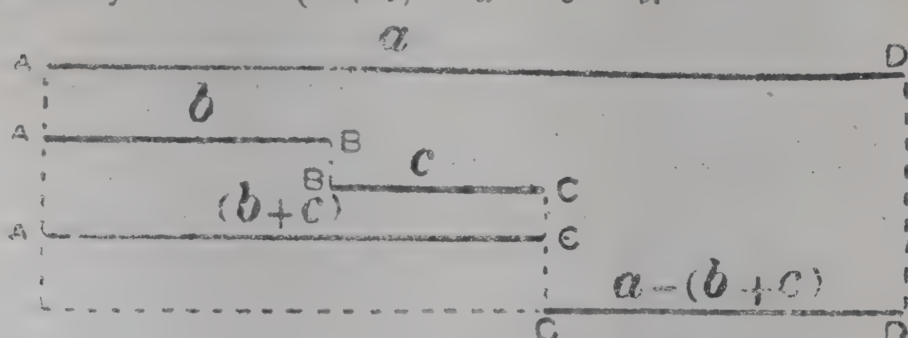


FIG. 20.

AD Fig. (20) represents a units.
and AB ,, ,, b ,,
and BC ,, ,, c ,,
Then AC will represent $(b + c)$,,
∴ CD ,, ,, $a - (b + c)$,,

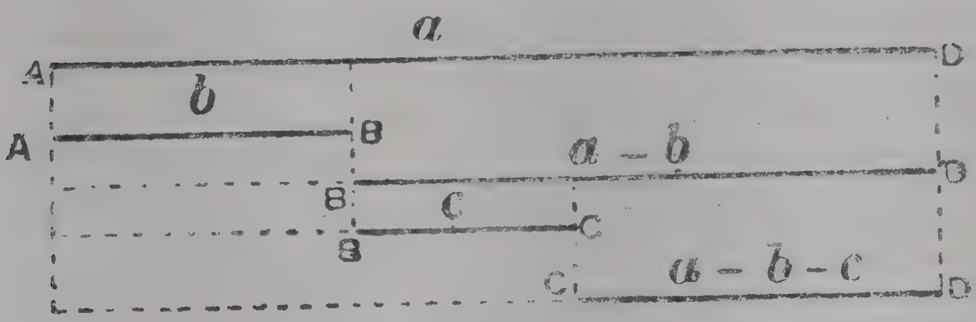


FIG. 21.

Again in Fig. (21)

	AD represents	a	units.
	AB ,,	b	,,
∴	BD ,,	$a - b$	,,
	CB ,,	c	,,
∴	CD ,,	$a - b - c$	,,

Since in both the cases the final result CD is of the same length, $a - (b + c) = a - b - c$.

We can similarly show from Figs. (22) and (23) that

$$5 + (4 - 3) = 5 + 4 - 3$$
or $a + (b - c) = a + b - c$.

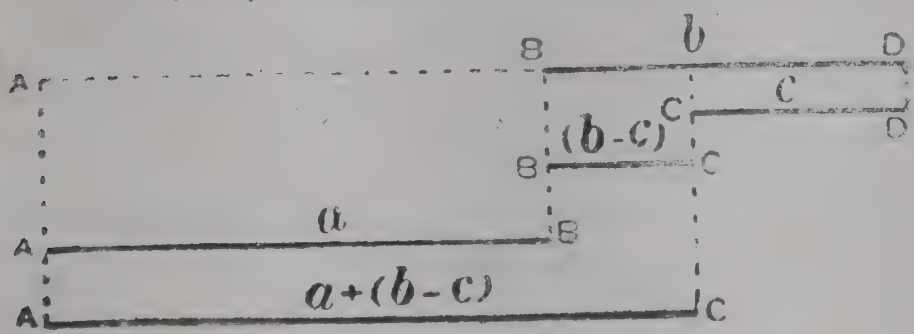


FIG. 22.

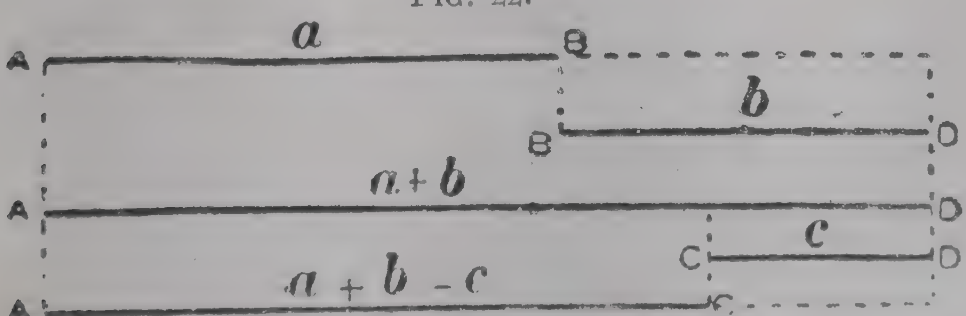


FIG. 23.

Similarly it may be shown from Figs. (24) and (25) that

$$5-(4-3) = 5-4+3$$

or $a-(b-c) = a-b+c$.

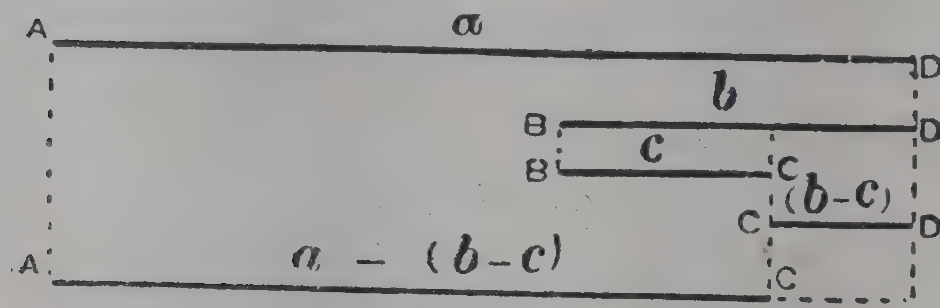


FIG. 24.

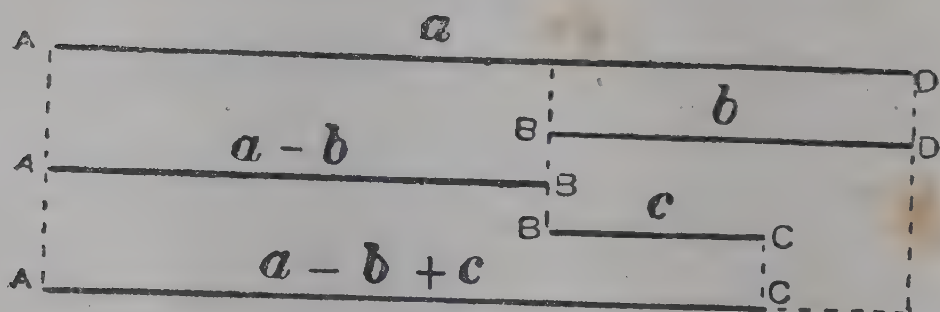


FIG. 25.

The explanation in these two cases is left as an exercise to the student.

We have thus shown that

$a + (b + c) = a + b + c$ (see page 28) ... (1)

$a - (b + c) = a - b - c$ ... ... (2)

$a + (b - c) = a + b - c$ ... ... (3)

$a - (b - c) = a - b + c$ ... ... (4)

In (1) and (3) when the brackets are removed, the signs of b and c on the right-hand side are the same as within brackets; but in (2) and (4), the signs of b and c when there are no brackets are different from what they have when within the brackets; whence we get the following two important rules for the removal of brackets.

(1) To remove the brackets when preceded by the + sign, omit the brackets and write the numbers within the brackets as they are without changing the sign.

(2) To remove the brackets when preceded by the—sign, omit the brackets and write the numbers within the brackets changing the sign of each.

Exercise—Oral.

Find the value of each of the following in 2 ways : (i) performing first the operation within brackets ; (ii) first removing the brackets and then performing the operation ; and compare your results :—

1. $8 + (7 + 5)$.

2. $9 + (7 - 6)$.

3. $16 - (8 + 4)$.

4. $16 - (8 - 4)$.

5. $7 + (6 - 2) - (9 - 5)$.

Exercise—Symbolic.

1. A has 5 marbles, B gives A 6 and C gets from A 3 marbles. Represent the operations in symbols.

2. A has p rupees with him. He gives 4 rupees to B. Represent in symbols what is left with him.

3. A man earns p rupees in a month and spends q (q is less than p) rupees. What does he save in a month ?

4. A purchases p mangoes from one bazaar, q mangoes from another bazaar and gives away r mangoes to his brother. How many has A ?

5. Two bags contain cocoanuts ; the first bag has a cocoanuts; the second has b more than the first. How many are there in the second bag ?

6. A's age is now a years. How old will he be 6 years hence ?

7. B's age is now b years. How old was he 10 years ago ?

8. A man draws his salary of a rupees on the first day of the month, spends b rupees on the second, and c rupees on the third out of the remainder. How many rupees has he left (1) on the second day, (2) on the third day ?

9. A man has Rs. 50 with him. He lends his friend Rs. 10 who returns the amount afterwards. Represent this in symbols and find how much he has with him. What do you learn from this example ?

10. In a library there are p books. Before the summer vacation, q books are lent out and after the re-opening the q books are returned. Express this in symbols. What do you infer from this example ?

* Negative Quantities.

§ **25.** We remarked in one of the preceding articles about the impossibility of subtracting 10 from 8. We also noticed that such an operation as that indicated by 8—10 was meaningless to us then. We shall see presently what meaning may be given to 8—10.

Suppose we want to find the sum of 3 and 1 graphically. We take a straight line AB 3 inches long and produce it to C so that BC = 1 inch. Then AC = AB + BC = 3 + 1 or 4 inches. On the other hand if we want to represent the subtraction of 1 from 3 graphically, in the figure instead of producing AB to C, measure BD = 1 inch in BA. Then AD = AB—BD = 3—1 or 2 inches. To add 1 to 3, *i.e.*, to find 3 + 1, we measure 1 inch to the right of B. To subtract 1 from 3, *i.e.*, to find 3—1, we measure 1 inch to the left of B, *i.e.*, in an opposite direction. + and — are thus opposed to each other and denote exactly opposite directions. If + denotes walking to the right (as we understood it to be at the beginning)—may be taken to signify walking to the left, *i.e.*, in an exactly opposite direction ; similarly if + 5 denotes the earnings of a person to be 5 rupees, — 5 would signify the expenses of the person to be 5 rupees ; if + 500 Rs. denotes that a person has a property worth Rs. 500,—500 Rs. signifies a debt of Rs. 500 ; if + 800 years denotes that an event took place after the birth of Christ, — 800 years signifies 800 years

before the birth of Christ; if + 50 denotes an ascent of 50 yards up a hill, — 50 denotes a descent of 50 yards down the hill. The quantities — 800, — 50, &c., in the above are called **negative** quantities and they have a meaning exactly opposite to that of the corresponding **positive** quantities + 800, + 50, &c.

Thus corresponding to the numbers 1, 2, 3,...... we have the numbers — 1, — 2, — 3, with a meaning quite opposite to that of 1, 2, 3, ... respectively.

Exercise—Oral.

1. If + 3 means walking 3 miles north, what is the meaning of — 3; — 6; — 7?

2. If + 2 means double class promotion of a boy, what is the meaning of — 2; — 1?

3. If + 50° denotes the longitude of a place to the east of Greenwich, what is the meaning of — 50°; — 60°?

4. If + 40° means the latitude of a place north of the equator, what is the meaning of — 40°?

5. If + 3 means 3 hours after noon, what is the meaning of — 3? What name is usually given to the former? And what to the latter?

6. Produce (1) BA to a point C and (2) AB to C. How is (1) different from (2)?

§ **26. Graphical Illustration.**—A person starts

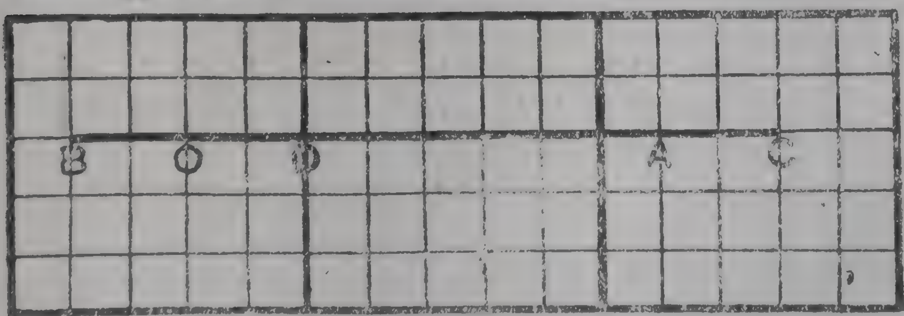


FIG 26.

from O and walks 8 divisions to the right reaching a point A as shown in the figure. After reaching A let him turn back and walk 10 divisions from A in the

direction AO. Finally he reaches a point B whose distance from O is 2 divisions to the left of O. Suppose the same person walks from O 10 divisions to the right reaching C and then 8 divisions to the left. He reaches the point D 2 divisions to the right of O, *i.e.*, (1) walking 8 divisions to the right and thence 10 divisions to the left takes him to the same point as walking 2 divisions to the left of O; (2) walking 10 divisions to the right and thence 8 divisions to the left takes him to the same point as walking 2 divisions to the right. If distances walked to the right are *positive*, the distances walked to the left may be regarded as *negative*. On this supposition (1) and (2) in the above will be symbolically represented as (1) $+8-10=-2$, and (2) $+10-8=+2$.

It is also clear that if a person walks 8 divisions to the right and thence 8 divisions to the left he will be at O, *i.e.*, the result is the same as if he remained at O without walking, *i.e.*, symbolically we have $+8-8=0$.

Thus if the positive numbers $+1, +2, +3$......be represented by lengths measured to the right of O (*vide* art. 13) then the negative numbers $-1, -2, -3$,......will be represented by lengths measured to the left of O, respectively equal to the lengths corresponding to $+1, +2$,......on the right of O, as shown in Fig. 27.

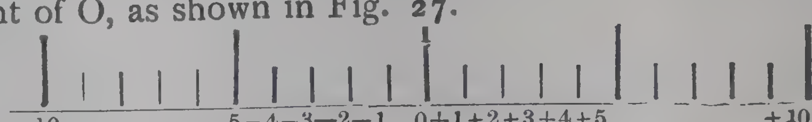


FIG. 27.

Note.—The sign — as used here has nothing whatever to do with subtraction. It is part of the name of the new numbers and indicates that they are less than 0; *i. e.*, they are reached by counting backwards from 0.

Exercise—Graphical and Symbolic.

Illustrate graphically :

(1) $-4+5=1$.

(2) $+7-5=2$.

(3) $-8-2=-10$.

(4) $10-10=0$.

1. A man is worth Rs. 300 and borrows Rs. 500 for a marriage. Represent his financial condition symbolically.

2. A man walks 30 miles to the north and 58 miles to the south. How far is he from the starting point and on which side of it? Represent his path graphically.

3. What is the range of temperature for the year, if the maximum temperature is 90° (Fahrenheit) and the minimum 8° below 0 (Fahrenheit)?

4. A labourer gets 5 annas for every day he works and forfeits 4 annas every day he is absent. How is his earning affected by a day's absence?

5. A gets 35 marks below the minimum and B 15 marks above the minimum. What is the difference between A's and B's marks?

6. Alexander invaded India in the year 327 B.C. The first battle of Paniput was fought in 1526 A.D. What time elapsed between the two events? (Note that there is no year called 0 B.C. or 0 A.D.)

7. Every day I reckon in Rs. the money earned and the money spent, writing + for earnings and — for expenses. The results for each week in a month are as follow:

1st week $+2+6-5-7+8-9$;
2nd week $+3-4-1+7-3-4$;
3rd week $+6-8+7+3-4-5$;
4th week $-8+9-3-4+8+6$.

Fill up the blanks in the table:

Week.	1	2	3	4	For the month.
Money saved.					

8. The road from A to B rises first 15 ft., then falls 20 ft., and then falls again 3 ft., and finally rises 9 ft. How high is B above A?

9. Express a gain of Rs. 5 followed by a loss of Rs. 3 symbolically : (1) as a gain, (2) as a loss.

10. Express a loss of Rs. a followed by a gain of Rs. b (1) as a gain, (2) as a loss.

Addition and Subtraction combined.

§ 27. When we have a series of additions and a subtraction involved in a problem the third method of subtraction explained in art. 22 will be very useful.

Example 1.—Add together the numbers 3287, 4108, 3596 and subtract the total from 21385. The working is shown in the margin and the mental process is as follows :—

	21385
14, 21 + **4** = 25 (set down 4)	
and carrying 2, 11, 19 + **9** = 28 (set down 9)	3287
and carrying 2, 7, 8, 10 + **3** = 13 (set nown 3)	4108 3596
and carrying 1, 4, 8, 11 + **0** = 11 (set down 0)	
and carrying **1** + **1** = 2 (set down 1)	10394

The figures in big type are set down as they are mentally thought of.

* The following is anothor method† of subtraction which is specially useful in a series of additions and subtractions combined.

Definitions.—(1) Numbers of 1 digit such as 2, 3......9 may be called numbers of the first grade ; numbers having 2 digits such as 43, 78 may be called numbers of the second grade and so on.

(2) By the complement of a number is meant the number that should be added to it in order to get the lowest number of the next higher grade ; thus 3 and 7 are complements for they together make up 10 ; 34 and 66 are complements for they together make up 100, the lowest number of the next higher grade. The complement of 523 is 477, etc.

† Adapted from an article in the *Educational Review* contributed by Mr. S. Chinnaswamy Aiyar.

From these examples it will be found that the complement is written down readily thus. In the units place, write down the figure which with the units figure of the given number will make up 10 and in each of the other places write down the figure which with the corresponding figure in the given number will make up 9, *e.g.*, the complement of 6927 is 3073.

Now it will be clear, that to subtract any number is the same as adding its complement and subtracting the lowest number of the next higher grade. For instance subtracting 523 is the same as adding 477 and subtracting 1000, *i.e.*, -523 is equivalent to $+\bar{1}477$ meaning thereby that 477 is to be added and 1 which stands for 1000 is to be subtracted. Thus

$$1428-523 = \begin{array}{r} 1428 \\ \bar{1}477 \\ \hline 0905 \end{array}$$

$$= 905$$

Again

$$25389-4683+982-346 = \begin{array}{r} 25389 \\ \bar{1}5317 \\ 982 \\ \bar{1}654 \\ \hline 21342 \end{array}$$

Here the total in the 3rd column is 23, 3 is set down and 2 is carried; and in finding the total of the 4th column $\bar{1}$ is considered as -1 and the total consequently is 11, etc.

Exercise III (e).

The following sums from 1 to 4 should be worked out in one operation as in example 1, art. (27) :—

1. From 18,423, subtract the sum of 3645, 8302 and 3450.
2. From 36,457, subtract the sum of 8034, 12031, 14024.
3. From the sum of 860425, 362684 and 920365, subtract 1,31,025.

4. From the sum of 1602638, 8603284 and 20,00099, deduct 9999999.

5. 3 railway carriages are linked together. The length of each carriage is 24 ft. and the distance between two consecutive carriages is 3 ft. Find the distance between the front of the first carriage and the rear of the last carriage.

6. In 1901 the number of persons in Madras who could speak only Telugu was 9,687,421, only Tamil 15,784,093, both Tamil and Telugu 275,420. How many people were able to speak (1) Telugu, (2) Tamil?

7. B 300 yds. north of A, C is 650 yds. south of B, and D 840 yds. north of C. Show that D is 490 yds. north of A. Represent this graphically taking 1 mm. to stand for 10 yds.

8. B is 600 yds. north of A, C is 384 yds. south of B, and D 1020 yds. north of C. How much is D north of A? Represent this graphically as in question 7.

9. The thermometer read 61° Fahr. at 10 A.M.; by 1 P.M. it went up 13°, between 1 P.M. and 2 P.M. it went up 5°, between 2 P.M. and 5 P.M. it went down 4°, and between 5 P.M. and 10 A.M. it went up 7°. What was the last reading?

10. The following table gives the number of scholars on the rolls on 31st March in Bengal for each of the 5 years commencing from 1903-04. Without copying out these figures write down the increase or decrease each year writing + for an increase and — for a decrease.

1903-04	...	...	...	...	1,745,296
1904-05	...	...	...	...	1,711,458
1905-06	...	...	...	...	1,152,237
1906-07	...	...	...	...	1,184,335
1907-08	...	...	...	...	1,220,973

Check your work by adding up the results; the total should be equal to the difference between the first and the last of the given figures.

11. In a certain year 12,00,000 rupees, 6,00,000 half rupees, 4,00,000 quarter rupees and 2,00,000 two-anna pieces were coined; but 18,000 rupees, 6,000 half rupees, 2,050 quarter rupees and 3,000 two-anna pieces were withdrawn from circulation. Find the increase in the number of silver coins in circulation during the year.

12. In the British Parliament the Conservatives just before a certain election were in a minority of 60, and just after the election they were in a majority of 122; how many votes did they win from their opponents at this election?

13. The total number of votes given for two candidates at an election was 18745 and the successful candidate had a majority of 8471; how many votes did each get?

14. In a cricket match the first man scored 15 runs, the second 25 runs, the third 0, the fourth 18 runs, the fifth 23 runs, the sixth 9, the seventh 60, the eighth 0, the ninth 29, the tenth 43 and ths last man 86. How many runs were scored by the team?

15. The following are the distances in miles of some stations on the Madras and Southern Mahratta Railway from the immediately preceding stations. Find the distances of the different stations from Madras and also the distance of the last station from Renigunta :—

Madras	0	(from Madras)
Renigunta	84	
Mamanduru	9	
Settigunta	10	
Kodur	6	
Nandalur	28	
Cuddapah	25	
Tadpatri	66	
Gooty	30	
Guntakal Junction	18	
Adoni	32	
Raichur	43	

16. The following are the distances in miles from Madras of some stations on the North-east line from Madras to Calcutta. Fill up the table by giving the distance of each station from the preceding station :—

		Distance from the preceding station.
Madras	0	
Ennore	11	
Ponneri	23	
Gudur Junction	86	
Nellore	110	
Bitragunta	131	
Ongole	182	
Bapatla	222	
Bezwada	268	
Ellore	305	
Rajahmundry	361	
Samalkot	392	
Cocanada	402	
Tuni	426	
Waltair	485	
Calcutta	1031	

17. The following table gives the expenditure on public instruction in a certain province for the year 1897-98; fill in the columns of totals and check your result by adding up the totals vertically and horizontally :—

Object of Expenditure.	From Provincial Revenue.	From Local Funds.	From Municipal Funds.	From Fees.	From Endowments.	Total.
	Rs.	Rs.	Rs.	Rs.	Rs.	
Colleges ...	84,318	977	400	7,852	1,040	
High Schools ...	44,709	...	...	2,619	174	
Middle Schools ...	47,862	7,131	...	19,156	...	
Elementary Schools	16,634	7,314	...	5,853	...	
Oriental Schools ...	15,650	1,45,684	...	2,448	...	
Special Schools ...	7,361	11,367	...	3	40	
Total ...						

*18. The following table gives the classification of some of the Brahmins in the Madras Presidency according to their occupation. Fill in the column of totals and check your results as in the last question :—

Sections.	Land-holders.	Public Service.	Priests.	Tenants.	Others.	Total.
Tamil ...	25,130	3,547	5,424	625	15,239	
Telugu ...	31.151	3,078	3,721	842	12,378	
Malayalam.	3,583	1,574	99	279	2,137	
Canarese ...	13,614	353	1,279	2,228	4.358	
Oriya ...	15,010	192	3,854	9,029	10,345	
Others ...	102,458	8,472	18,237	18,577	96,247	
Total ...						

*19. In the following three columns of numbers subtract from the top number the sum of the remaining numbers :—

(1)	(2)	(3)
16,835	25,389	37,496
8,437	9,762	12,349
3,925	7,943	8,453
1,764	2,421	9,675

*20. In the following additions, the last number is unknown write down that number and check your results by addition :—

(1)	(2)	(3)
4 329	2,479	3,875
3,476	8,342	14,493
873	1,010	9,746
		12,349
9,765	15,375	45,437

*21. From the following table find the number of scholars in Primary Schools in the years 1903-04, 1904-05, 1905-06 :—

Scholars in	1903-04.	1904-05.	1905-06.
Colleges ...	24,618	26.103	25,122
Secondary Schools ...	662,287	679,769	689,583
Special Schools ...	35,220	39,308	41,984
Primary Schools ...			
Total ...	4,235,281	4,375,335	4,565,939

***22.** Find, by the method of art. 27, the value of

(1) $833+749-(395+29-297)$.

(2) $3,985-(2,478+1,345-2,143)+807$.

(3) $8,935-(4,309+2,307-3,085)-1,090$.

***23.** The following table gives the amount of sales in rupees for each day of a week in the Triplicane Urban Co-operative Stores :—Find (1) the total sales for the week ; (2) the increase or decrease each day compared with the previous day, denoting increase by $+$ and decrease by $-$; (3) the total of the figures you get in working (2), adding the $+$'s and subtracting the $-$'s.

	Rs.
March 2nd	2,439
3rd	3,542
4th	2,942
5th	1,245
6th	963
7th	857

Note that the result in (3) equals the difference between the first and the last of the given figures.

***24.** The following table gives the population per sq. mile of the town of Negapatam for the years 1871, 1881, 1891, 1901. Find the increase or decrease during the three intervening decades and find the total increase or decrease in the 30 years from 1871 to 1901 (by two different methods) :—

1871... 9,775
1881...10,771
1891...11,844
1901.. 11,438

***25.** From the following table, find the total number of emigrants to Ceylon from each of the districts of Tanjore, Madura and Trichinopoly, and from all the three districts put together ; also find the totals of men, women and children for the three districts put together :

	Men	Women	Children
Tanjore	66,764	13,763	5,298
Madura	269,186	70,157	30,259
Trichinopoly	569,666	94,659	71,015

***26.** In the Province of Madras, the number of boys in Primary Schools increased by 31,179 between 1903-04 and 1904-05 ;

by 19,269 between 1904-05 and 1906-07, and by 39 865 between 1906-07 and 1907-08. If the total number of boys in 1907-08 was 659,716, what was the number in 1903-04?

***27.** In the following table showing internal emigration, fill in the 2nd column against each district and also the row of totals :—

Districts.	Immigrants.	Emigrants.	+ denotes excess of immigrants over emigrants, and −, excess of emigrants.
Ganjam	22,678		+4,821
Vizagapatam .	20,675		−146,084
Godavari ...	158,235		+129,940
Kistna ...	315,673		+ 81,207
Nellore ...	37,911		−67,108
Chingleput ...	82,319		−33,310
Total ...			

***28.** In the Triplicane Co-operative Stores the value of stock in hand in the beginning of January 1909 was Rs. 23,469. In January, fresh stock was purchased for Rs. 36,085 and stock valued at Rs. 35,944 was sold. In February the corresponding figures were Rs. 49,397 and Rs. 49,483. In March the figures were Rs. 46,487 and Rs. 46,782. In April the figures were Rs. 38,407 and Rs. 36,087. Find the value of stock at the end of each of these four months.

***29.** The deposit in a certain bank during the year 1906 amounted to Rs. 43,385 and the deposits withdrawn amounted to Rs. 40,897. At the end of the year the total deposit was Rs. 52,765. How much was there at the beginning?

***30.** In the Triplicane Co-operative Stores, the amount of loans issued during the year 1909 was Rs. 10,483, the loans repaid was Rs. 9,379. If the total of the outstanding loans at the end of the year was Rs. 8,705, what was the total of the outstanding loans at the beginning of the year?

CHAPTER IV.

ANGLES AND THEIR MEASUREMENT.

§ **28. Direction.** Suppose a person starts from A and walks in a straight line to a point B. (Fig. 28). His course will be represented by the straight line AB. But the mere straight line AB is not a sufficient representation because it cannot tell you whether the person started from A and walked to B or started from B and walked to A. Something more is necessary to indicate the **direction** of walking; this is done by placing an arrow mark similar to the arrow heads on sign-posts which you find placed where three roads meet for the guidance of travellers.

FIG. 28.

The Fig. (28) with the arrow head shows that the person starts from A and goes to B.

Here are drawn a number of straight lines from A to several points D, E, F, G, H...... each marked with an arrow head. A person starting from A may walk along any of these straight lines, *i.e.*, he may take any **direction.** Each of these straight lines indicates a different direction.

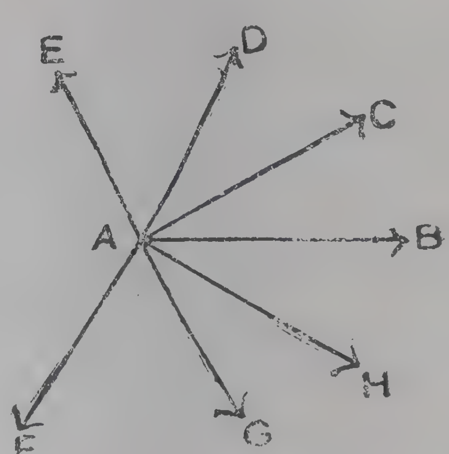


FIG. 29.

If a man walks along a straight road running northwards he will always find himself facing north; whereas if he is walking along a curved path, say $\overline{OBE}$, (Fig. 30) he will find that sometime afterwards as at B, he faces north-east and sometime after, east. Thus a straight line shows one constant direction, whereas a curved line does not.

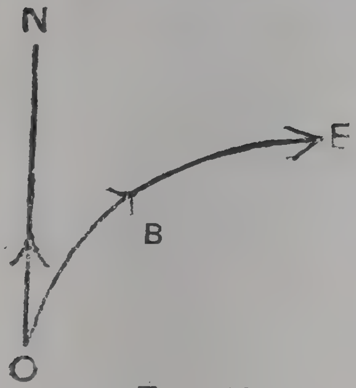


FIG. 30.

§ **29. Change of direction.** Stand at A (Fig. 31) facing the east and look straight before you. Draw a line AB on the ground indicating the direction in which you look. Partly turn round and look straight before you. Draw another line AC on the ground to show the direction in which you now look. You see therefore that you have to do some amount of turning before you can change the direction which you face, *viz.*, from AB to AC in this case.

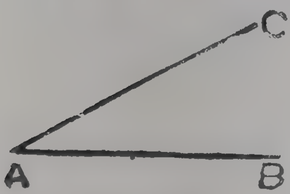
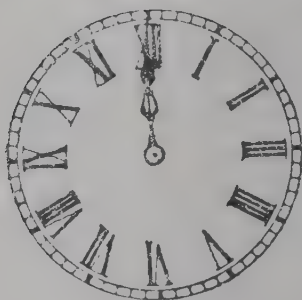


FIG. 31.

One change of direction may require a greater amount of turning than another. Suppose three persons start from X and walk along XY reaching Y; let the first

FIG. 32.

person then travel along YA, the second along YB, the



third along YC. Here the first and the third change their directions at Y; and the first, *i.e.*, the person going along YA has to turn more than the third, *i.e.*, the person going along YC. You know that you have to turn more in a right turn than in a half-right turn.

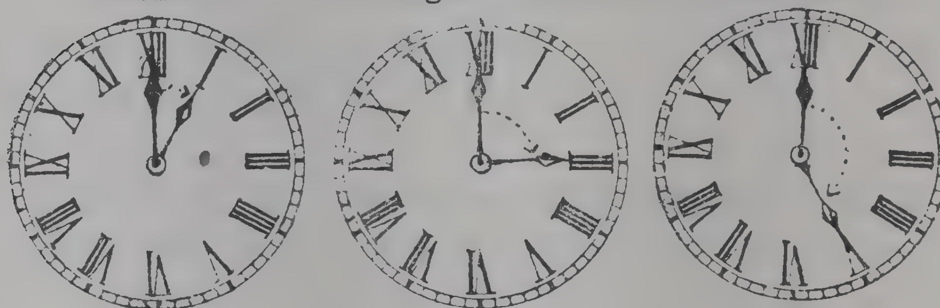


FIG. 33.

Again notice the motion of the hands of a clock. They are continually changing their direction. At 12 o'clock both the hands are together and both the hands show the same direction. Look at the position of the hands at 1 o'clock, 3 o'clock and at 5 o'clock (Fig. 33). To show 3 o'clock the hour hand should be turned from its position at 12 o'clock more than is necessary to show 1 o'clock and more still to show 5 o'clock.

§ **30. Angle.** When a line turns about one of its extremities, as in the cases described above, it traces out an **angle**; and the amount of turning necessary to change from one position to another, *e.g.*, from OA to OB (Fig. 34) gives the *size* of the angle. When two straight lines meet as in (Fig. 31) they *form an angle* and its size is given by the turning necessary to change the direction from AB to AC.

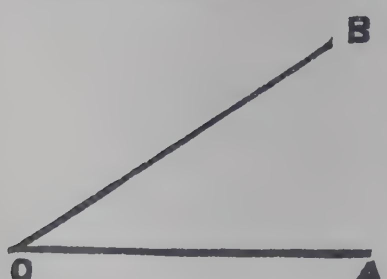


FIG. 34.

Take your pair of dividers. Widen the legs; you then form an angle; keeping one leg fixed, you may go on turning the other about the fixed leg; as it revolves, the angle gradually increases. Fig. 35 shows the angles formed between the legs corresponding to different positions; here the greater the amount of turning the greater is the angle between the legs.

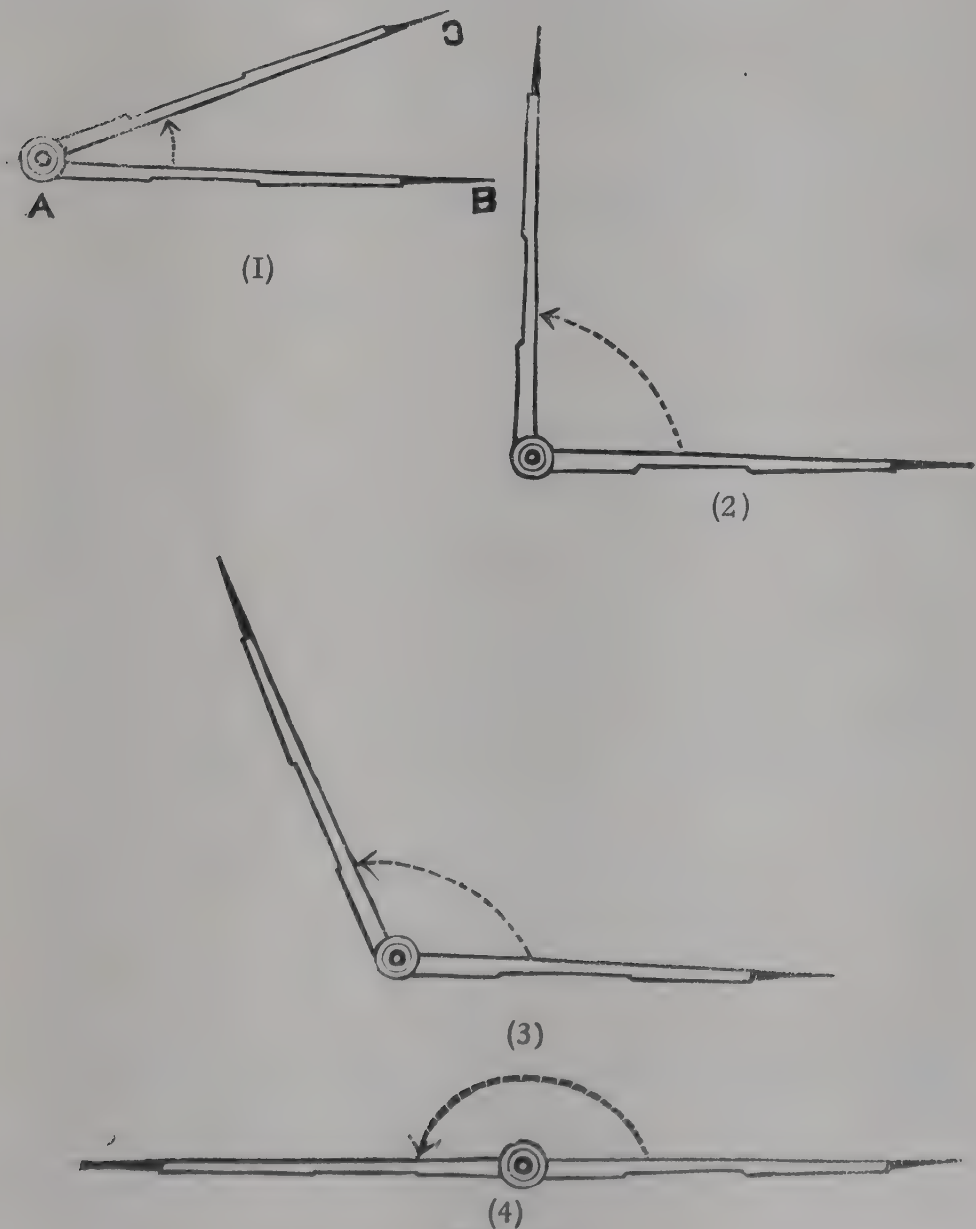


FIG. 35.

Exercise—Oral.

1. Compare the angles in positions (1) and (2).

2. Do. do. (2) and (3).

3. Do. do. (3) and (4).

4. In which position is the angle between the dividers largest, and in which smallest?

5. How is change of direction brought about?

6. Stand facing the north and then take a right turn, in what direction do you look now?

7. Take another right turn, in what direction do you look now?

8. How many right turns more should you take to make you face the north again?

9. Thus from the north, to again come to face the north, how many right turns have you to take in all?

10. Suppose you call the amount of turning in a right turn, a right angle; how many right angles have you n a complete revolution?

11. How many right angles have you in a right about turn?

We see then that where two straight lines meet we get an angle. If OA and OB are the two straight lines the angle is named AOB or BOA, the letter in the middle being always the letter at the point of intersection (meeting) of the two lines. The point of intersection of the two lines is called the **vertex** of the angle and the two straight lines forming the angle are called the **arms** of the angle.

If we produce a line, we change its length but not its direction. OA and OC in this figure have the same direction and OB and OD also have the same direction; therefore the amount of turning in changing from the direction OA to OB, *i. e.*, the angle AOB is the same as in changing

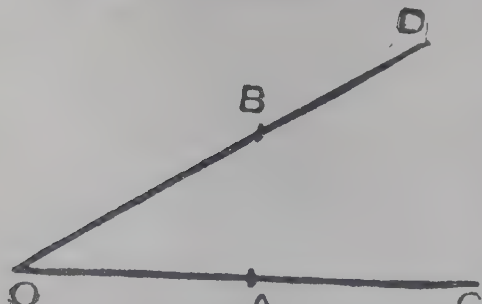


FIG. 36.

from OC to OD, *i.e.*, the angle COD. Thus you see that even though the arms are produced the angle remains the same. Compare also the angles between the hands of a watch and those of a clock at 1 o'clock. They are the same though the hands of a clock are longer than those of a watch. Thus **the size of an angle is independent of the lengths of the arms.**

∠ is the symbol for 'angle' and 'the angle ABC' is generally written as ∠ ABC.

§ **31. Comparison of angles.** Let us take two angles AOB and A′ O′ B′. To compare them, make a trace of ∠ AOB on the tracing paper and

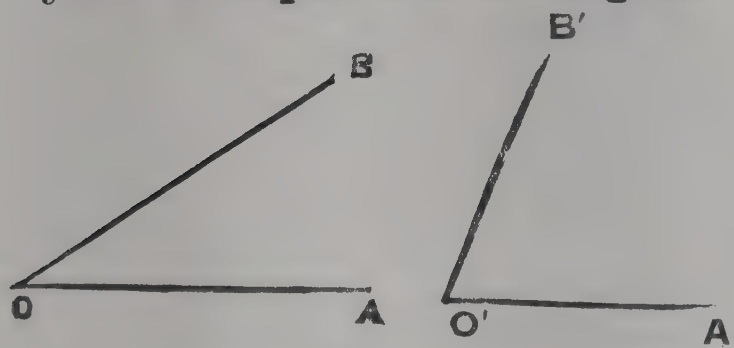


FIG. 37.

put the trace on ∠ A′O′B′ so that OA on the tracing paper falls on O′A′; then if OB on the tracing paper falls exactly on O′B′, the two angles are equal; if not, they are unequal. If OB falls between O′A′ and O′B′, then ∠ AOB is less than ∠ A′O′B′. If however OB falls outside ∠ A′O′B′ then ∠ AOB is greater than ∠ A′O′B′. This method of comparing angles is known as the method **of superposition.**

§ **32. Right-angle.** Face the north; then turn to the east. The angle through which you turn is called a **right-angle** (the motion, you know, is called a **right** turn in

'drill'). Note the position of the hands of a watch or clock

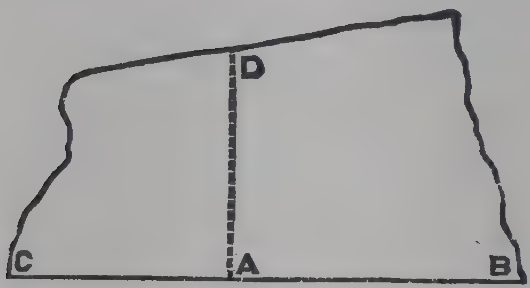


FIG. 38.

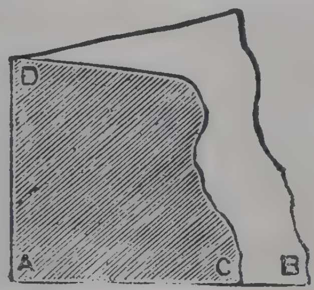


FIG. 39.

at 3 or 9 o'clock. The angle between the hands then is also a right angle. Take a bit of thick paper with a straight edge (Fig. 38). Fold the paper so that the straight edge is bent back on itself (Fig. 39). Open out the fold and mark the line of the crease with your pencil (Fig. 38). How many angles are formed? and what do you know about their size? Two are formed, *viz.*, BAD and DAC; they are equal. Why? Because on folding, the two arms of the one angle exactly fall on the two arms of the other angle, *i.e.*, the angles *coincide*. Each of these angles, like the angle formed by the hands of a clock at 3 or 9 o'clock, is a right-angle.

Fold a bit of thick paper and get a straight edge by pressing along the fold with your fingers, fold it again so that the edge is bent back on itself as in the previous exercise; unfold the paper completely and mark the two creases AB, CD (Fig. 40). Let them cut at O. Note that four angles are formed at O. They are all equal because on folding as before they all coincide; and each angle is a right-angle. Thus you have 4 right-angles formed; you know you have to make 4 right turns about yourself for a complete revolution or in

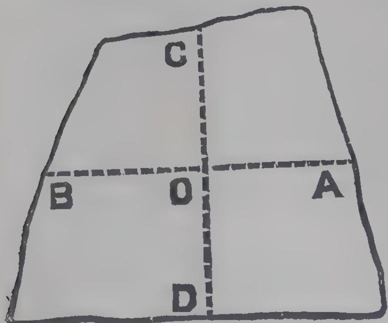


FIG. 40.

other words 4 right-angles make one complete revolution. The amount of turning round O from OA to OB, *i.e.*, ∠AOB (Fig. 40) contains two right-angles (*viz.* AOC and COB) and is called a **straight angle,** because its arms AO and OB form one straight line; and it is half a complete revolution as in right-about turn in 'drill.' Compare the angle in Fig. 35 (4). The angle between the hands of a watch or a clock at 6 o'clock is a straight angle.

Look at the square; (Fig. 41) you see it has a right-angle at A because that angle when put on BAD (Fig. 39) will be found to coincide with it. By using this square we can draw a right angle. First draw any straight line AB. Place the set square so that one of the edges of the right-angle of the set square may fall along AB; or put your flat rule along AB and on this rule place the set square (Fig. 41). Draw a line AC along the other edge of the set square. You get a right-angle, *viz.*, BAC. AC is said to be drawn **at right-angles** to AB and AC is called a **perpendicular** to AB; and AB is also said to be at right-angles or perpendicular to AC. The symbol ⊥ is generally used for 'perpendicular.'

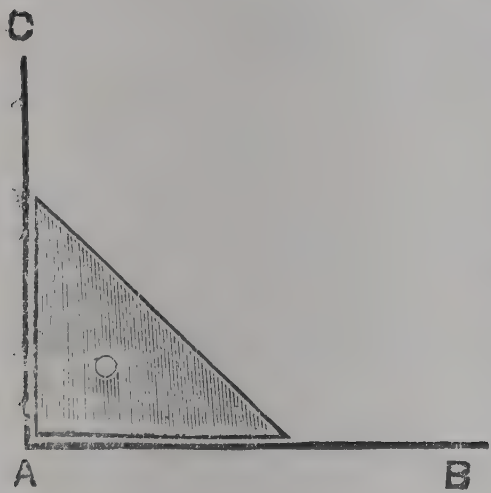


FIG. 41.

A neater method of drawing a line ⊥ to another line by means of the set square will be given later on.

Exercise—Practical.

1. Draw a straight line AB. Mark any point C in it and draw (using your set square) CD at right-angles to AB.

2. Draw a straigt line AB. Take points L, M, N, P, Q, R, S in the straight line AB. At these points draw perpendiculars LL′, MM′, NN′, PP′, QQ′, RR′, and SS′ making each of these lengths equal to 2 inches. Join (using a straight edge) the points L′, M′, N′, P′, Q′, R′, and S′. Note that these points all appear to lie on a straight line.

3. In the same figure draw perpendiculars on the other side of AB, and call the perpendiculars LL″, MM″, NN″, PP″, QQ″, RR″, and SS″; let each be made equal to 2 inches. Note that the lines LL′ and LL″ appear to be in one continuous line, similarly MM′ and MM″ and so on. Join the points L″, M″, N″, P″, Q″, R″, S″. What do you notice about these points?

4. Draw a straight line AB 4 inches long. Draw BC 3 inches long at right-angles to AB. Join AC. Measure AC.

5. Represent a section of the steps of a staircase one rising above another, the breadth of each step being 6 inches and the height 5 inches (take 1 mm. to represent 1 inch).

§ **33. Squares and rectangles**. Take a straight line AB = 1 inch. Fig. (42). At A draw AD at right angles to AB (using your set square) and make AD = 1 inch. At D draw DC at right angles to AD using your set square) making DC = 1 inch. Join BC. Then by applying the set square ∠s DCB and ABC will be found to be right angles; also if you measure BC you will find it to be 1 inch. Thus the figure ABCD is bounded by four equal straight lines (each being 1 inch long) called its **sides**, and has all its angles right-angles. It is thus **a four-sided figure having all its sides equal and all its angles right-angles**; such a figure is called a **square**.

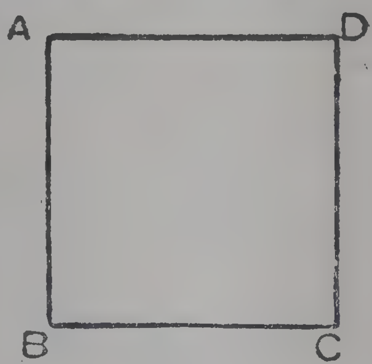


FIG. 42.

Take a half-sheet of paper.—Fold it so that the shorter edge falls completely along the longer and press along the fold with your fingers. Cut off with a penknife the portion that has no double. Unfold the paper and you will find it to be a square. Measure with your foot-rule the lengths of the sides. You will find them to be equal. Also by using your set square you will see that the angles are right angles. Fig. (43) is that of such a bit of paper folded along AC. The line of the fold, *i.e.*, AC is called a **diagonal** (that which joins *two* opposite *angles*) of the square ABCD.

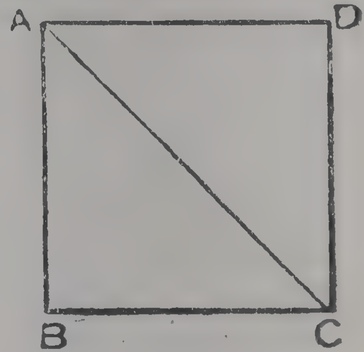


FIG. 43.

When the angles of a four-sided figure are all right angles, it is called a **rectangle**. The ordinary half-sheet of paper is a rectangle. The pages of this book are in the form of a rectangle. The opposite sides of a rectangle will be found (by measurement) to be equal ; and if all the sides are equal it is called a square.

Exercise—Graphical and Practical.

1. Construct a square with sides 2 in. long.

2. Do. do. 2 cm. ,,

3. Do. do. 3·1 in. ,,

4. Do. do. 6·2 cm. ,,

5. Construct a rectangle 3 in. by 2 in., *i.e.*, whose length is 3 in., and whose breadth is 2 inches.

6. Construct a rectangle 5 cm. long and 3 cm. broad.

7. Do. do. 5·3 cm. by 3·8 cm.

8. Take half a sheet of paper. Show by folding how to get a square whose side is 4 inches, and with your scissors cut off the square.

9. Take another half-sheet of paper. Show how by measuring and folding you can get a rectangle 4·5 in. by 3·6 in.; and with your scissors cut off the rectangle.

10. Construct a square whose sides are each 4 in. as in Fig. (44). Take points E, F, G and H in AB, BC, CD and AD respectively so that AE = BF = CG = DH = 2 in. Join with a straight edge EF, FG, GH and HE. Show by measuring the sides and angles that EFGH is a square.

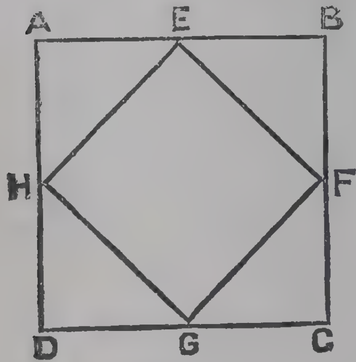


FIG. 44.

11. In the same figure, take E, F, G, and H in AB, BC, CD and DA so that AE = BF = CG = DH = 3 in.; and show that EFGH is a square.

12. Construct a rectangle ABCD having AB = 3 in. and BC = 2 in. Take points E, F, G and H in AB, BC, CD and DA so that AE = 1·5 in. BF = 1 in. CE = 1·5 in. and DH = 1 in. Is the figure EFGH a square? If not, why not?

13. Draw a plan of the floor of your class-room paying attention only to the form of the room (leaving the exact length and breadth out of consideration).

14. Draw a plan of the floor of your school-room showing the position of 2 opposite door-ways (in the breadth-wise walls).

15. Draw a plan of the floor of a school-room which is 20 ft. long and 16 ft. broad, taking 5 mm. to represent a foot.

16. The length of a room is 30 feet, the breadth 20 feet. Draw a plan of the room (taking 1 inch to represent 10 feet.)

17. Cut off from a piece of paper a rectangle 3 in. by 2 in. Mark off inches along both the length-wise edges and also along both the breadth-wise edges (using your foot rule). Fold the paper along the marks on the length-wise edges so that there may be 2 folds, corresponding to the 2 marks, one lying above the other. Then unfold the paper and fold it along the marks on the breadth-wise edges. Now unfold it and see if the figures formed by the creases are all squares. How many such squares have you?

18. Take a rectangle ABCD having AB = 4 in. BC = 2 in.

In AB, take E, F, G, so that AE = EF = FG = GB = 1 in., *i.e.*, mark off inches; similarly take K in AD so that AK = KD = 1 inch. With the help of your set square, draw lines at right-angles to AB at E, F and G and also to AD at K. Verify that the whole figure is now divided into squares. How many such squares have you in all?

§ **34. Circles.** Look at the two end faces of the cylinder (Fig. 3) in the second Chapter; they are **Circles.** The dial of a clock or watch is a circle. We shall now learn how to describe such figures. Using the foot-rule, on the paper, set your compasses so that the pencil point is 2 inches from the metal point and keeping the metal point fixed on the paper at O, turn the other arm round so as to draw a curved line with the pencil point.

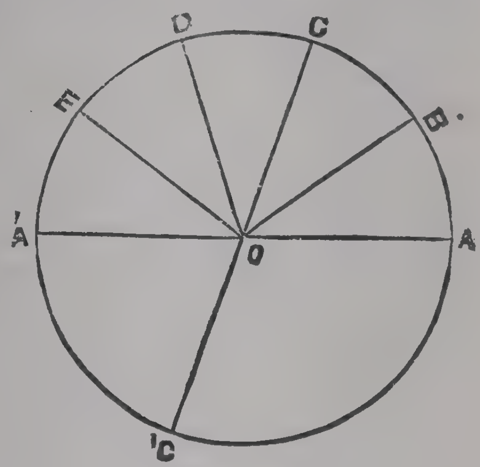


FIG. 45.

As the curved line is being traced, notice that the pencil point always keeps the same distance from O, *viz.*, 2 inches. Notice also that the pencil returns to the starting point so as to close the curve.

The curve traced out by this pencil point is a **circle** and the fixed point O is called its centre. Sometimes the word *circle* means the space enclosed by the curve and then the curve itself is called the **circumference** of the circle. The points B, C, D, E, &c., taken on the curve are said to be on the **circumference** of the circle. A line drawn from the centre to any point on the circumference is called a **radius**, so that OA, OB, OC, &c., are all radii. From the

way in which the curve was traced it is evident that **all radii are equal to one another.** Now produce AO to meet the circumference again at A′. The line AOA′ passing through the centre and terminated both ways by the circumference is called a **diameter** of the circle. Measure the diameter. You will find it to be 4 inches. AOA′ = AO + OA′ = a radius + a radius. ∴ a diameter is equal to twice the radius. Similarly produce CO to C′. Then CC′ is a diameter.

Draw a circle and draw any diameter AOA′. Fold the circle about AA′. Note that the two parts of the circle into which AA′ divides it do fall exactly one upon the other. This shows that the two parts are equal. Each part is half the circle and is called a **semi-circle**.

Exercise—Practical and Graphical.

1. Describe a circle of radius 2·3 in.

2. Describe two circles of radii 3·4 cm. and 4·5 cm.

3. Take a point P and with P as centre describe a circle of radius 3·4 inches.

4. Take two points P and Q 3 inches apart. With P and Q as centres describe circles having their radii 4 in. and 5 in. respectively. Note that the circles cut one another.

5. Take two points R and S 6 inches apart. Describe circles with centres R and S having their radii respectively equal to 4 and 2 inches. What do you notice about these circles ?

6. Take two points X and Y 8·3 cm. apart. With X and Y as centres describe circles whose radii are 3·2 cm. and 4·1 cm. respectively. What do you notice about these circles ?

7. Construct a square ABCD having each of its sides = 4·5 cm. Join AC and BD. Let them intersect at O. With O as centre

and OA as radius describe a circle. Notice that this circle passes through B, C, and D. What is its radius? Mention two diameters of this circle.

8. Construct a rectangle ABCD having its sides = 8·3 cm. and 6·5 cm. Join AC and BD. Let them intersect at O, With O as centre and OB as radius describe a circle. What do you notice about this circle? Measure its radius.

9. With O as centre and a radius = 3 cm., describe a circle; with the same centre and a radius = 4 cm., describe another circle. Draw two straight lines OAB, OCD meeting the first at A and C, and the second circle at B and D. Show that AB = CD by calculation, and verify by measurement.

10. Cut out a circular bit of paper and fold it about a diameter and fold it again so that one extremity of the diameter coincides with the other extremity. Unfold and see how many angles are formed at the centre; what name is given to each of these angles? How will you fold the paper so as to get eight equal angles, 16 equal angles and 32 equal angles at the centre?

§ 35. Measurement of angles. In order to compare angles when we cannot place them one upon another, we measure them in terms of some standard, as in the case of lengths of lines. The standard or the unit usually employed for measuring angles is the right angle; but to measure small angles, we want a smaller unit for which purpose, the right angle is divided into 90 equal parts, each part being called a *degree*.

A **protractor** is an instrument for measuring angles in which the divisions of a right angle into degrees are marked. It is generally either semi-circular or rectangular. The fig. (46) is that of a semi-circular protractor. There is also a rectangular protractor placed in it so as to show how the rectangular shape can be obtained from the semi-circular.

In measuring an angle, you should place the protractor

with the middle point of its base, *viz.*, O at the vertex of the angle and the base along one of the arms of the angle and

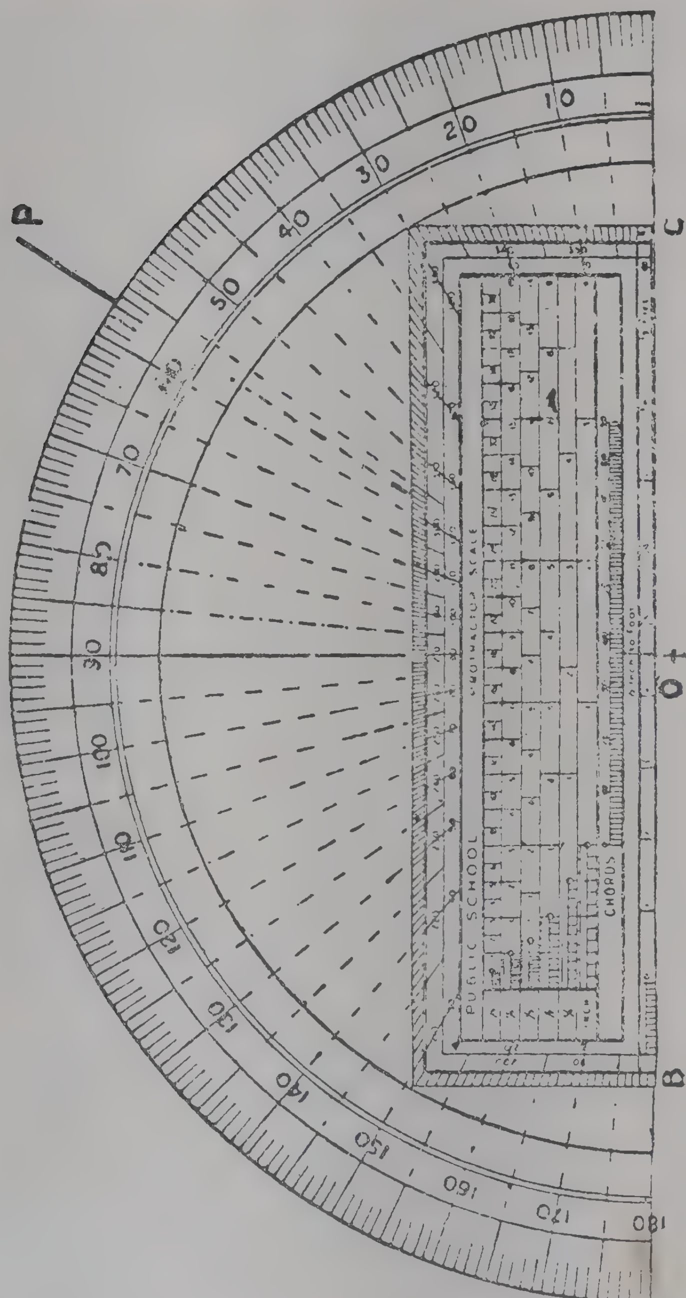


FIG. 46.

see through what reading the other arm passes. ∠COP in Fig. (46) = 57 degrees (generally written 57°).

Exercise IV (a).

1. Measure the angles ABD, DBE, EBC, ABC, DBC, ABE; GFH; KLM, NOP; QRT, RTS, TSU, SUV, ZYX and XYW.

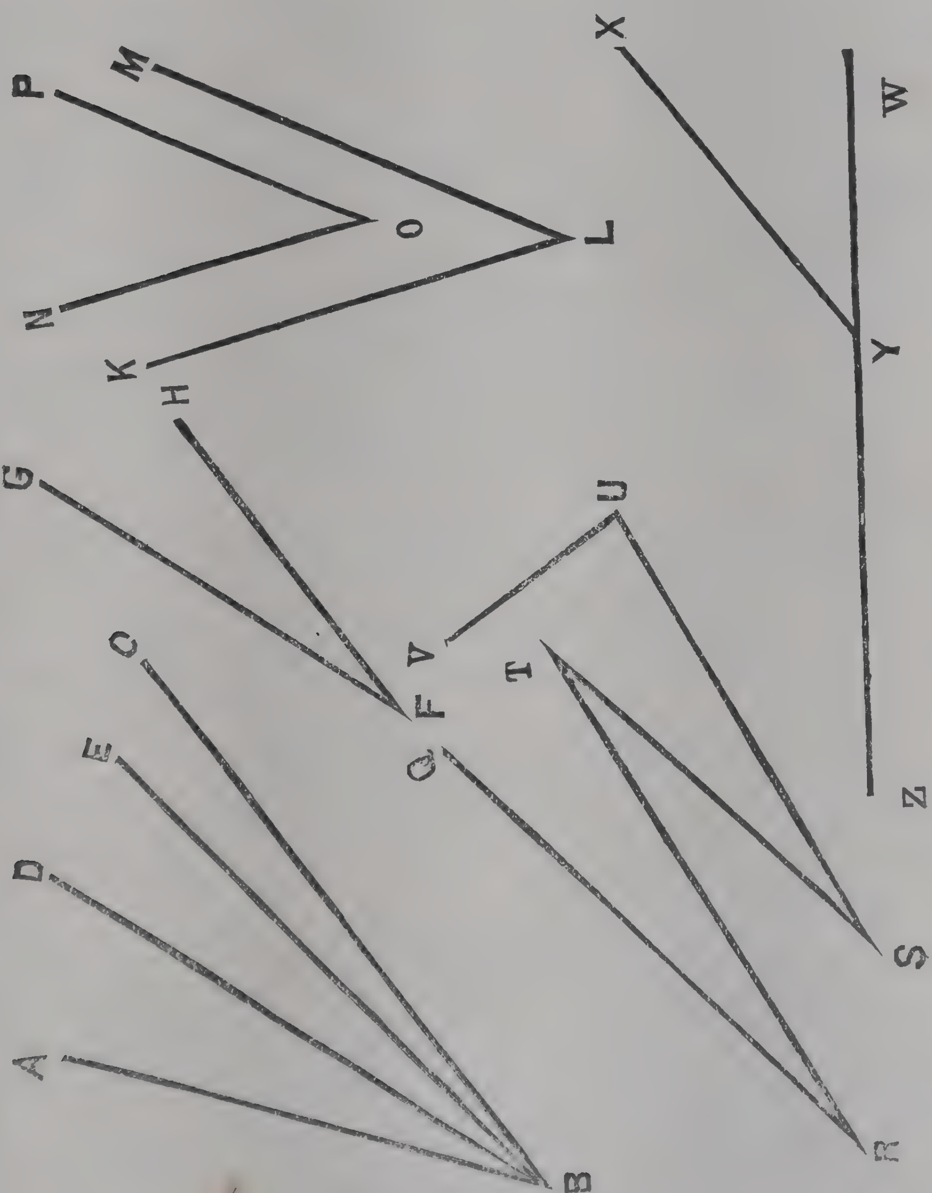


FIG. 47.

2. Draw a square ABCD whose sides are 3 inches. Join AC and BD cutting at O. Measure the angles OAB, OBA, OAD, ODA, ODC, OCD, OBC and OCB.

3. Repeat experiment (2) for any other square. What inference can you draw?

4. Draw a rectangle ABCD 6 cm. by 5 cm. Join AC and BD cutting at O. Measure the angles OAB, OBA; OAD, ODA; ODC, OCD; OBC and OCB. Repeat the experiment with another rectangle 4·3 cm. by 3·2 cm. What inference can you draw about these pairs of angles?

5. How many minute divisions apart are the two hands of a clock when they are at right angles? You know that a right-angle is divided into 90 degrees. How many minute divisions correspond to 90°?

6. Through how many degrees does the minute hand of a clock revolve in 25 minutes; 20 minutes; 35 minutes; 30 minutes?

7. Describe two circles with centres A and B and having their radii each = AB. Let C be one of the points of intersection of the two circles. Join AC, BC and AB. Measure the $\angle$s ABC, ACB, BAC.

8. Repeat the experiment with another pair of circles similarly described, varying the length of AB.

9. In Example 7, if D is another point of intersection of the circles, join AD, BD and measure the $\angle$s ABD, ADB and BAD.

10. Fig. (48) shows the points of the compass. What are the angles between (1) N and E, (2) between W and N-W, (3) between N and E-N-E, (4) between East and E by S, (5) S-E and S-S-W, (6) S-W and S by W?

§ **36. Kinds of angles.** When an angle is less than a right angle or 90° it is called an **acute** angle; when an angle is greater than a right angle it is called an **obtuse** angle. If you examine your protractor carefully you see that at each division in it there are 2 readings, the larger reading denotes an obtuse angle, the smaller reading an acute angle and you also see that in each case the two readings when added together give you 180°. When the sum of two angles is equal to 180° or 2 right angles, the angles

are said to be **supplementary**; or one angle is said

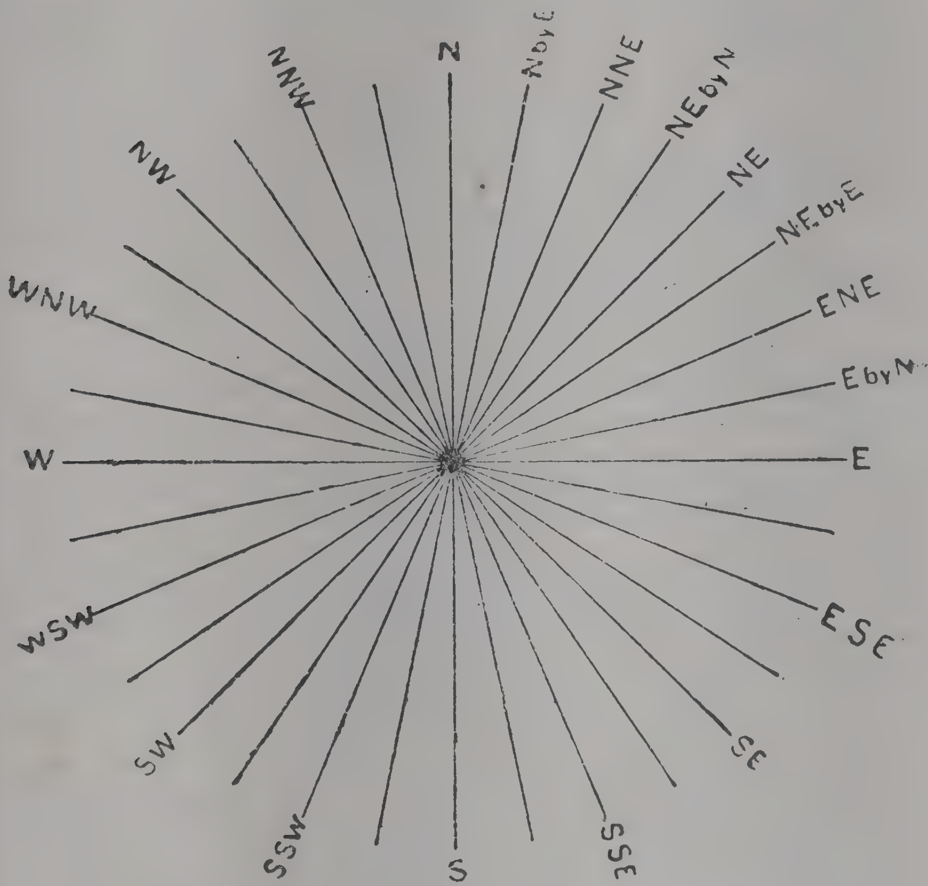


FIG 48.

to be the **supplement** of the other. *E.g.*, the angle 70 is called the supplement of the angle 110° and 110° is called the supplement of 70°, and 110° and 70° are supplementary.

Similarly when two angles are together equal to one right-angle or 90° the angles are said to be **complementary**; or each is said to be the **complement** of the other; *e.g.*, 40° and 50° are complementary.

In using a protractor care should be taken to take the proper reading. The following hints will be very useful. In measuring the angle BAC for instance, if the base of the

protractor lies along AB the set of readings to be taken is the one in which the numbers increase as the line AB turns from AB to AC.

Exercise—Oral.

1. Give the supplement of
$37^\circ, 43^\circ, 86^\circ, 92^\circ, 100^\circ, 120^\circ, 60^\circ, 11^\circ, 35^\circ, 43^\circ, 170^\circ, 179^\circ$.

2. Give the complement of
$23^\circ, 26^\circ, 32^\circ. 44^\circ, 46^\circ, 64^\circ, 80^\circ, 87^\circ, 36^\circ, 40^\circ, 60^\circ$.

3. By what construction will you find the supplement of the angle ABC ?

4. By what construction will you find the complement of the same angle ABC?

5. If an angle contains x degrees how many degrees are there (1) in its supplement: (2) in its complement ?

6. The supplement of an angle is greater than twice the angle by 6°. How many degrees does the angle contain?

Exercise IV (b)—Practical and Graphical.

Measure the angles and fill up the following tables :—

1.

$\angle ABD =$
$\angle DBC =$
$\therefore\ \angle ABC =$

Check your result by measuring $\angle ABC$.

2.

$\angle DBC =$
$\angle CBK =$
$\therefore\ \angle DBK =$

Check your result by measuring $\angle DBK$.

3.

$\angle ABD =$
$\angle DBC =$
$\angle CBK =$
$\therefore\ \angle ABK =$

Check your result by measuring $\angle ABK$.

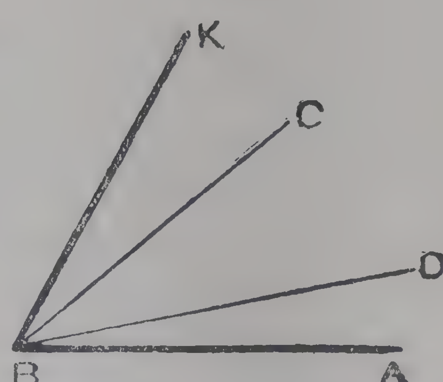


FIG. 49.

4.

$$\begin{array}{rl} & \angle OML = \\ & \angle OMN = \\ \hline \therefore & \angle NML = \end{array}$$

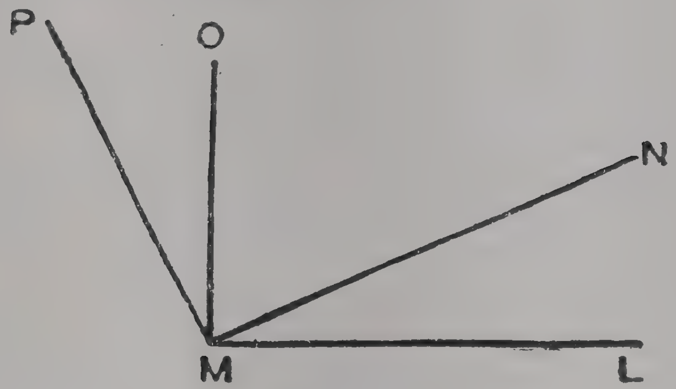


FIG. 50.

Check your result by measuring $\angle$NML.

5.

$$\begin{array}{rl} & \angle PMN = \\ & \angle PMO = \\ \hline \therefore & \angle OMN = \end{array}$$

Check your result by measuring $\angle$OMN.

6.

$$\begin{array}{rl} & \angle PML = \\ \hline & \angle OMN = \\ & \angle NML = \\ \hline \therefore & \angle PMO = \end{array}$$

Check by measuring $\angle$PMO.

§ 37. Construction of angles with the help of the protractor. To draw an angle equal to 23° at the point A in AB. There are two ways :--

(1) Put the middle point of the base of the protractor at A. Mark a point opposite to the graduation 23°. Join the mark to A.

(2) Arrange the protractor so that the middle point of the base of the protractor is at A and the 23° graduation lies on the line AB. Then rule a line along the base of the protractor.

The second is better than the first because we have two operations in the first, marking a point and joining, whereas in the second we draw a line at once.

Exercise IV (c).—Graphical and Practical.

1. Set out with the help of your protractor angles of 13^0, 23^0, 30^0, 35^0, 47^0, 60^0, 80^0, 95^0, 130^0, 145^0, 178^0.

2. Draw a straight line AB 5 cm. long. At the points A and B make angles BAC, and ABC = 50^0 and 60^0 respectively. Find by measurement the size of the angle ACB.

3. Draw a straight line PQ 6·8 cm. long and at the points P, Q, make the angles QPR and PQS each equal to 30^0. Let PR and QS meet at O. Find the magnitude of the angle at O.

4 A person starts from A and walks due north 10 miles. He then takes a north-eastern direction and walks 5 miles and reaches C. Represent his path graphically. Find by measurement the distance of C from A. Measure the angle which AC makes with the north and express the bearing of C from (*i.e.*, direction of C as seen from A). (Take 1 inch to represent 5 miles.)

5. A person starts from A and walks due north 10 miles. He then turns north-east and walks 5 miles. Again he turns due south and walking 10 miles reaches a place C. How far is he from the starting point? What angle does AC make witn the eastern direction?

6 A person starts from O and walks 12 miles due south. He then walks due east 16 miles. How far is he from the starting point? What is the bearing of his final position from the starting point? (Take 1 inch to represent 4 miles.)

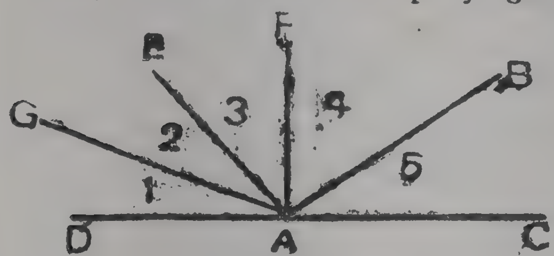
7. Two persons start from the same point O, one of them proceeds due north 10 miles and then walks east 8 miles. The other proceeds due south 10 miles and walks 8 miles to the west. What is the distance between the two persons at the end of the journey?

8. From a point A draw a set of straight lines as in Fig. 29. Guess the size of the angles so formed. Verify by measurement. Make a table thus:—

Angle.	Guessed.	Measured.
BAC	5^{0}_{0}	4^0
CAD	2	13^0
...	...	...

§ 38. Angles at a point.

Exp. 1. In the accompanying figure measure ∠BAC; also ∠BAD, find their sum.



FIG, 51.

Exp. 2. In the same figure measure ∠ CAE and also ∠ DAE and find their sum.

What do you infer from these experiments regarding the sum of the two angles formed when a straight line stands on another? They are together equal to two right angles. This must be so; because CAB + BAD means turning from the direction AC to AB and thence to AD; in effect this is the same as turning from AC to AD which we have seen is two right angles (*i.e.*, the angles CAB and BAD are supplementary). (Any two angles having a common vertex and a common arm and lying on opposite sides of the common arm are called **adjacent angles,** *e.g.*, EAD and EAC are adjacent angles, and also the pair BAD and BAC). Thus we have

1st *Proposition.*—**When one straight line stands on another straight line, the adjacent angles formed are together equal to two right-angles,**

Exp. 3. In the same figure measure the angles DAF, FAB, BAC. Find their sum.

Exp. 4. In the same figure measure the angles DAE, EAF, FAC. Find their sum.

Exp. 5. In the same figure find the sum of the four angle DAE, EAF, FAB, BAC.

These experiments shew that:—

2nd *Proposition.*—**When any number of straight lines meet a straight line on one side of it at a point, all the angles so formed are together equal to two right-angles.**

Exp. 6. In the accompanying figure formed by two intersecting straight lines measure and compare the angles 1 and 3; and 2 and 4.

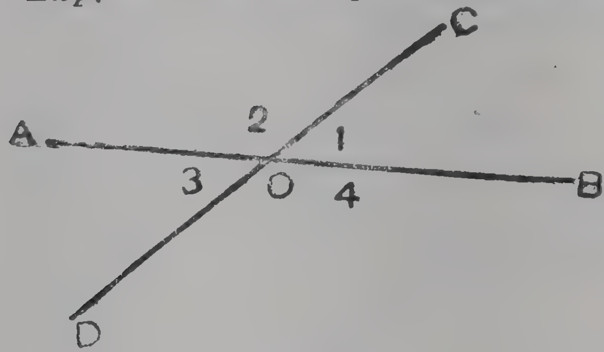


FIG. 52.

Exp. 7. Repeat the experiment with another pair of intersecting lines.

What do you infer from Exp. 6 and 7 ?

These pairs of angles are equal.

The opposite angles such as 1 & 3 and 2 & 4 of (Fig. 52) made by two intersecting straight lines are called **vertically opposite angles** (*vertically opposite* because they have the same vertex and are oppositely situated) ; and we have

3rd *Proposition.*—**If two straight lines cut each other the vertically opposite angles are equal.**

Exp. 8. As in the accompanying figure draw five straight lines meeting at a point O. Measure all the angles so formed, *viz.*, 1, 2, 3, 4 and 5 and find their sum.

Exp. 9. Repeat the exercise taking six straight lines meeting at a point.

What do you infer from these two experiments ?

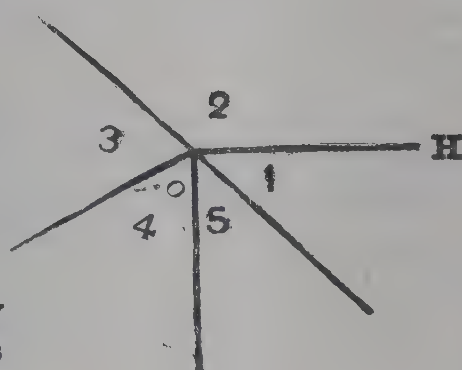


FIG. 53.

4th. *Proposition.*—**If any number of straight lines meet at a point all the angles so formed are together equal to four right-angles.**

Exercise.—Oral.

1. In (Fig. 51) suppose that a straight line starts from the position AC and revolves round A until it occupies the position AD. What portion of a revolution does it make ? What can you infer

from this about the sum of all the angles at the point A in the figure ?

2. Fig. (52) represents two rods AOB, COD freely jointed at O, so that they can turn about O. Now suppose that COD starts from the position AOB and turns about O and comes to the position COD, then compare the amount of turning of CO from the position AO with the amount of turning of DO from BO. What inference can you make about the vertically opposite angles 2 and 4 ? and likewise the angles 1 and 3 from a similar consideration ?

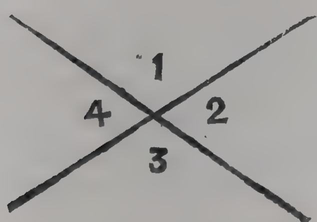


FIG. 54.

3. (*a*) If $\angle 1$ (Fig. 54) = 113°, what are the remaining angles ?

(*b*) If $\angle 1$ were 80°, what would be the remaining angles in the figure ?

4. How many degrees does $\angle 1 + \angle 2$ contain ? Similarly $\angle 1 + \angle 4$? Hence what is the relation between $\angle 1 + \angle 2$ and $\angle 1 + \angle 4$? Removing $\angle 1$ what do you infer about $\angle 2$ and $\angle 4$?

5. In (Fig. 53) suppose a straight line starts from the position OH and revolves about O coming again to the position OH. How many half revolutions does it make ? Hence what can you infer about the sum of the angles at O ?

6. In Fig. (53), if $\angle 1 = 50^\circ$, $\angle 2 = 120^\circ$, $\angle 3 = 48^\circ$, $\angle 4 = 60^\circ$, what is the magnitude of $\angle 5$?

7. If, in a similar figure, $\angle 1 = 80^\circ$, $\angle 3 = 90^\circ$, $\angle 4 = 78^\circ$, $\angle 5 = 89^\circ$, find the magnitude of $\angle 2$.

8. $\angle AOB = 30^\circ$. AO and BO are produced to C and D. Give the magnitude of each of the angles of the figure so formed.

9. At O in the line AO make $AOB = 30^\circ$ and AOC (on the other side of AO) $= 150^\circ$. What do you know about the lines OC and OB ?

10. In the figure of the Mariner's compass (p. 77) how many straight lines meet at the point O? How many angles are formed and what is the magnitude of each of the angles?

11. If the angles AOC and BOC (Fig. 52) contain x^0 and y^0 respectively, what is $x+y$ equal to? if x is 60^0 what is y?

12. If in (Fig. 54) $\angle 1$ contains D^0, how many degrees do the angles 2, 3, 4 respectively contain?

Exercise IV (d).

1. Without using your protractor draw angles, as nearly as you can judge, to contain 30^0, 45^0, 60^0, 75^0, 90^0, 120^0, 135^0, 160^0, 226^0. Measure the angles drawn and tabulate the results noting the errors.

2. Draw a straight line AB of length 5 cm. From A draw a line making an angle of 64^0 with AB. From B draw a line making an angle of 64^0 with BA meeting the first line in C. (Both lines are to be drawn on the same side of AB). Measure SAC and BC.

3. Repeat Exercise (2) but make the angles at A and B (i) 28^0; (ii) 82^0; (iii) 156^0. Do this in a single figure and note that the points of intersection of the pairs of lines thus drawn are all in a straight line.

4. Draw a straight line PQ of length 9 cm. From P draw two lines one on each side of PQ each making an angle of 48^0 with it. Repeat the process making angles of 76^0 and 130^0 on each side of PQ. (Do this in a single figure).

Fold your figure about PQ, and note how the lines on one side of PQ fall with regard to those on the other side.

5. A and B are two forts 4 miles apart and contain guns whose range is 3 miles. Draw a figure to show the space exposed to guns of the two forts.

6. A cow is tethered to a stake by a rope whose length is 4 yds. Show by means of a diagram the space left to the cow for free grazing. (Take 1 inch to represent 1 yd.)

7. Describe two equal circles of radius 2 inches, so that the circumference of each may pass through the centre of the other.

8. Describe three unequal circles so that the centre of each may be on the circumference of one of the others.

9. A man walks 3 miles, turns to his left through an angle of 60^0 (*i.e.*, he turns through an angle 60^0 from the direction in which he would have gone, if he had gone straight on). He then walks 3 miles, turns to his left through the same angle and again walks 3 miles. Find by a diagram how far is he from where he started ? (Take 1 inch to represent 1 mile).

10. A man walks 2 miles, turns through 45^0 to his left, goes half a mile, turns through 40^0 to his left and goes 1 mile further. How far is he from the starting point ?

11. The height of a wall is 24 feet ; a ladder is placed reaching to the top of the wall while the foot of the ladder is 18 feet from it, Draw a figure and find the length of the ladder.

12. A is a lighthouse, B and C are 2 ships 4·5 miles apart. B is due north of A, C due east of B and north-east of A. Find from a diagram the distance of both the ships from the lighthouse. (Take 1 cm. to represent a mile).

13. With centre O and a radius equal to 4 cm. describe a circle. Draw three radii OA, OB, OC so that the angle between each pair = 120^0. Join AB, BC, CA, measure AB, BC, CA and also each of the angles at A, B, C and $\angle$ OAB, $\angle$ OBA, $\angle$ OAC, $\angle$ OCA, $\angle$OCB, and $\angle$OBC,

14. With centre O and a radius = 5 cm. describe a circle. Draw 2 diameters AB and CD at right angles to each other. Join AC, CB, BD and DA. What kind of a figure is ACBD ? Give reasons for your answer.

15. In the figure of the previous example, draw a diameter making an angle of 45^0 with OD and cutting AD and BC in X and Y. Draw a diameter at right angles to the former meeting AC and BD in Z and W. What kind of a figure is WXZY ?

16. Describe a circle of any radius having O as centre. Draw 5 radii OA, OB, OC, OD and OE so that the angle between every two consecutive radii may be 72^0. Join AB, BC, CD, DE and EA, measure and compare these lines and the angles A, B, C, D and E.

17. Describe a circle of any radius having O as centre. Draw radii OA, OB, OC, OD, OE and OF so that the angle between

every two consecutive radii may be 60°. Join AB, BC, CD, DE, EF and FA. Show by measurement that these lines are equal and measure the angles A, B, C, D, E and F.

18. Repeat the process making (1) 8 angles at the centre each $= 45^\circ$. (2) 10 angles at the centre each equal to 36°. (3) 12 angles at the centre each equal to 30°. Measure the sides and angles of the figures so obtained.

19. The legs of a pair of compasses are 10 cm. long. Open them to an angle of 60°. Find by a diagram the distance between the compass points.

20. From a point O, draw lines OA, OB, OC, OD, and OE each equal to 8 cm. In OA, OB, OC, OD, OE take points A′, B′, C′, D′, and E′ so that $OA' = OB' = OC' = OD' = OE' = 5$ cm. Join the points successively. What can you say about the sides and angles of the figure so formed ?

CHAPTER V.

DECIMALS, ADDITION AND SUBTRACTION.

§ **39. Meaning of Decimals.**—In Chapter II, Art. (9) the measurement of the line EF which is 2 inches and 2-tenths of an inch long is written 2·2 in. (read '2 point 2 inches'). The main part of the measure is 2 inches and so, 2 is put down first and then comes the measure of the portion of length left over, which is a part or a **fraction** of an inch, having the same length as 2-tenths of an inch and so, 2 is put down next; and to avoid confusion with 22 or twenty-two, a dot or point is placed between the two 2's. Here the first 2 denotes two whole inches and the next 2 denotes a *part or fraction* of an inch, *viz.*, 2-tenths; and the point serves to *separate* the figure denoting whole inches from the figure denoting a fraction of an inch; just as in Rs. 2-4 the dash separates whole rupees from annas or fractions of a rupee. Similarly if a line be 3 inches and 7-tenths of an inch long, its length may be expressed as 3·7 in. Here also 3 denotes three whole inches and 7 denotes a fraction of an inch, *viz.*, 7-tenths of an inch.

Again if a line be 5 centimetres and 6 millimetres, (*i.e.*, 6-tenths of a centimetre) long, its length may be expressed as 5·6 cm., 5 denoting five whole centimetres and 6 denoting a part or fraction of a centimetre, *viz.*, 6-tenths of a centimetre.

Thus in each of the numbers, 2·2, 3·7, 5·6 the figure before the point represents units and the figure after the point denotes a fraction of such unit generally called a **decimal** fraction because it denotes so many **tenths** of the unit; and the dot which marks off the fractional part is known as the **decimal** point or simply the **point.**

Exercise—Oral.

1. The following are the lengths of a number of lines. Read the lengths as whole inches and tenths of an inch :—

   (*a*) 2·5 in.; (*b*) 3·7 in.; (*c*) 14·6 in.: (*d*) 22·2 in.

2. The following are the lengths of a number of lines; read their lengths as whole centimetres and tenths of a centimetre :—

   (*a*) 3·7 cm., (*b*) 4·9 cm.; (*c*) 13·5 cm.; (*d*) 3·33 cm.

3. In question 1 (*d*), state what each 2 stands for.

4. In question 2 (*d*), state what each 3 stands for.

§ 40. Place value in decimal notation.—In the adjoining figure ABCD is a square and it is cut up into small equal squares by means of cross lines. The whole figure contains 10 equal strips like the one shaded in it, so that, each strip is a tenth part of the whole figure. Also since the whole figure contains 100 small squares (as you can see by counting), and each strip 10 small squares, we see that 10 small squares put together is a tenth part of 100 small squares put together; or taking a small square as the unit the figure illustrates that 10 units is a tenth part of a hundred units or ten is a tenth part of a hundred. Also since each strip contains 10 small squares, a small square is a tenth part of a strip, (*i.e.*, ten small squares put together) *i.e.*, a unit is a tenth part of a ten.

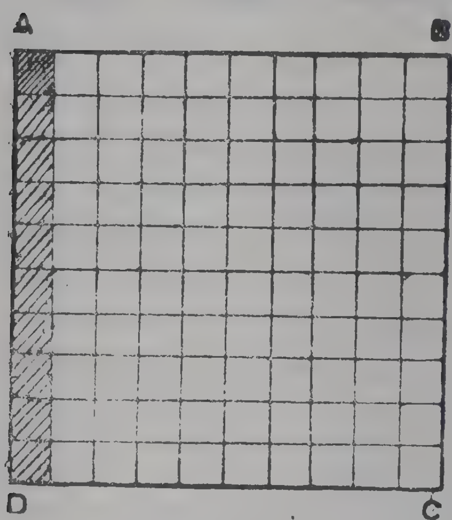


FIG. 55.

Again since the whole figure contains 100 small squares, each such square is a hundredth of the figure. But each square is a tenth of a strip which again is a tenth of the whole figure. ∴ We have *hundredth part is a tenth of a tenth part.*

We have seen that in the number 313, in the ordinary notation, the 3 to the extreme left stands for 3 hundreds, the next 3 for 3 tens (*i.e.*, tenths of a hundred) and the next 3 for 3 units (*i.e.*, tenths of a ten) *i.e.*, *in the local value of each 3 is a tenth of the local value of the 3 to its left.*

And now in the length 33·3 cm. of question 2 (*d*) above, we see that the 3 after the point denotes 3-tenths of a centimetre, *i.e.*, of the unit in this case, which shows that the law regarding the local values of figures is continued on to the figure after the point, *i.e.*, to the figure which denotes a **fraction.**

Exercise—Oral.

Express—

1. 25 mm. as centimetres and fractions, (*i.e.*, decimals) of a centimetre.
2. 37 as tens and decimals of a ten.
3. 457 as tens and decimals of a ten.
4. 250 or 25 tens, as hundreds and decimals of a hundred.
5. 3250 as hundreds and decimals of a hundred.

§ **41. Metric table of length.**—In the second chapter you have been told that 100 centimetres make 1 metre which is the standard unit of length in the French system. A metre is divided into ten equal parts, each part being called a decimetre, *i.e.*, a decimetre is a tenth part of a metre and contains ten centimetres. Thus a decimetre, a centimetre, and a millimetre are all parts or *fractions* of a metre and are used for measuring small lengths.

To measure great lengths or long distances we use the following lengths as standards, *viz.*, decametre,

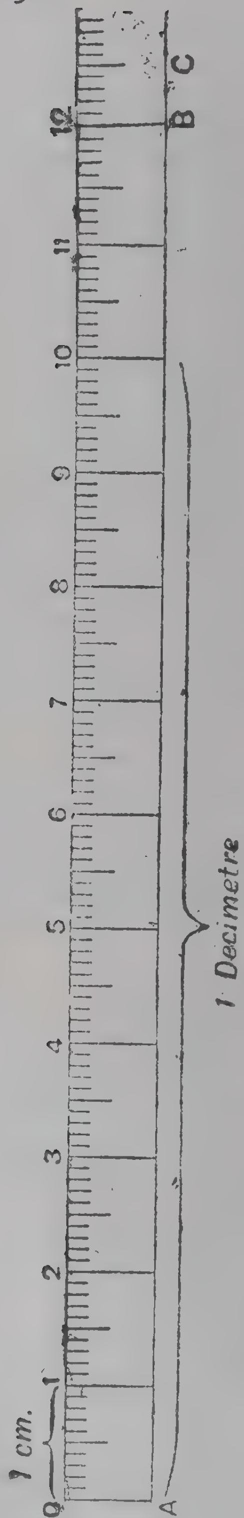


FIG. 56.

hectometre, kilometre, which respectively mean 10, 100 and 1000 metres. Thus we have the following table of linear measure in this system :—

10 millimetres (*mm.*)	=	1 centimetre (1 *cm.*)
10 centimetres	=	1 decimetre (1 *dm.*)
10 decimetres	=	1 **metre** (*m.*)
10 metres	=	1 decametre (*Dm.*)
10 decametres	=	1 hectometre (*Hm.*)
10 hectometres	=	1 kilometre (*Km.*)

§ 42. The second decimal place.—The line AB (Fig. 56) is 12 cm. long. Express this as decimetres and centimetres, *i.e.*, as decimetres and fractions of a decimetre. When thus expressed it is 1 dm. and 2 cm. or 1·2 dm. Now suppose the length of a line, (say AC, Fig. 56) is not exactly 12 cm. but 12 cm. and 5 mm. When expressed as a decimetre and fractions of a decimetre, it is 1·2 dm. and 5 mm. or 1·2 dm. and 5-hundredths (*i.e.* tenth of a tenth) of a dm. and may therefore be written 1·25 dm. assuming that each figure after the decimal point represents a tenth of the value which it would have if it occupied a place immediately to its left.

Thus in 73·85 metres 3 in the units place denotes 3 metres. Therefore 7 represents 70 metres, 8 represents 8-tenths of a metre, (*i.e.* 8 decimetres) and 5 represents 5-hundredths of a metre, *i.e.* 5 centimetres.

Example 1. Express 18926 as hundreds and decimals of a hundred. Here a hundred is to be the unit; therefore the decimal point must come after the figure in the hundreds place. ∴ 18926 = 189·26 hundreds.

Example 2. Express 326 cm. as metres and decimals of a metre. Here a metre is to be the unit; therefore the decimal point must come after the figure which denotes metres, *viz.*, 3, thus 326 cm. = 3·26 m.

Example 3. Express 7 Dm. 8 m. 9 dm. 5 cm. 6 mm. as decimetres and decimals of a decimetre. Here a decimetre is to be the unit; therefore the decimal point must come after the figure denoting decimetres. Thus the given length is 789·56 dm.

Exercise—Oral.

1. The following are the lengths of some lines; read them as metres, decimetres, &c. :—(1) 2·5 m.; (2) 2·67 m.; (3) 4·05 m.; (4) 6·98 m. (5) 7·50 m.

2. Express the following measurements as metres and decimals of a metre:—(1) 5 metres and 4 decimetres; (2) 4 metres 3 decimetres and 4 centimetres; (3) 3 metres and 5 centimetres; (4) 5 m. 6 dm. 9 cm.; (5) 6 m. 8 cm.

3. The following are the lengths of some lines. Read the lengths in inches and fractions of an inch:—(2) 2·3 in.; (2) 3·51 in.; (3) 4·09 in.; (4) 7·68 in.; (5) 8·75 in.

4. Express the following measurements as metres and decimals of a metre:—(*a*) 2 Dm. 3 m. 4 dm. 5 cm.; (*b*) 4 Dm. 3 dm. 7 cm.; (*c*) 3 Hm. 7 Dm. 4 m. 8 cm.; (*d*) 5 Hm. 3 m. 6 dm. 4 cm.

5. Express the following as directed :—

(1) 4 Dm. 5 m. 4 dm. 7 cm. as (*a*) metres and decimals of a metre; (*b*) dm. and decimals of a dm.

(2) 5 Hm. 3 Dm. 4 m. 7 cm. as (*a*) dm. and decimals of a dm.; (*b*) metres and decimals of a metre.

You have seen that 8·37 metres means 8 full metres 3-tenths of a metre and 7 hundredths of a metre. Similarly 2·45 inches means 2 full inches 4 tenths and 5 hundredths of an inch. It is not lengths alone that are thus expressed as decimals.

Other quantities as well may be expressed as decimals, *e.g.*, we may have 4·5 rupees; 8·9 lb; £3·75, respectively, meaning 4 full rupees and 5-tenths of a rupee; 8 lbs and 9-tenths of a lb; and £ 3 and 7-tenths and 5-hundredths of a £.

Again in 125·27 lakhs the units place denotes lakhs; 2 in the tens place therefore denotes tens of lakhs and 1 in the hundreds place hundreds of lakhs or crores; also 2 in the first decimal place, coming after the units place represents tenths of lakhs, or 2 ten thousands and 7 in the second decimal place represents 7-hundredths of lakhs or 7 thousands. Thus Rs. 125·27 lakhs means Rs. 1,25,27,000.

Exercise—Oral.

Express—

1. 4 m. 5 dm. 8 cm. as metres and parts of a metre.

2. 4875 as tens and decimals of ten.

3. 4875 as hundreds and decimals of hundred.

4. 5·17 metres as (*a*) decimetres, (*b*) centimetres.

5. 8 m. 4 cm. 5 mm. as (*a*) metres, (*b*) decimetres, (*c*) centimetres.

6. 23·78 m. as dm.

7. 44·12 m. as cm.

8. 6224 cm. as m.

9. Express the length indicated by the figures marked with an asterisk in the following :—

(*a*) $78\cdot\overset{*}{2}4$ cm. in cm. (*b*) $5\overset{*}{3}\cdot14$ m. in cm.

(*c*) $63\cdot9\overset{*}{8}$ m. in cm. (*d*) $18\cdot2\overset{*}{5}$ m. in mm.

(*e*) Express 1,84,000 as millions and decimals of a million.

§ **43. Decimal notation—General.** Suppose a line AB is 125 cm. Express this as metres and fractions of a metre. When thus expressed it is 1 metre 2 decimetres and 5 cm. or 1·25 m. Now suppose that the length of the line is 125 cm. and 8 mm. Express this as a fraction of a

metre. It is 1·25 m. and 8 mm. or 1·25 m. and 8-thousandths (or tenths of a hundred) of a metre and may therefore be written 1·258 m. assuming that each figure after the decimal point represents a tenth of the value, which it would have, if it occupied a place immediately to its left. We have seen that the first decimal place denotes tenths of a unit, the second hundredths of a unit, the third thousandths of a unit. This decimal notation may be extended so as to denote fractions smaller than thousandths by taking the fourth decimal place to signify ten-thousandths of a unit, the fifth hundred-thousandths of a unit and so on. Thus we have the following tabular form of the decimal notation extended to fractions :—

Thousands.	Hundreds.	Tens.	Units.	Tenths.	Hundredths.	Thousandths.	Ten-thousandths.
			2	3	4		
		7	4	5	2	6	
	8	9	6	0	4	3	
2	7	[illegible]	3	6	0	7	8

Exercise—Oral.

1. The following are the lengths of some lines ; read them as metres and parts of a metre :—

(1) 3·625 m ; (2) 6·789 m. ; (3) 8·032 m. ; (4) 8·009 m. ; (5) 9·003 m.

2. Express the following lengths in decimal notation taking a metre as the unit :—

(*a*) 3 dm. 9 cm. 8 mm. (*b*) 1 m. 2 dm. 9 mm,

(*c*) 5 m. 8 cm. 9 mm. (*d*) 10 dm. 9 cm. 7 mm.

3. What is the difference between—

(*a*) ·1 and ·01 of an inch? (*b*) 3·2 in. and 3·02 in.?

(*c*) 4·05 in. and 4·50 in.? (*d*) 0·2 in., ·02 in. and ·20 in.?

4. What is the value of 8 in (*a*) 6·4518, (*b*) 1·3848?

5. Give the local values of the digits marked with an asterisk in

(*a*) $3\cdot824\overset{*}{5}$; (*b*) $4\overset{*}{0}\cdot8034$; (*c*) $2\overset{*}{7}\cdot03824$.

6. Read the numbers given in the tabular form given above and express them in decimal notation without the columns.

ADDITION OF DECIMALS.

§ 44. Decimals with one decimal place.

Ex. 1. The straight line AB is 5·4 cm. long and CD 3·2 cm. long. Find the sum of the lengths of the two straight lines.

AB = 5·4 cm. or 5 cm. 4 mm. = 54 mm.

CD = 3·2 cm. or 3 cm. 2 mm. = 32 mm.

∴ AB + CD = 86 mm. (by addition) = 8 cm. and 6 mm. or 8·6 cm.

Thus 5·4 cm. + 3·2 cm. = 8·6 cm.

Similarly 5·4 in. + 3·2 in. = 8·6 in.

The work may be arranged thus—

inches.	tenths of an inch.	
5	4	= 5·4 in.
3	2	= 3·2 in.
8	6	= 8·6 in.

Ex. 2. To find the sum of two decimals 3·8 and 4·7 graphically. Draw AB 3·8 cm. long. Produce AB to C making BC 4·7 cm., (being the length of the other line) read off the length of AC, *viz.*, 8·5 cm., *i.e.*, AC or AB + BC = 8·5 cm., *i.e.*, 3·8+4·7 = 8·5 as it should be by addition,

Exercise—Graphical.

1. Find (and check your results graphically) the sum of (1) 2·4 in. and 1·6 in. (2) 3·6 in. and 2·3 in. (3) 8·4 cm.

and 2·6 cm. (4) 8·2 cm. + 6·4 cm. + 1·2 cm. (5) 3·8 cm. + 2·6 cm. + 3·9 cm.

2. Find the sum of 3·5 cm., 4·3 cm., and 6·8 cm. by reducing them to mm. and then express the result as a decimal. Check your result by graphical representation.

§ 45. Addition: General Case.

Add together—

Metres.	dm.	cm.	mm.
8	4	7	8
10	5	9	6
11	2	3	4
21	0	8	3
51	3	9	1

The total of the mm. column is 21 mm., *i.e.* 20 mm. (= 2 cm.) and 1 mm. Set down 1 in the mm. column of the sum and carry 2 to the cm. column. The cm. column, with this carried 2 gives 29, *i.e.* 20 cm. + 9 cm. or 2 dm. + 9 cm. So, set down 2 and carry 2, to the dm. column.

The dm. column gives 13. Set down 3 and carry 1 to the metre column. The metre column gives 51.

The sum is 51 m. 3 dm. 9 cm. 1 m. Thus it will be seen that this addition is exactly similar in every respect to addition of ordinary numbers, *i.e.* as if you had thousands, hundreds, tens and units, instead of m., dm., cm., mm., respectively. Now every one of the addends may also be expressed as so many metres and decimals of a metre and the addition may be regarded as addition of decimals and the working may be arranged as in the margin.

Metres
8·478
10·596
11·234
21·083
51·391

Care must be taken that the decimal points are all under one another so that units may be under units

tens under tens, tenths under tenths and so on (for which reasons have already been given in the Chapter on Addition.) Thus arranged it will be seen that addition of decimals may be performed as if there are no points and in the answer the point may be put exactly under the points in the addends.

Example. Add together 146·83, 683·29, 936·7, 1403·8378.

The work is shown in the margin. In this example there are no thousandths or ten thousandths in the first two numbers and not even hundredths in the third number. In all these cases you may suppose that zero is understood in those places as there are no figures there; but whether you write zeros or not the sum is not affected. Proceeding as before we get 3170·6578 as the sum.

146·83
618·29
936·7
1403·8378
3170·6578

Exercise V (a).

Add together questions (1) to (5). Also set down the lengths to be added as decimals of a metre and add up again, and check the results.

1.

m.	dm.	cm.	mm.
3	4	5	6
8	0	0	2
3	9	3	2
14	8	7	2

2.

m.	dm.	cm.	mm.
15	2	8	9
14	3	0	6
283	4	2	3
121	3	6	8

3. 100 m. 2 cm. 6 mm.; 24 m. 8 dm. 6 mm.; 38 m. 4 mm.

4. 200 m. 3 dm. 4 cm. 8 mm.; 135 m. 4 cm. 2mm.; 36 m. 4 dm. 8 cm. 9 mm.; 40 m. 8 cm. 9 mm.

5. 49 m. 5 mm.; 6 dm. 7 mm.; 8 m. 7 dm.; 16 m. 9 dm. 8 cm.; 2 mm.; 15 m. 9 cm. 6 mm.

Find the value of

6. ·683 + ·0932 + 15·654 + 21·0098.

7. 6·33 + 7·18 + 0·87 + 0·28 + 7·814.

8. 79·3 + 92·6 + 92·07 + 0·92 + 167·6 + 2·31.

9. 236·2 + 2·89 + 108·341.

10. 632·4365 + 38·0932 + 1084·365 + 769·835 + 534·23 + ·00098.

SUBTRACTION OF DECIMALS.

§ 46. Decimals with one decimal place.

A straight line measures 5·4 cm. and another measures 3·8 cm. Find their difference.

5·4 cm. = 54 mm.
3·8 cm. = 38 mm.

∴ difference = 16 mm. = 1·6 cm.

The working may be arranged in decimal notation as in the margin.

$$\begin{array}{r} 5{\cdot}4 \\ 3{\cdot}8 \\ \hline 1{\cdot}6 \end{array}$$

Explanation.—8 tenths cannot be taken from 4 tenths and so out of 5 units, one unit is broken into 10 tenths, and we thus get 14 tenths from which taking away 8 tenths we have 6 tenths; so, 6 is set down in the tenths place and subtracting 3 units from 4 units left we have 1 unit. Thus the difference is 1·6.

This shows that subtraction is the same as in ordinary numbers and the decimal point in the difference comes directly below the point in the given numbers.

The difference can be graphically obtained thus:

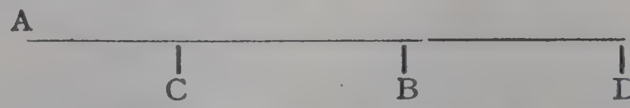
Take a line AB 5·4 cm. long. In AB take a point C so that AC = 3·8 cm.

Then AB − AC = BC. Measure BC and you will find it to be 1·6 cm.

Exercise—Graphical.

1. Find graphically the difference between 6·3 cm. and 4·9 cm. Check your result by expressing them in millimetres and then finding their difference.

2. Find graphically the difference between 9·8 and 8·9.

3. 

(*a*) Measure AB and AC with the centimetre scale and find their difference.

(*b*) Find the difference between AC and DB.

(*c*) Find AC + CD — BD.

(*d*) Find CD + DB — CB.

4. A train starts from A to go to B a distance of 57 miles. After travelling 39 miles she reaches a place C. Find the distance CB graphically (take 1 division to represent 10 miles).

5. Find the difference between the length and breadth of your slate.

§ 47. Subtraction: General Case.

Example.—The length of a string is 2 m. 0 dm. 9 cm. 8mm.; it is cut into two pieces one of which is 1 m. 3 dm. 4 cm. and 9 mm. Find the length of the other.

(1)				(2)		(3)
m.	dm.	cm.	mm.	mm.	*i.e.*	m.
2	0	9	8	2098		2·098
1	3	4	9	1349		1·349
	7	4	9	749		·749

(1) shows the subtraction as in ordinary compound quantities; (2) gives the subtraction expressing the lengths in mm.; (3) gives the subtraction expressing them in metres and decimals of a metre.

In subtraction the numbers are arranged so that the decimal points come below one another for the same reasons as in addition of integers.

Ex. Subtract 857·6958 from 9086·3.

Over 9, 5, 8 there are no figures; supply zeroes and proceed as in the subtraction of ordinary numbers. The work is shown in the margin.

$$\begin{array}{r} 9086{\cdot}3000 \\ 857{\cdot}6958 \\ \hline 8228{\cdot}6042 \end{array}$$

Exercise V (b).

Do the following by working in decimals of a metre :—

1. From 14 m. 3 dm. 3 cm. 4 mm. take 4 m. 8 dm. 9 cm. 3 mm
2. From 8 m. 3 dm. and 4 mm. take 7 m. 5 dm. 9 cm and 2 mm.
3. From 849 m. 7 cm. 5 mm. subtract 350 m. 3 dm. 4 cm. 8 mm.
4. Find 635 m. 3 dm. 4 cm. 2 mm. take 384 m. 9 cm. 4 mm.

Find the value of—

5. 7·685 — 3·849.
6. 612·8 — 83·365.
7. 7·36 + 8·649 + 9·345 — 10·859.
8. 10·687 — 3·98 + 8·476 — ·938.
9. 162·8437 + ·39847 — ·3849 + 6·3847.
10. 162·3678 + 398 — 46347 — 638·4378.

§ **48.** * **Approximations.** The measurement of straight lines is only approximate. Sometimes it appears to be an exact number of inches, and sometimes an exact number of inches and tenths of an inch. But it often happens that a straight line measures something (a fraction of a tenth) more than the exact number of tenths, *e.g.*, 2·5 inches + a fraction of a tenth. In this case the correct length can be obtained only if we know the exact value of the fraction. Otherwise the length can be given as 2·5

inches or 2·6 inches according as the extra fractional part is or is not less than half a tenth.

To obtain the more accurate measure, divide the tenth part of an inch after the reading 2·5 into 10 equal parts and see how many of the new minor divisions (each $\frac{1}{100}$ of an inch) are contained in the extra length above spoken of, If, for instance, it contains 4 of these new sub-divisions, then the length measured is 2·54 inches. If it does not contain an exact number of these, the result may be expressed as 2·54 or 2·55 inches, in accordance with the principle followed already in the case of tenths. If we are not satisfied with even hundredths and want to measure the portion beyond this new sub-division 4, we use a thousandth of an inch and so on.

When we express a result to one decimal figure (got by taking the nearer of the two limits between which the result lies) the result is said to be *correct* to 1 decimal place or *correct* to ·1 or *correct* to the nearest tenth; similarly when we express the result to two decimal figures (the second decimal figure being the nearer of two limits between which the result lies), the result is said to be correct to 2 decimal places or correct to ·01 or correct to the nearest hundredth. You may similarly understand what is meant by finding a a result correct to 3, 4......decimal places.

We have these approximations not merely in the measurement of a line but also in all kinds of measurements. Thus suppose the weight of an iron truck is found to be 14·8763 tons, one who does not care to know the weight so very accurately would find it to be between 14·8 and 14·9 tons and finding it to be nearer 14·9 would give the weight as 14·9. Another who cares for a more accurate

result would give it as 14·88 (because it lies between 14·87 and 14·88 and is nearer 14·88 than 14·87). Thus the weight expressed *correct* to a tenth of a ton is 14·9 and correct to a hundredth it is 14·88 and to a thousandth it is 14·876.

Exercise—Oral.

Express the following :—

1. 83·42875 correct to (*a*) 4 places of decimals.
(*b*) 3 places ,,
(*c*) 2 places ,,
(*d*) 1 place ,,

2. 4563·8 correct to (*a*) Units.
(*b*) Tens.
(*c*) Hundreds.
(*d*) Thousands.

Exercise V (c).

1. The rainfall in a certain place for the six months commencing from January is as follows :—

January	...	18·38″
February	...	20·67″
March	..	9·7″
April	...	8·3″
May	...	16·4″
June	...	26·9″

Find the total rainfall during the six months.

2. The weights of iron bars in 4 railway trucks are 14·83, 12·78, 16·2 and 15·89 tons. Find the total weight of the iron bars in all the trucks.

3. The following are the sums of money expressed in thousands of rupees expended on the different kinds of secondary schools for boys and for girls in the Punjab :—

Secondary Schools for Boys—	
High Schools	705·08
Middle English Schools	292·96
Middle Vernacular Schools	131·18
Secondary Schools for Girls—	
High Schools	93·08
Middle English Schools	39·09
Middle Vernacular Schools	29·02

What was the total expenditure on general secondary schools: (1) for boys alone, (2) for girls alone, and (3) for both boys and girls?

4. 4 rods whose lengths are 2 m. 3 dm. 4 cm., 5m. 4dm. 5cm., 7 m. 8 dm. 4 mm., and 8 m. 5 cm. 4 mm. are jointed together so as to form one rod. What is the length of the whole rod?

5. The weight of a truck filled with coal is 20·893 tons. The weight of the truck alone is 14·8 tons. Find the weight of the coal.

6. A rod whose length is 3·076 m. is divided into two pieces one of which is 2 m. 8 cm. 6 mm. in length. Find the length of the other and express it as a decimal of a metre.

7. The length of a certain rod is 23·01 ft. On heating, it expands ·0094 ft. What is the length of the rod after expansion?

8. One kerosene oil drum contains 40·73 gallons; a second contains 42·23 gallons, and a third contains 38·97 gallons. Find the total number of gallons contained in the 3 drums.

9. The income of a certain railway company in the year 1903-1904 was 89·37 lakhs of rupees. In the year 1904-1905 it was 94·47 lakhs. Give the difference in income in the two years, (correct to a lakh).

10. A line is 18·3 in. long. What length should be cut off from it so that the remainder may be equal to the difference of the two lines 3·4 in. and 12·9 in. long?

11. Find the value of $a - b$, when $a = 7{\cdot}125$ and $b = 6{\cdot}839$.

12. Find the value of $a + b - c$, when $a = 3{\cdot}128$, $b = 6{\cdot}928$, and $c = 7{\cdot}830$.

13. $a + 625 = 2·5$. What is the value of a ?

14 $a - 6·258 = ·9834$. Whatis a ?

15. $7·682 + b = 13$. What is b ?

16. $·364 = a$ tenths $+\ b$ hundredths $+\ c$ thousandths. What is a, what is b and what is c ?

17. A tank contains 700 gallons of water. One pipe empties 63·47 gallons in an hour. A second pipe empties 132·87 gallons in an hour. If when the tank is full both the pipes are left open for an hour, how many gallons will there be in the tank after one hour ?

18. The sum of three lines X, Y and Z is 8·67 dm. The sum of X and Y alone is 5·32 dm. The difference between X and Z is 1·32 dm. (X being greater than Z). Find their lengths separately

19. The length of a rod is 16·36 inches. Express the result correct to (*a*) 1 place of decimals ; (*b*) an inch.

20. From a cloth 22 yds. long 3 pieces respectively equal to 4·12 yds., 6·82 yds. and 7·32 yds. are cut off. What is the remainder, correct to one place of decimals ?

CHAPTER VI.

MULTIPLICATION.

§ **49. Meaning of Multiplication.** If we buy 6 cases of kerosene oil, each case containing 2 tins, how many tins have we purchased on the whole? 2 + 2 + 2 + 2 + 2 + 2, or 12, *i.e.*, we have to write down 6 twos and add. Similarly if we buy 10 bags of rice each bag containing 31 measures to find the total number of measures bought, we have to write 31 ten times as 31 + 31 + 31 + 31 + 31 + 31 + 31 + 31 + 31 + 31 and find, by addition, the total to be 310. Suppose there are 580 bags, each bag containing 49 measures. Then we have to write 49 five hundred and eighty times and add. It is a very tedious process and besides it involves a waste of our precious time. So a shortened method called **Multiplication,** whereby the tedious process of adding the same number repeated a number of times as in the above, may be avoided, has been discovered. Thus **Multiplication is a brief method of finding the sum of a number of repetitions of the same number.** In the first example finding the sum of 6 two's is otherwise spoken of as multiplying 2 by 6. In order, however, to employ this short method in the case of large numbers, we must form, by actual addition, a table of results (called the Multiplication table) of multiplying small numbers and commit it to memory.

The sign × (read 'into') is the sign of multiplication and denotes that the number preceding it is to be multiplied by the number succeeding it, *i. e.*, the number preceding it is to be repeated for addition as many times as is denoted by the number succeeding it. The

number preceding the sign × is called the **multiplicand** (because it is the number to be multiplied or repeated); the number following the sign × is called the **multiplier** (because it tells us how many times the number is to be repeated). The final result is called the **product.**

Thus in $2 \times 6 = 12$ and $31 \times 10 = 310$ — 2 and 31 are multiplicands, 6 and 10 are multipliers and 12 and 310 are products.

The multiplicand and the multiplier are both called **factors** of the product, *e.g.*, 2 is a factor of 12; 6 is also a factor of 12.

§ 50. Graphical illustration of multiplication.

6 × 5 according to our definition means that 6 must be

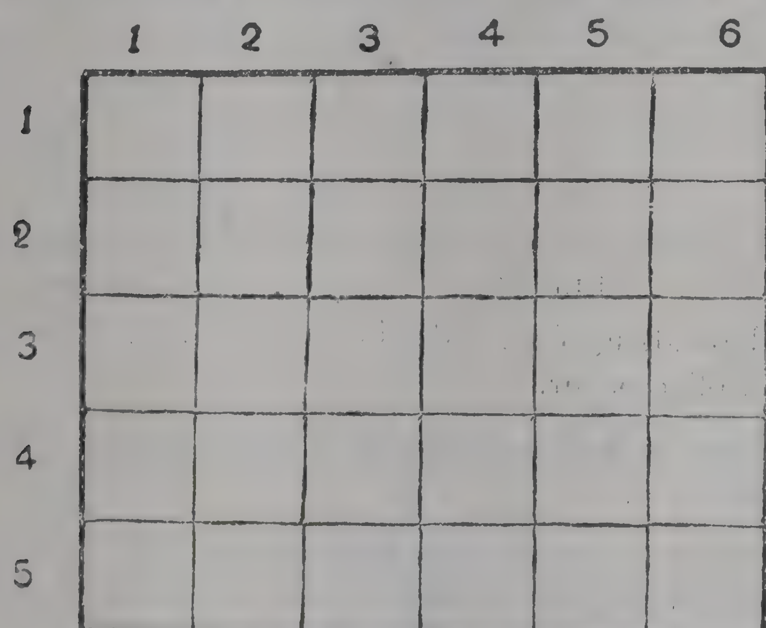


FIG. 57.

written down 5 times and added. Construct a horizontal row of 6 squares as shown in Fig. 57 and put 4 more such rows below one another. This amounts to writing 6 five times.

Now the total number of squares $= 30$ (by counting).
$\therefore 6 \times 5 = 30$.

Similarly to find 5×6, we have to write the number 5 six times and add. Take a horizontal row of 5 squares, as shown in Fig. 58. Put 5 more such rows below one another thus obtaining 6 rows of 5. Now the total number of squares $= 30$ (by counting).

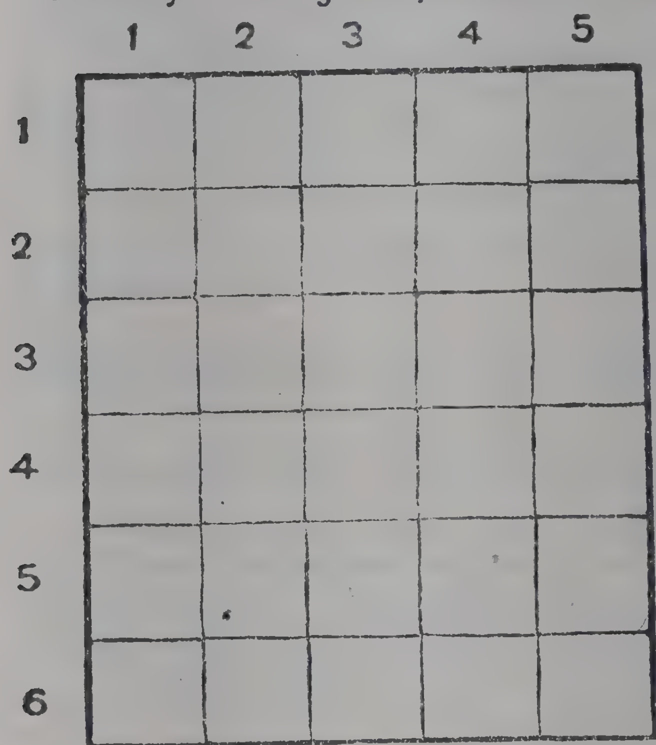


FIG. 58.

Whether we multiply 6 by 5 or 5 by 6 the result or the product is the same, *viz.* 30, and therefore independent of the order in which the multiplicand and the multiplier occur.

Similarly we may prove that $6 \times$ any number $a =$ that number $a \times 6$.

$\therefore 6 \times a = a \times 6$. Similarly, if we take b to represent any number, we can show that $b \times a = a \times b$.

The product $a \times b$ is generally written $a.b$ or ab.

The product of 6 and a is written $6 \times a$ or $6.a$ or $6a$ as explained before. But $a \times 6$ is not written $a.6$ or $a6$. The form $a.6$ is not in use. When a product consists of a number and a letter the number is always placed before the

letter. Whether it is the product of 6 and a or a and 6, the product, being the same in each case, is written $6a$.

6 in $6a$ is called the co-efficient or the co-factor of a.

Note.—It is only in the case of letters the form ab is used to denote the product of a and b. If $a=4$, $b=5$, the product is not written 45 (45, we have already seen, denotes forty-five) but is written 4×5 or $4 \cdot 5$. Then point here is placed lower down so as to be distinguishable from the decimal point.

To show that $6\,(3+2) = 6 \cdot 3 + 6 \cdot 2$.

Now $6\,(3+2)$ means 6×5, *i.e.* 6 must be repeated 5 times and we have seen in the graphical representation of 6×5 that we get 5 horizontal rows of 6 squares each and the result is 30. The same block or rectangle of 30 squares may now be divided into two blocks as shown in Fig. 59

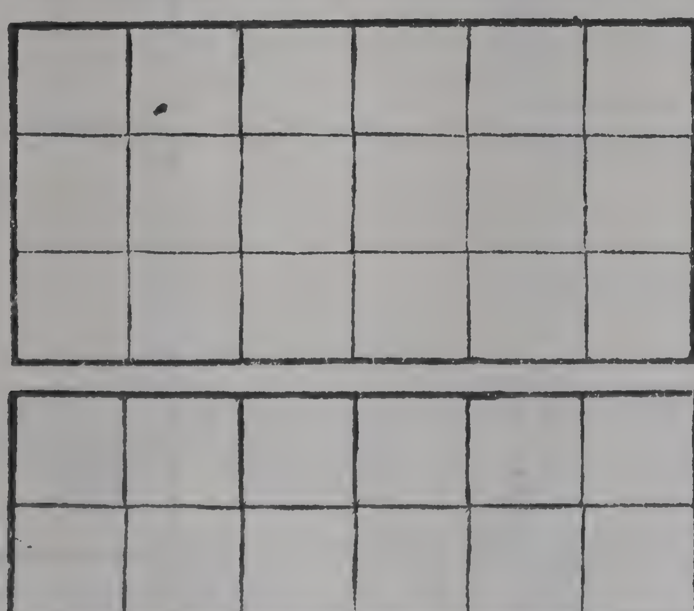


FIG. 59.

the first block containing 3 horizontal rows of 6 squares each and therefore representing 6×3 and the second block containing 2 horizontal rows of 6 squares each and repre-

senting 6×2. Since the whole block is equal to the two blocks we conclude that 6×5 or $6\,(3+2) = 6 \times 3 + 6 \times 2$.

Similarly we may prove that $7 \times (3+2) = 7 \times 3 + 7 \times 2$ and in general, we have $a \times (3+2)$ (where a is any number) $= a \times 3 + a \times 2$.

Similarly $a\,(4+5) = a \times 4 + a \times 5$.

More generally if we take b and c to represent any numbers, we have $a\,(b+c) = a \times b + a \times c$.

Whether you multiply the sum of two numbers by a third number, or multiply the two numbers separately by the third and add the products, the result is the same.

Again if $8a$ is subtracted from $13a$ we get $5a$ or $13a - 8a = 5a$ and $(13-8)\,a = 5a$.

$\therefore (13-8)\,a = 13a - 8a$, and more generally if b and c be any two numbers (c less than b) we have $a(b-c) = ab - ac$.

Also from the above, $(3+2)(5+1) = 3\,(5+1) + 2\,(5+1) = 3.5 + 3.1 + 2.5 + 2.1$. This is illustrated graphically in the adjoining Fig. 60.

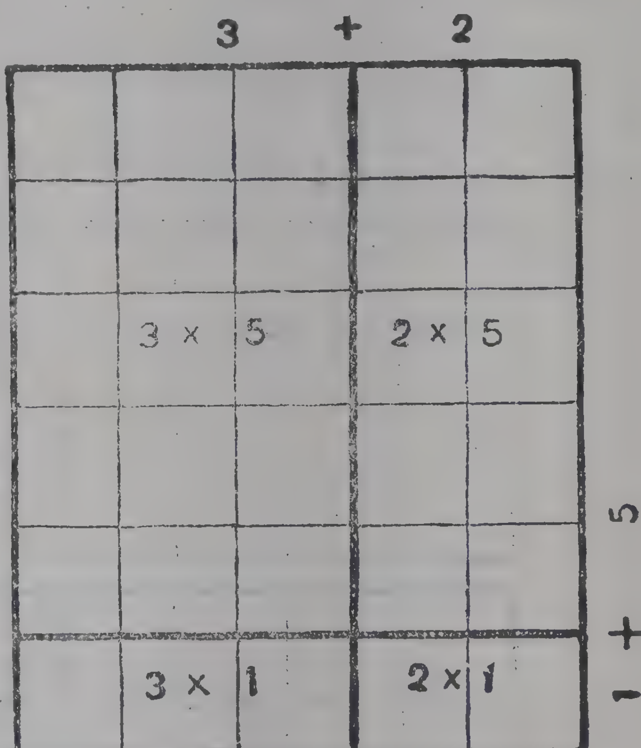


FIG. 60.

Similarly we can show that $(a+b)(c+d) = ac + ad + bc + bd$

And more generally we can show

$$k\,(a+b+c+\ldots\ldots) = ka + kb + kc + \ldots\ldots$$

Exercise—Oral.

1. What are the factors of the following : (*a*) 18, (*b*) 24, (*c*) 32, (*d*) 45, (*e*) 48, (*f*) 42 ?

2. $8 \times a = 72$, what is a ?
$13 \times b = 117$, what is b ?
$a \times 16 = 256$, what is a ?
$b \times 12 = 84$, what is b ?
$8 \times 13 = a$, what is a ?
$16 \times 12 = b$, what is b ?

3. Express in symbols the product of 12 and a ; of b and 13 ; of c and d ; of a and c ?

4. What is the difference between $(8+7) \times 2$ and $8 + 7 \times 2$?

5. What is the cost of 5 books at Rs. 4 each ?

6. What is the cost of p pencils at 2 annas each ?

7. If each man earns Rs. 5, how many rupees do m men earn ?

8. What is the cost of s sheep at r rupees each ?

9. If p men earn q rupees each, what is the total sum earned ?

10. What is the cost of x tickets at y annas each ?

Exercise—Practical and Graphical.

1. Show graphically that $8 \times 4 = 4 \times 8$; $6 \times 9 = 9 \times 6$.

2. Represent graphically—
(1) $6 \times 9+3$ and also $6\ (9+3)$. (2) $6 \times 6+3$. (3) $5 \times 5-3$.
(4) $8 \times 2+8 \times 3$. (5) $8 \times 3-6 \times 3$. (6) $7 \times 3+4 \times 3$.

3. Show graphically that $(8+2)\ 2 = 8.2+2.2$ and that $(3+1)(2+4) = 3.2+1.2+3.4+1.4$.

§ 51. Multiplication by a single digit.

Example.—Multiply 537 by 4.

This means finding the sum of 537 written 4 times. By actual addition as shown in the margin we have 2148 for the product.

$$\begin{array}{r} 537 \\ 537 \\ 537 \\ 537 \\ \hline 2148 \\ \hline \end{array}$$

Now by multiplication.

As we added 4 sevens in addition, so we take 4×7 making 28 and setting down 8 in the units' column in the product we carry 2 tens; again corresponding

to adding 4 threes and this 2, we take the value of $4\times3+2$, *viz.*, 14, set down 4 and carry 1 and lastly we find $4\times5+1 = 21$ and thus the product is 2148.

The work is generally arranged thus

$$\begin{array}{r} 527 \\ 4 \\ \hline 2148 \\ \hline \end{array}$$

Multiplication of a number by 10, 100,...

Example.—To multiply 8532 by 10.

8532 = 8 thousands + 5 hundreds + 3 tens + 2 units.

When multiplied by 10, units become tens, tens become hundreds, hundreds become thousands and so on.

$\therefore$ 8532×10 = 8 ten thousands + 5 thousands + 3 hundreds + 2 tens = 85320.

Therefore the result of multiplying a number by 10 is the same as placing a zero after the units figure so that the place value of each figure gets increased ten-fold.

If we multiply a number by 100, the value of each figure is increased a hundred-fold; for, when multiplied by 100, units become hundreds, tens become thousands, hundreds become ten thousands and so on.

So, multiplying by 100 is effected by placing two ciphers after the units figure in the number.

Generally to multiply any number by 10, 100, 1000, etc., we place after the units figure as many ciphers as there are zeroes in the multiplier.

Multiplication by factors.

Example.—Multiply 8763 by 15.

Now, $15 = 5\times3$. The question means writing down 8763 fifteen times and adding. These 15 numbers may be taken in 3 groups of 5 each and each group of 5 numbers may be added separately and the three sums thus obtained

(which will all be equal) be finally added to get the total. Adding each group of 5 numbers is the same as multiplying the number by 5 and adding the three sums is the same as multiplying the first result by 3. Thus multiplying by 15 is the same as multiplying by 5 and that result by 3. Thus $8763 \times 15 = (8763 \times 5) \times 3$

$$= 43815 \times 3 = 131445.$$

Directly multiplying 8763 by 15 (using the multiplication table) we get the same result, *viz.*, 131445.

Exercise—Oral.

(1) Multiply 873 by 10, 100, 1,000.

(2) Multiply 3936 by 10,000, 100,000.

(3) What is the value arising from the multiplication of the digits dotted in (1) $3\overset{*}{8}46$ by 7. (2) $4\overset{*}{2}389$ by 3. (3) $6\overset{*}{2}38$ by 100.

Exercise VI (a).

Multiply—

1. 3284 by 3, 4, 6, 7.

2. 68,346 by 50, 30, 500, 300, 4,000, 8,000.

Multiply (by factors)—

3. 832, 459 by 12, 14, 16, 18, 20.

4. 34,67,489 by 21, 24, 28, 30, 35, 42.

5. 84,38,475 by 54, 56, 64, 81.

§ 52. Multiplication: General Cases.

Example.—To multiply 647 by 382.

$647 \times 382 = 647 \times (300 + 80 + 2) = 647 \times 300 + 647 \times 80 + 647 \times 2.$

The working is shown below:

647			
382			
194100	product	by	300
51760	,,	,,	80
1294	,,	,,	2
247154	,,	,,	382

It is usual to place the multiplicand and the multiplier with their units digits below one another. Generally the ciphers added at the end of the partial products to increase the local values of digits are omitted. The working may be shown in the adjoining shortened form. It should be noted that the first figure of every partial product comes directly below the corresponding figure of the multiplier.

```
   647
   382
 ------
 1941
  5176
   1294
 ------
 247154
 ------
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Note.—To avoid confusion, beginners are recommended to use squared paper for working such questions.

Although the order in which the partial products (*i.e.*, products by 300, 80 and 2) are arranged is immaterial, it is advisable to multiply beginning with the highest figure of the multiplier as is shown above; for, then the partial products will get *arranged in the order of their importance.*

It must be clear to the student that if the figure in the hundreds place of the multiplicand be multiplied by the digit in the hundreds place of the multiplier the product will be so many ten thousands, for $100 \times 100 = 10000$. In this example, 6 of the multiplicand is in the hundreds place and 3 of the multiplier is also in the hundreds place. Therefore the product must be 18 ten thousands. Similarly if a digit in the lakhs place be multiplied by a digit in the hundreds place the result must be so many hundred lakhs or crores. In this way the place value of the product arising from the multiplication of any digit in the multiplicand by any digit in the multiplier can be determined.

Rough Checks. Before beginning to actually multiply the two numbers it is desirable that the beginner finds

certain limits between which the product lies, *e.g.*, in the example worked out above, 647 lies between 600 and 700 and 382 lies between 300 and 400 ∴ the product should lie between 180,000 and 280,000. Such a determination of limits will be a safeguard against any obvious mistakes he may commit in the course of working. A pupil who gets 326,390 as the answer for the above question, could easily discover its absurdity, if he had taken the trouble to form a rough estimate of his answer as indicated above. Such processes are called *rough checks*.

Exercise—Oral.

1. A foot (1 ft.) contains 12 inches. How many inches are there in (1) 6 ft., (2) 5 ft., (3) 4 ft., (4) 10 ft. ?

A yard (1 yd.) contains 3 feet. How many feet are there in (*a*) 6 yds., (*b*) 3 yds., (*c*) 9 yds., (*d*) 8 yds. ?

2. One 2-anna piece has the same value as 2 annas. How many annas should be exchanged for (*a*) 9 two-anna pieces, (*b*) 8, (*c*. 19 ?

3. Each packet of Belmont candles contains 24. How many candles are there in (1) 4 packets, (2) 5 packets ?

4. In a quire of paper you have 24 sheets. How many sheets are there in 5 quires ; 8 quires ; 9 quires ?

5. The length of a certain line is 8 cm. What is the length of a line 5 times as long ; 8 times as long ; 9 times as long ?

6. An angle contains 32°. What is the magnitude of another angle 4 times as great ; 6 times as great ?

7. How many days are there in (1) 4 weeks, (2) p weeks ?

8. There are p beggars and I give q pies to each. How many pies do all the beggars together get ?

Exercise VI (b).

Multiply :—

1. (*a*) 256 by (1) 2, (2) 4, (3) 5. (*b*) 625 by (1) 4, (2) 5,

2. Give all the factors of (1) 42, (2) 36, (3) 44, (4) 80,

3. Write down the partial products in the following multiplication exercises, corresponding to the digits dotted in :—

(*a*) 634 by $12\overset{*}{6}$. (*b*) 836 by $\overset{*}{1}26$.

(*c*) 643 by $2\overset{*}{3}8$. (*d*) 836 by $\overset{*}{2}038$.

4. Write down the products of the pair of numbers represented by the digits dotted in :—

(*a*) $87\overset{*}{5} \times 32\overset{*}{4}$. (*b*) $\overset{*}{6}49 \times 3\overset{*}{4}3$.

(*c*) $\overset{*}{8}39 \times \overset{*}{9}01$. (*d*) $9\overset{*}{6}12 \times \overset{*}{8}033$.

5. Multiply by factors :—

(*a*) 6,748 by 49. (*b*) 8,348 by 63.
(*c*) 25,384 by 121. (*d*) 16,486 by 144.
(*e*) 248,609 by 112. (*f*) 983,093 by 256.
(*g*) 893,876 by 500. (*h*) 893,876 by 1,100.

6. Multiply and check your results in each of the following :—

(*a*) 84,381 by 2,363. (*b*) 96,001 by 5,684.
(*c*) 10,328 by 4,832. (*d*) 106,428 by 4,382.

7. (1) Find the value of :—

(*a*) $11 \times 234 + 2 \times 234$. (*b*) $13 \times 456 + 7 \times 436$.
(*c*) $(261 \times 356) + (82 \times 356) + (14 \times 356.)$
(*d*) $(18 \times 960) - (12 \times 960)$.
(*e*) $(18 \times 140) + (13 \times 140) - (9 \times 140)$.

(2) Simplify :—

(*a*) $10a + 15a - 3a$, (*b*) $12b + 13b - 8b - 9b$.
(*c*) $14p + 10p + 8p - 9p$.
(*d*) $16x + 8x - 3x + 2x - 6x$.

8. An acre is worth Rs. 4,650. What is the value of 156 acres?

9. What is the weight of 413 waggon loads of rice each one of which weighs 1,643 maunds?

10. If a train travelled for a week at the uniform rate of 32 miles per hour, what distance would it describe?

11. If each cask of wine contains 42 gallons, how many gallons are there in 423 casks?

12. The interval between two consecutive telegraph posts is 38 yds. What distance does a train travel in passing through 889 such intervals?

§ 53. Shortened methods of Multiplication.

Example 1.—Multiply 17,283 by 99.

17,283 × 99 = 17,283 (100 − 1) = 17,283 × 100 − 17,283 = 1,728,300 − 17,283 = 1,711,017. *Ans.*

The product is obtained by placing 2 ciphers to the right of the multiplicand and subtracting the multiplicand from the resulting number.

Example 2.—Multiply 683,245 by 9,999.

683,245 × 9,999 = 683,245 (10,000 − 1) = 6,832,450,000 − 683,245 = 6,831,766,755.

Example 3.—Multiply 38,364 by 498.

38,364 × 498 = 38,364 (500 − 2) = 38,364 × 500 − 2 × 38,364 = 19,182,000 − 76,728 = 19,105,272.

Example 4.—Multiply 876,428 by 864 in two lines.

876,428 × 864 = 876,428 (800 + 64)

876,428 (= N say)	
864	
701,142,400	= N × 800
56,091,392	= N × 64
757,233,792	= N × 864

Example 5.—Multiply 969,832 by 16,644 in 3 lines.

969,832 × 16,644	
3,879,328	N × 4
15,517,312	N × 4 × 4000 or N × 16000
620,692,48	N × 4 × 160 or N × 640
16,141,883,808	N × 16644.

Example 6.—Multiply

48,324 by 4 and subtract the result from 634,829

Mental Process.

4 times 4, 16 and 3 = 19; (carry 1)

4 times 2, 8 + 1, 9 and 3 = 12 (carry 1)
4 times 3, 12 + 1, 13 and 5 = 18 (carry 1)
4 times 8, 32 and 1, 33 and 1 = 34 (carry 3)
4 times 4, 16 and 3, 19 and 4 = 23 (carry 2)
2 and 4 = 6

the answer is 441,533.

The student is recommended to practise the following method for multiplying mentally two numbers of two digits each.

Example 7.—Multiply 38 by 74, *i.e.*, $(30+8)(70+4) = 30 \cdot 70 + 8 \times 70 + 4 \times 30 + 8 \times 4 = 3 \times 7 \times 100 + (8 \times 7 + 4 \times 3)\, 10 + 8 \times 4 = 2100 + 680 + 32 = 2812$.

The mental process stands thus : 3 times 7, 21   2100

$\left.\begin{array}{l}\text{8 times 7, 56}\\ \text{4 times 3, 12}\end{array}\right\}$ 56 and 12, 68; 680

2100 and 680; 2780

8 times 4, 32; 2780 and 32, 2812.

This method begins the multiplication with the highest or the most important figures in the multiplicand and the multiplier and if we want only an approximate result we may roughly say the result is nearly 40 times 70 or 2800.

Exercise—Oral.

1. If each form contains 38 boys, how many boys are there in 15 forms?

2. If each case contains 55 books, how many books are there in 13 cases?

3. In each basket there are 79 mangoes, how many mangoes are there in 89 baskets?

4. How many letters are there in a page of 22 lines if each line contains 36 letters?

5. A book has 480 pages with 36 lines in each page. What is the total number of lines?

6. What is the weight of 64 bales of cotton if each bale weighs 148 seers?

7. If a furlong contains 220 yds., how many yards are there in 360 furlongs?

Exercise VI (c).

Multiply

1. 876, 823 by 999.
2. 468, 325 by (*a*) 9,999, (*b*) 9,995, (*c*) 998, (*d*) 497, (*e*) 896.
3. 684,325 by (*a*) 728, (*b*) 864, (*c*) 14,412, (*d*) 1,296, each in 2 lines.
4. 687,238 by (*a*) 637,189, (*b*) 763,189, (*c*) 189,637, (*d*) 718,963, (*e*) 189,763, (*f*) 631,897, each in 3 lines.
5. All operations should be done in one line in the following:—

   (*a*) 976,354 × 4 + 143,128.
   (*b*) 1,803,296 — 837,654 × 2.
   (*c*) 12,611,528 + 5 × 317,629.
   (*d*) 10,000,000 — 6 × 153,204.
   (*e*) 2,000,000 — 5 × 142,857.
   (*f*) 1,260,839 + 8 × 9,740,625.

§ **54. Squares, cubes, ...**.When a number is multiplied by itself it is said to be **squared,** *e.g.*, 7 × 7 or 49 is said to be the **square** of 7. The operation 7 × 7 is, generally represented by 7^2 (read '7 squared').

Similarly 7 × 7 × 7 is represented symbolically by 7^3 (read '7 cubed') and 7^3 or 343 is said to be the **cube** of 7.

When three sevens are thus multiplied together, 7 is said to be **cubed.**

Similarly 7 × 7 × 7 × 7 is represented by 7^4 (read '7 to the fourth') and is called the fourth power of 7.

In general 7^n (where n is any whole number) indicates that n sevens are to be multiplied together.

In general $a \times a$ is represented by a^2; $a \times a \times a$ by a^3; $a \times a \times a \times a$ by a^4 and $a \times a \times a$......n a's by a^n (read 'a to the power of n').

In each of these cases, 2, 3, 4, n is called the **index,** because it *indicates* how many a's are multiplied together and a^2, a^3, a^4, a^n are respectively called the second, the third, the 4th and the nth power of a.

The student should clearly understand the difference between the terms *co-efficient* and *index*. 2 in $2a$ is the co-efficient of a, whereas 2 in a^2 is the index of a. ($2a$ means

$2 \times a$ or $a + a$) whereas a^2 means $a \times a$, *i.e.*, $a + a + a + \ldots\ldots$(a a's being added together). Again if $a = 3$, $2a = 6$ and $a^2 = 9$.

Similarly $3a$ is different from a^3. $3a$ means $3 \times a$ or $a + a + a$ whereas a^3 means $a \times a \times a$.

Again $2^3 = 2 \times 2 \times 2$ and $2^2 = 2 \times 2$. $\therefore 2^3 \times 2^2 = (2 \times 2 \times 2) \times (2 \times 2) = 2 \times 2 \times 2 \times 2 \times 2 = 2^5 = 2^{(3+2)}$.

Thus when we multiply the 3rd power of 2 by the 2nd power of 2 we get the fifth power of 2. Similarly the 4th power of 3 × the 3rd power of 3 = the 7th power of 3, *i.e.*, $3^4 \times 3^3 = 3^7 = 3^{(4+3)}$. Generally $a^3 \times a^2 = a^5$ and $a^m \times a^n = a^{m+n}$ or in words, **the product of two powers of the same number is a power of the same number whose index is the sum of the indices of the two given powers.**

Exercise—Oral.

1. Write down in index notation—
   (a) $2 \times 2 \times 2 \times 2 \times 2$.
   (b) $3 \times 3 \times 3 \times 3 \times 3 \times 3$.
   (c) $a \times a \times a \times a \times a \times a$.
   (d) $b \times b \times b \times b \times b \times b \times b \times b$.
   (e) $a \times a \times a \times b \times b$.
   (f) $x \times x \times x \times x \times x$.
   (g) $y \times y \times y$.
   (h) $z \times z \times z \times z \times y \times y \times y$.
2. What is the value of—
   (a) $2^2 \times 3^2$. (b) $2^3 \times 3^3$. (c) $2^4 \times 3^2$.
   (d) $2^5 \times 6$. (e) $2^8 \times 9$. (f) $2^3 \times 2^4$.
   (g) $3^8 \times 3^9$. (h) $4^8 \times 4^2$. (i) $5^{10} \times 5^8$.

Exercise VI (d).

1. Construct a table showing the powers of 2, 3, 4 and 5 up to the tenth in each case.

2. In the tables of powers of 2 and 4, find out what powers of 2 and 4 have the same value. Hence construct another table for powers of 2 that are also powers of 4.

3. From the table of powers of 2 and 5, multiply similar powers of 2 and 5, and express the products as powers of 10, *e.g.*, $2^3 = 8$; $5^3 = 125$; $2^3 \times 5^3 = 1000$ or 10^3.

4. Find the value of :– using the table constructed in exercise 1, VI (*d*).

(*a*) $2^3 \times 3^4$. (*b*) $2^8 \times 3^7$. (*c*) $3^8 \times 4^6$.
(*d*) $3^8 \times 4^5 \times 5^4$. (*e*) $4^8 \times 3^6 \times 5^2$.

5. Construct a table of squares for the 1st 25 numbers.

6. From the table in 5 find what must be added to the following numbers to get the nearest squares (*a*) 12 ; (*b*) 39 ; (*c*) 26 ; (*d*) 34 ; (*e*) 38 ; (*f*) 63 ; (*g*) 89 ; (*h*) 97 ; (*k*) 124 ; (*l*) 248 ; (*m*) 347 ; (*n*) 484 ; (*o*) 526 ; (*p*) 613.

7. From the same table find numbers whose squares are nearest the following : (*a*) 11 ; (*b*) 24 ; (*c*) 36 ; (*d*) 42 ; (*e*) 48 ; (*f*) 124 ; (*g*) 156 ; (*h*) 265 ; (*k*) 321 ; (*l*) 584.

8. Examine the table of squares and find in what digits the numbers end. Is there any law that you can discover ?

9. Construct a table of cubes for numbers from 1 to 15.

10. Find the value of a^2b ; a^3b^2 ; $a^2b^3c^4$; a^2bc^2 ; given that $a = 2, b = 3, c = 4$.

11. Find the value of $a^2b + ab^3$; $abc + a^2b^3$; $ab^2c^3 + a^2b^4c$; $a^3 + b^2c + ac^3$; given $a = 1, b = 2, c = 3$.

Exercise—Practical and Graphical.

1. The figure in the margin represents the square of 3. Draw similar figures to represent the squares of 4, 5, 6, 11.

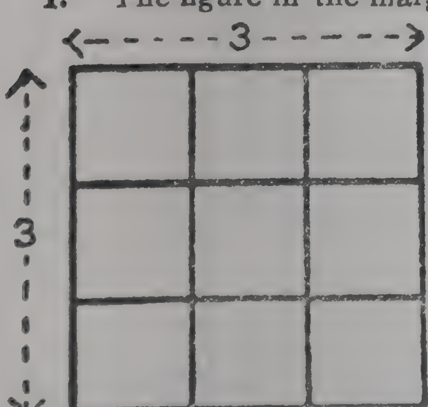


FIG. 61.

2. Show by drawing figures that
(*a*) $12 \times 5 + 12 \times 7 = 12^2$.
(*b*) $14 \times 8 + 14 \times 6 = 14^2$.

3. Show by drawing figures that
(*a*) $2 \times 12 = 2 \times 10 + 2^2$.
(*b*) $3 \times 14 = 3 \times 11 + 3^2$.

4. Show by drawing figures that
(*a*) $5 \times 2 = 7 \times 2 - 2^2$.
(*b*) $3 \times 8 = 11 \times 3 - 3^2$.

5. Show graphically that

(a) $(6+5)(6+5)$ or $11^2 = 6(6+5) + 5(6+5)$.

(b) In the same fig. show that $6(6+5) = 6^2 + 6.5$ and $5(6+5) = 5^2 + 6.5$.

(c) Show from the figures thus divided that $11^2 = 6^2 + 5^2$ + twice 6.5.

6. Similarly draw figures shewing that

(a) 9^2 or $(4+5)^2 = 4^2 + 5^2$ twice 4×5.

(b) 12^2 or $(10+2)^2 = 10^2 + 2^2 +$ „ 2×10.

7. Generally if we put a and b for any two numbers, show that $(a+b)^2 = a^2 + b^2 + 2ab$.

8. Making use of the above result, find the squares of the following numbers :—

11, 15, 21, 35, 41, 103, 204, 305, 1001, and 1005.

9. The accompanying figure (Fig. 62) represents 3^2 and 2^2 and also $3^2 + 2^2$. Similarly draw figures representing—

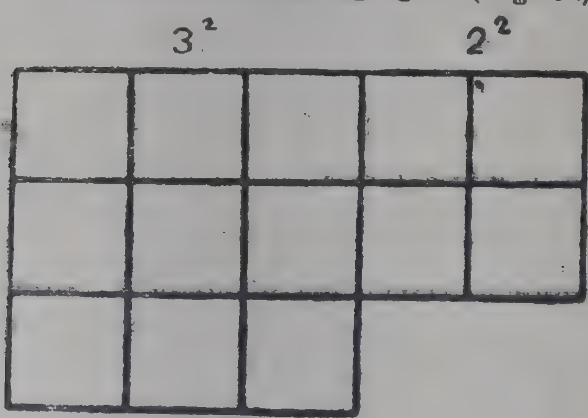


FIG. 62.

(a) 4^2 and 2^2 and $4^2 + 2^2$.

(b) 4^2 and 3^2 and $4^2 + 3^2$.

10. The whole figure (Fig. 63) as in question (9) represents $5^2 + 3^2$ and contains two rectangles each being 5×3 and a square 2^2, *i.e.*, the square, *viz.*, $2^2 =$ the whole figure — the two rectangles.

$= 5^2 + 3^2 -$ twice 5×3.

Similarly draw that

(a) 3^2 or $(7-4)^2 = 7^2 + 4^2 -$ twice 7×4.

(b) 8^2 or $(10-2)^2 = 10^2 + 2^2 -$ twice 10×2.

11. If a and b be any two numbers, show that $(a - b)^2 = a^2 + b^2 - 2ab$.

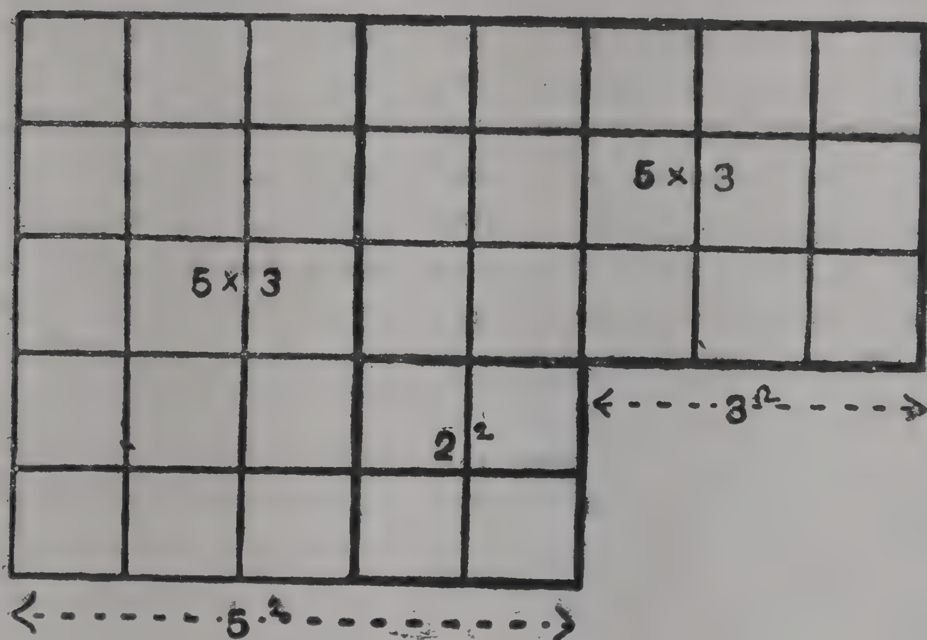


FIG. 63.

12 Make use of the above result to find the squares of 15, 19, 25, 35, 39, 99, 993.

Significant figures.

§ 55. Suppose a gentleman's property including his house, lands, jewels, cloths, furniture, and everything he possesses is worth 16,483 rupees. If questioned how much he is worth, we generally say he is worth about 16,000 rupees, omitting 483 rupees which is small when compared with 16,000 rupees. Similarly if another gentleman's property is worth 1,25,689 we say roughly the property is worth a lakh and 25 thousands. Here we retain the first 3 figures. Suppose the revenue of a country is 63,47,25,089 rupees. This revenue may be roughly taken to be about 63 crores omitting even lakhs. In this case the lakhs is small when compared with 63 crores. In expressing results we do

often give only the first two or three figures omitting the rest, so as to give a rough idea of the magnitude of the results without troubling ourselves with minute details. In all these cases we are said to express the number giving only the first two or three **significant** figures. These figures are said to be *significant* because they denote the most important part of the number as opposed to the remaining figures which denote the *insignificant* or negligible part of the same number. Suppose you have in your purse Rs. 6-4-11; when asked how much you have, you will say 6 rupees and odd, meaning thereby that 6 rupees is the appreciable part of the sum you have and the rest, though it has value, is not worth mentioning; for it is small when compared with the rupees in the purse.

Example 1. Express Rs. 82,95,101 correct to the first two significant figures.

Since the sum lies between 82 lakhs and 83 lakhs and since it is nearer 83 than 82 it is approximately given as 83,00,000 or 83 lakhs.

Example 2.—Multiply 88,654 by 16 giving the result to (1) three significant figures : (2) five significant figures.

The product is 1,418,464; the answer correct to 3 significant figures is 1,420,000 and correct to 5 significant figures is 1,418,500.

Exercise—Oral.

1. Read off the following correct to 4 significant figures :—

(*a*) 34,249. (*b*) 85,043. (*c*) 4,897,453. (*d*) 5,678,765.

2. If the equatorial radius of the earth be 6,356,548 metres, give its value correct to 3 significant figures.

3. A kilogram = 15,432·3 grains ; express a kilogram in grains correct to 4 significant figures.

4. The revenue of the United States in a certain year was 695,389,058 dollars; express the revenue correct to million dollars

5. A kilometre is 39,370·1 inches ; express a kilometre correct to an inch.

Exercise VI (e).

1. Multiply 38,769 by 6 giving the result to 3 significant figures.

2. Multiply 46,879 by 11 giving the result correct to thousands.

3. Multiply out to the nearest thousand 56,098 by 13.

4. Multiply out to the nearest ten thousands 154,985 by 700.

5. Multiply correct to 3 significant figures—

(*a*) 35,069 by 23. (*b*) 48,097 by 37. (*c*) 34,896 by 374.

6. The population per square mile of the Madras Presidency is 274 and its area is 139,698 square miles; find the total population correct to a lakh.

7. The total number of educational institutions in the Mysore Province in 1899 was 3795 and the average number of pupils for each institution was 34; find the total number of pupils correct to a ten thousand.

8. If the time taken by light to travel from the sun to the earth be 408 seconds and if light travel 186,330 miles in a second; find the distance of the sun from the earth correct to 3 significant figures.

9. The radius of the sun is 109 times that of the earth. If the radius of the earth is 6378 kilometres; find the radius of the sun correct to 100 kilometres.

10. Find the value of $(3142)^2$ correct to 3 significant figures.

11. Find the value of 560 bicycles of which 250 are worth Rs. 320 each and the rest Rs. 260 each.

12. A wall is built of 138 layers of bricks. Each layer is 2 bricks wide and 2250 bricks long. Find the number of bricks in the wall to the nearest hundred.

13. In a page of the newspaper 'The Hindu' there are four columns, and in each column 146 lines, and in each line 42 letters on an average. How many letters are there in a page?

14. Of every 400 years 97 contain 366 days; the remainder contain 365 days. Find the total number of days in a period of 400 years.

15. The population of the Madras City in 1891 was 509,346. If it increased on an average by 10,786 every year for the next 10 years, what was the population in 1901?

16. The number of scholars attending educational institutions in the Madras Presidency increased on an average by 9,836 every year for the 5 years from 1903 to 1908. If the number in 1908 was 10,57,170, find the number in 1903.

17. In a street there are 168 houses. Of these 46 have 3 families of 5 persons each living in each of them, 18 have 4 families of 4 persons each, and the rest 2 families of 6 persons each. What is the population of the street?

18. A wall is built of 34 layers of bricks. Each course is 3 bricks wide and 1540 bricks long. Find the number of bricks in the wall.

19. There are 2350 hackney carriages in Madras of which 685 are jutkas, 1234 are bullock carts and the rest broughams. Each jutka costs on an average Rs. 125, each bullock cart Rs. 90, and each brougham Rs. 480. Find the cost of all the hackney carriages in Madras.

20. Find the value of 836 cawnies of land of which 432 are wet (Nunja) and the rest dry (Punja). The Nunja lands cost Rs. 845 a cawny and the Punja Rs. 625 a cawny.

21. There are on the Indian Railways 22,985 locomotives, 85,654 passenger coaches, and 892,487 goods wagons. An engine costs Rs. 37,500, a coach Rs. 6,750 and a wagon Rs. 1,500. Find the total value of the rolling stock.

22. In a certain school there are 623 boys belonging to the Upper Secondary Department of whom 500 boys pay fees at the standard rate of Rs. 38 per annum: 73 Mahomedans pay at the rate of Rs. 28 per annum and the rest, *viz.*, backward classes are allowed to pay at half the standard rate. Find the total collection of fees in the Upper Secondary Department.

CHAPTER VII.

DIVISION.

Suppose a father has 100 rupees in his possession and he pays his son a monthly instalment of 15 rupees. For how many months will this sum last? By a series of subtractions we can find the answer as shown in the margin. The working shown here tells us that the instalments can be paid for 6 months and there is a sum of rupees ten left over. Similarly if we want to find how many fifteens there are in a thousand we may find it by a number of subtractions which however, is very tedious. Continued subtractions of the same number may be effected by a short method called **Division** by making use of the multiplication table. **Division** is thus a method by means of which we ascertain **how often one number can be subtracted from another or how often one number is contained in another.** Division may be regarded as repeated subtraction facilitated by the use of the multiplication table.

100	
15	I
85	
15	II.
70	
15	III.
55	
15	IV.
40	
15	V.
25	
15	VI.
10	

The symbol $\div$ (read 'divided by') is the symbol of division and when placed between two numbers means that the number preceding it is to be divided by the number following it. The number preceding it is called the **dividend** and that following it is called the **divisor.** The number of times the divisor is contained in the dividend is called the **quotient** and what is left over is called the **remainder.** In the example we have taken

viz., $100 \div 15$, 100 is the dividend, 15 the divisor, 6 the quotient and 10 the remainder. 15 is contained 6 times in 100 and 10 is left over. We have seen that 6 times 15 is written 15×6. Thus the above statement may be symbolically expressed as $100 - 15 \times 6 = 10$, which is the same as $100 = 15 \times 6 + 10$, and generally we have

$$\text{Dividend} = \text{Divisor} \times \text{Quotient} + \text{Remainder}$$
$$\text{or } D = d \times p + r$$

where D, d, q and r stand for Dividend, Divisor, Quotient and Remainder respectively.

Note.—The operation of dividing 100 by 15 is also expressed as $\frac{100}{15}$ (read '100 by 15'). Thus when a is to be divided by b, the operation is expressed by $\frac{a}{b}$.

§ 56. Graphical representation of Division.

To represent graphically 14 divided by 4.

Draw a straight line AB containing 14 sub-divisions as shown in the figure. Draw another line CD containing

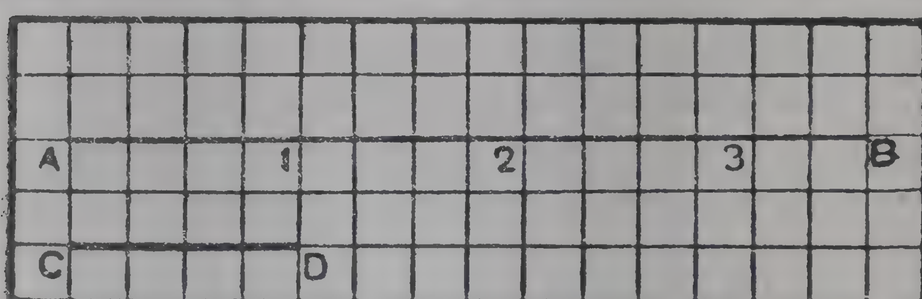


FIG. 64.

4 sub-divisions. With your dividers step off lengths along AB each equal to CD. How many times can you do so and what is the portion left over? You find there are 3 CD's in AB and 2 sub-divisions extra, *i.e.*, 4 divides 14 three times leaving 2 for the remainder.

§ 57. Division by a single digit.

Example.—Divide 4763 by 8. The work arranged in the margin thus :

8)4763
595—3

∵ 4 is less than 8, regard 4 thousand 7 hundred as 47 hundreds; and divide by 8; the quotient is 5 hundred and 7 hundred is the remainder.

Regard this remainder 7 hundred and 6 tens as 76 tens and divide by 8; the quotient is 9 tens and the remainder 4 tens.

Regard this remainder 4 tens and 3 units as 43 units; and divide by 8; the quotient is 5 units and 3 units is the remainder.

The total quotient is 595 and the remainder 4.

The mental process in the above division is as follows:

8 goes into 47, **5** times and 7 over;
8 goes into 76, **9** times and 4 over;
8 goes into 43, **5** times and 3 over.

The figures in thick type are set down as they are mentally thought of.

To check our answer let us see our quotient and remainder satisfy the relation.

Dividend = Divisor × Quotient + Remainder.

Here Divisor × Quotient + Remainder = 595 × 8 + 3 = 4763 which is the Dividend.

∴ we are correct.

Exercise—Oral.

1. How often is 7 contained in 35; 40; 80; 102? What is the remainder in each case?

2. How often can 12 be subtracted from 100; 150; 178; 200? What will be left over in each case?

3. If you distribute 130 annas among a number of beggars so that each gets 4 annas, how many beggars will be paid and what sum will remain?

4. By what number should you multiply 8 to get 32; 48; 56?

5. In a yard, how many 5 inches have you, and how much is left over?

6. What number should be divided by 8 to get 40 as quotient and 4 as remainder?

7. How many numbers less than 100 are exactly divisible by 8? Which is the greatest of such numbers?

8. A mull piece is 20 yds. long. How many cloths each 9 ft. long can be torn from it and what length of cloth will be left behind?

Exercise VII (a).—Practical and Symbolic.

1. Show graphically that (1) $25 \div 5 = 5$

   (2) $32 \div 4 = 8$

   (3) $33 \div 3 = 11$.

2. Find graphically the quotient and the remainder when (1) 17 is divided by 4, (2) 28 by 5, (3) 43 by 9.

3. What is the value of x if, when it is divided by 9, it gives 256 as quotient and 8 as remainder?

4. Find the quotient and the remainder when 83,245 is divided by 6.

5. Dividend = quotient × divisor + remainder may be written in the form : $D = q \times d + r$:
find the value of D if

(a) $\left.\begin{aligned} q &= 4 \\ d &= 3 \\ \text{and } r &= 2 \end{aligned}\right\}$ (b) $\left.\begin{aligned} q &= 5 \\ d &= 4 \\ \text{and } r &= 3 \end{aligned}\right\}$

6. In the same relation find q and r

   (a) if $D = 117$
   $= 8$.

   (b) if $D = 230$
   $d = 16$.

7. In the same result find dq if $D = 224$ and $r = 8$; and if d is 12, what is q? and if d is 18 what is q?

8. In the same result find dq if $D = 260$ and $r = 4$;

   and (a) if $q = 8$ find d

   (b) if $d = 16$ find q.

9. Divide 8325 by (1) 3; (2) 4; and check your result in each case.

10. Divide 62,750 by (a) 8; (b) 7; (c) 9; and prove the result to be true in each case.

11. There are 12 inches in a foot. How many feet are there in 4050 inches and how many inches over?

12. A vessel can contain 4 measures of water. How often have you to use it in order to empty a reservoir which holds 1250 measures of water.

§ 58. **Division by factors.** Suppose we wish to know how many times a yard is contained in 1793 inches. A yard equals 36 inches and ∴ the question means how many times 36 in. are contained in 1793 in., *i.e.*, we have to *divide* 1793 *by* 36.

This operation by the process of division is somewhat difficult. To simplify the operation, you first find how many feet (or 12 inches) are contained in 1793 in. which is done by dividing by 12, the answer being 149 ft. and 5 in. left over; and then find how many yds. (or 3 ft.) are contained in 149 ft., the answer being 49 yds. and 2 ft. left over. Thus the number of yds. in 1793 in. is 49 and 2 ft. 5 in. or $2 \times 12 + 5$ or 29 in. are left over. Thus the division of 1793 by 36 is effected by first dividing by 12 and then by 3, and we get 49 as the quotient and 29 as remainder.

12 and 3 are factors of 36 and division by 36 is thus shown to be equivalent to the successive division by its factors 12 and 3.

The work is shown in the margin; you notice how the total remainder 29 was arrived at; the remainder 2 corresponding to the divisor 3 was multiplied by 12, the previous divisor, and the product added to the first remainder.

$$\begin{array}{r l} 12) & \underline{1793} \\ 3) & \underline{149-5} \\ & 49-2 \end{array}$$

Similarly *divide* 5227 *by* 192 *by factors.*

The work is shown in the margin and the student is advised to regard this as the problem of finding how many rupees are contained in 5227 pies and how many pies are left over, and thus understand the meaning of the several steps. The quotient is 27 and the remainder is $12 \times 3 + 7$ or 43.

$$\begin{array}{r l} 12) & \underline{5227} \\ 16) & \underline{435-7} \\ & 27-3 \end{array}$$

It is immaterial in what order the factors are used as divisors. Thus the very same question is done in the margin by changing the order of the factors; the quotient is 27 and remainder $= 2 \times 16 + 11 = 43$.

$$\begin{array}{r} 16\underline{)\,5227} \\ 12\underline{)\,326}—11 \\ 27—2 \end{array}$$

The rational of the method of calculation of the remainder may be shown thus. In dividing 5227 by 12 we get 435 as quotient and 7 as remainder. $\therefore 5227 = 435 \times 12 + 7$ (according to the relation $D = d \times q + r$).

435 divided by 16 gives 27 as quotient and 3 as remainder. $\therefore 435 = 27 \times 16 + 3$

$$\begin{aligned} \therefore 5227 &= 435 \times 12 + 7 \\ &= (27 \times 16 + 3) \times 12 + 7 \\ &= 27 \times 16 \times 12 + 3 \times 12 + 7 \\ &= 27 \times 192 + 3 \times 12 + 7 \end{aligned}$$

$\therefore$ in dividing 5227 by 192 the quotient is 27 and the remainder is $3 \times 12 + 7$ or 43.

Similarly if a divisor has 3 factors, the division may be effected by dividing successively by the factors and the remainder calculated as before.

Example.—Divide 18,493 by 315.

$315 = 9 \times 7 \times 5$ and the work stands thus:

$$\begin{array}{r} 9\underline{)18{,}493} \\ 7\underline{)2{,}054}—7 \\ 5\underline{)293}—3 \\ 58—3 \end{array}$$

$\therefore$ The quotient is 58 and the remainder is $(3 \times 7 + 3) \times 9 + 7$ or 223.

The method of division by factors and calculation of the remainder is illustrated graphically thus:—

Divide 117 by 21 with factors 7 and 3

The accompanying figure contains 117 small squares arranged in 16 vertical columns of 7 squares each with 5

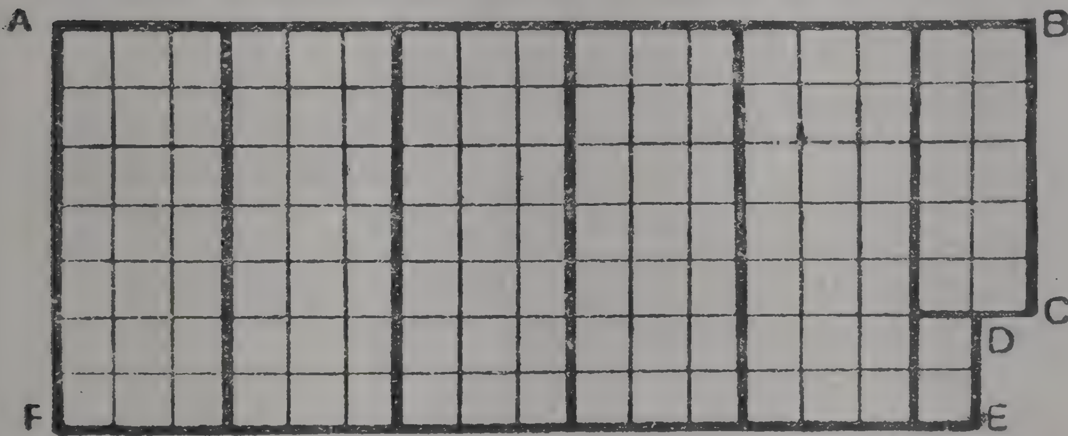


FIG. 65.

squares left in the extreme right-hand column showing the division by 7. Again these 16 vertical columns are arranged in 5 groups of three (marked by thick lines in the figure) leaving 1 vertical column of 7 squares to the right showing division of 16 by 3. This shows that when 117 is divided by 21 by the method of factors first by 7 and then by 3, the quotient is 5 and the remainder is $1 \times 7 + 5$ or 12.

Exercise VII (b).

1. Divide 26,048 by 5; 6; 8; 11; 12.

2. Divide 32,638 by 15; 18; 21; 24; using factors. Check your result by changing the order of factors.

3. Divide 36,009 by 32; 35; 44; 63. Check your result as in 2.

4. Divide 27,843 by 84; 96; 105; 112. Check as in 2.

5. Divide 680,729 by 135; 168; 180; 192.

Exercise—Practical and Graphical.

1. Take a length representing 73 and find the number of times a distance representing 7 can be marked off on this. Find what is left over.

2. Divide graphically 92 by 12.

3. Take a length representing 85. Find how many lengths each representing 3 can be marked off on this. Then take 5 such groups together. How many groups each representing 15 are there? and what is left over?

4. Similarly divide 93 by 15 and 21.

5. The accompanying figure contains 158 squares. Divide it

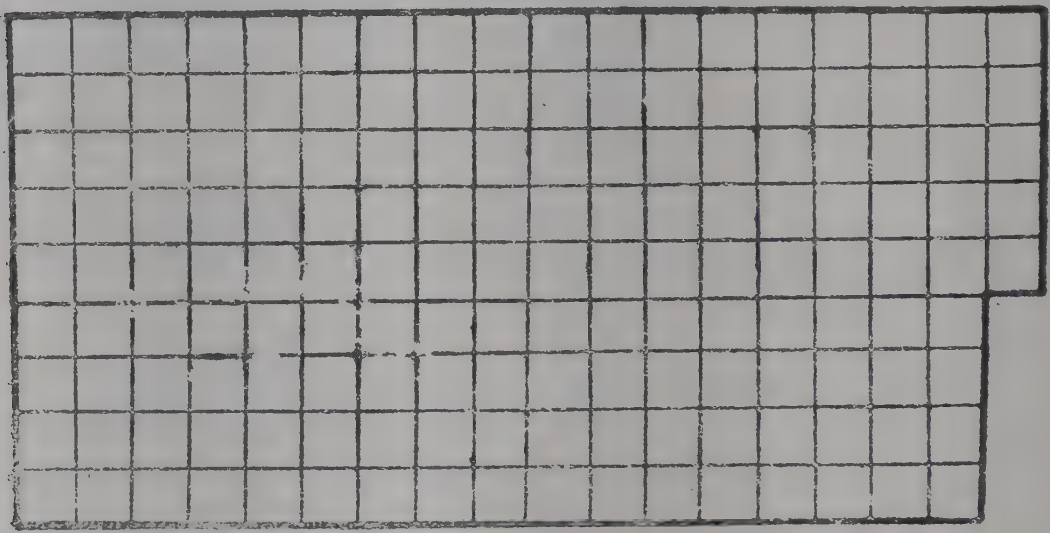


FIG. 66.

up so as to show that when 158 is divided by 27, by the method of factors first by 9 and then by 3, the remainder is of the form $2 \times 9 + 5$ and is equal to 23. What is the quotient?

6. Take a piece of squared paper and cut off from it 158 squares so as to show that when it is divided by 27 first by 3 and then by 9 the remainder is of the form $3 \times 7 + 2$ and equal to 23.

7 Draw figures as in (5) to show the form of the remainders, when 259 is divided by 28; 32; and 35.

8. Draw a line 5 inches long and see how many centimetres and millimetres there are in it.

9. Draw a line 10 cm. long and see how many inches and tenths of an inch there are in it.

10. To measure a pole, a stick 5 ft. long was used 7 times, and a length 2 ft. was left over. Rrpresent this graphically. [Take 1 inch to represent 10 ft.]

§ 59. Division by multiples of 10.

Example.—Divide 8374 by 10.

8374 = 8 thousands + 3 hundreds + 7 tens + 4 units.

When divided by 10, thousands become hundreds, hundreds become tens and tens become units.

∴ 8374 ÷ 10 = 8 hundreds + 3 tens + 7 units with 4 units left over;

∴ the quotient is 837 and the remainder is 4.

∴ the effect of dividing a number by 10 is to decrease the value of each digit to a tenth part of what it was. We may strike off the last figure to the right; the rest will be the quotient; the figure so struck off will represent the remainder

Similarly to divide by 100, 1000, etc., we strike off 2, 3, etc., figures to the right; the figures so struck off will be the remainder in each case.

Example.—Divide 748 by 200. The work may be arranged thus :—

$$2\cancel{0}\cancel{0})77\cancel{4}\cancel{8}$$
$$\overline{38-148}$$

striking off 48 amounts to division by 100. And the quotient 77 is divided by 2. The resulting quotient is 38 and 1 is the remainder and stands for 100. The total remainder is thus 148 as is shown in 'division by factors.'

Exercise—Oral.

1. Divide 873 by 10, 100.
2. Divide 8736 by 100, 1000.
3. Divide 9831 by 200, 300, 400.
4. Divide 6843 by 800, 700, 900.
5. Divide 4860 by 160, 240.

§ 60. Long Division.

When the divisor is above 12 or 16 and when it cannot be resolved into factors the process known as **Long Division** is employed. The method is best seen from the following example :—Divide 38,465 by 237.

Here 237 does not divide 3 or 38, so we take 384 and it is greater than 1 × 237 and less than 2 × 237. But 384 itself stands for 384 hundreds and it lies between 100 × 237 and 200 × 237 ∴ we first subtract

```
       162
237)38,465
    23,700
    ------
    14,765
    14,220
    ------
       545
       474
    ------
        71
```

100×237 or 23700 from the number and the remainder is 14765; again 237 does not divide 147 but goes into 1476 which stands for 1476 tens and is greater than 6 tens $\times 237$, and so we subtract 60×237 or 14220 from 14765 and the remainder is 545 which is greater than 2×237, so we subtract 2×237, *i.e.*, 474 from 545 and the remainder is 71 which is less than 237 and cannot be divided. Here 237 is subtracted successively $100 + 60 + 2$ times or 162 times and the remainder is 71 $\therefore$ the quotient is 162 and the remainder 71.

$$\begin{array}{r} 162 \\ 237)\overline{38465} \\ \underline{237} \\ 1476 \\ \underline{1422} \\ 545 \\ \underline{474} \\ \underline{71} \end{array}$$

Thus once again it will be seen that in division we have only applied repeated subtraction of the divisor. The work is generally condensed as shown in the margin. Here the zeroes are omitted and figures from the dividend are brought down only as they are wanted. In the first stage of the division, we took 384 hundreds and therefore the 1st figure of the quotiet represents hundreds and this is indicated by placing it above the place of hundreds, *i.e.*, over the last figure of the portion of the dividend taken in the 1st stage, and the other figures of the quotient will then follow in their proper places.

Also it will be seen that this method known as *long division* is the same in principle as the method of short division described in Art. (57). The only difference is that, in long division, since the multiplication and subtraction cannot be done mentally as the divisor is large, we write them down in detail. The writing work shown in the last para is still further abbreviated by giving only the re

mainders in the process as explained below. This is known as the *Italian method* of division.

Example.—Divide 82,787 by 349.

Ordinary method.

```
       237
349)82787
    698
    1298
    1047
     2517
     2443
       74
```

Italian method.

```
       237
349)82787
    1298
     2517
       74
```

The mental process in the Italian method is as follows twice 9 = 18, and **9** = 27 carry 2 ;

twice 4 + 2 = 10 and **2** = 12 ; carry 1 ;

twice 3 + 1 = 7, and **1** = 8.

The figures **9**, **2**, **1** given in large type are set down as they are thought of. This gives the first partial remainder 129. Bring down 8 and proceed as before.

N.B.—It is difficult to detect errors in the Italian Method of Division ; so, it is not recommended to beginners.

Exercise VII (c).

Find the quotient and the remainder in the following cases :—

1. When 83,453 is divided by (*a*) 23 (*b*) 57 (*c*) 121 (*d*) 483 (*e*) 598.
2. When 56,754 is divided by (*a*) 21 (*b*) 53 (*c*) 128 (*d*) 609 (*e*) 799.
3. When 183,264 is divided by (*a*) 123 (*b*) 324 (*c*) 489 (*d*) 1,084.
4. When 1,082,645 is divided by (*a*) 547 (*b*) 348 (*c*) 2,468 (*d*) 2,567.
5. When 199,999 is divided by (*a*) 3,047 (*b*) 2,086 (*c*) 3,999 (*d*) 8,444.

§ 61. Another aspect of Division. A number is said to be **concrete** when it is applied to definite objects and **abstract** when it is thought of apart from any object ; 7 books, 5 mangoes are concrete, whereas the mere numbers 7 and 5 without any reference to the unit are abstract.

[Probably it would be better to speak of abstract numbers and concrete *quantities* rather than of abstract and concrete *numbers*.]

We have defined multiplication as repeated addition, the multiplier denoting the number of repetitions. Therefore the multiplier must denote an abstract number; otherwise the definition of multiplication becomes meaningless, *e.g.*, it is meaningless to say 'multiply 5 Rs. by 3 books,' *i.e.*, 'repeat 5 Rs. 3 books times and add.' The multiplicand may be either abstract or concrete, and the product will be of the same kind as the multiplicand. Thus if a product consist of two factors a and b, one of the factors a or b must be abstract and the other concrete or abstract according as the product is concrete or abstract.

Now in division we have seen that divisor × quotient = dividend when there is no remainder, *i.e.*, dividend is a product and the divisor and the quotient are its factors. The process of division therefore may be taken to mean "given the product (*viz.*, the dividend) and one of the factors (*viz.*, the divisor) find the other factor." Thus it may be understood as the **reverse of Multiplication.**

The dividend may be abstract or concrete. If it be abstract both the divisor and the quotient must be abstract. But if concrete, the divisor may be (1) concrete and then the quotient will be abstract or (2) abstract and the quotient will then be concrete and of the same kind as the dividend. In the first case the division may be defined, as we have already done, to be the process of finding how many times the divisor is contained in the dividend, *e.g.*, to divide 12 in. by 3 in. means to find how many times the length 3 in. is contained in the length of 12 in., and the quotient is thus abstract. In the 2nd case division may be looked upon as **the process of breaking a given quantity into a given number of equal parts**, *e.g.*, to divide 12 in. by 3, means to break the length of 12 in. into three equal parts, and the quotient here is concrete.

Exercise—Practical and Graphical

1. Draw a straight line, guess its middle point and mark it. Test your guess by measuring the two parts.

2. Repeat Ex. 1 three or four times with lines of various lengths. Show by a table how far you are wrong.

3. Draw a straight line 3·2 in. long; bisect it (*i.e.*, divide it into 2 equal parts) by calculating the length of half the line and measure off that length from one end of the line; measure the remaining part.

4. Divide a straight line 10 cm. long into 4 equal parts.

5. Draw a line 4·2 cm. long and divide it into 8 equal parts.

* § 62. Laws of Multiplication and Division.

1. To prove that $36 \div 6 \times 2 = 36 \times 2 \div 6$.

[If we have a series of numbers connected by the signs $\div$ and $\times$ the operations are to be performed one after another from left to right. $36 \div 6 \times 2$ means that 36 is to be divided by 6 and the result multiplied by 2; and $a \times b \div c$ means that first a is to be multiplied by b and the result to be divided by c.]

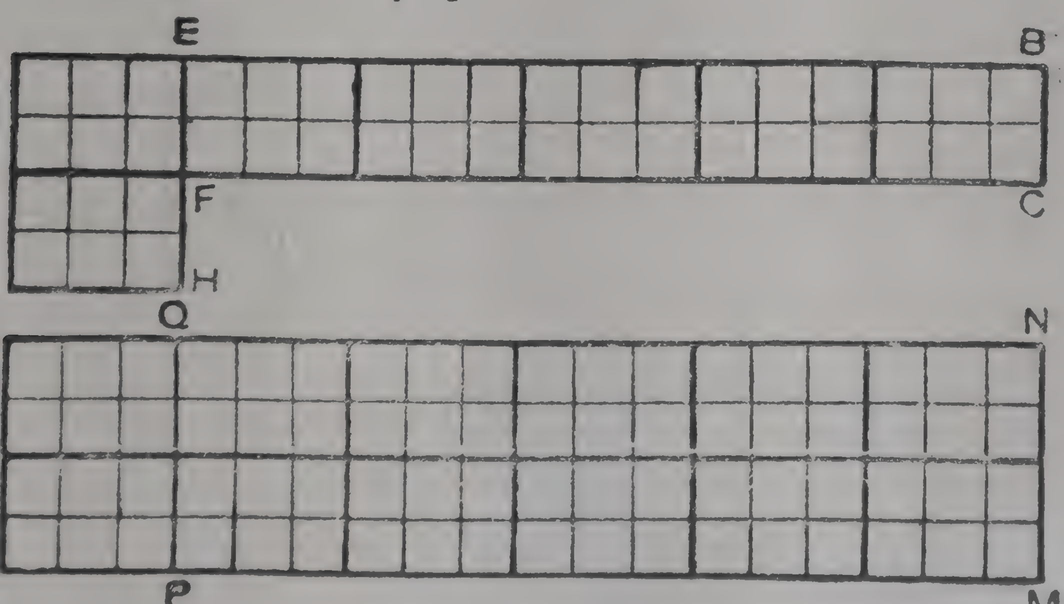


FIG 67.

In Fig. 67, ABCD represents 36. AEFD represents $36 \div 6$ and AGHE represents $36 \div 6 \times 2$.

In the same figure KLMN represents 36×2.

KLPQ represents $36 \times 2 \div 6$.

Now AGHE and KLPQ are equal

$\therefore\ 36 \div 6 \times 2 = 36 \times 2 \div 6.$

Or in general $\mathbf{a} \div \mathbf{b} \times \mathbf{c} = \mathbf{a} \times \mathbf{c} \div \mathbf{b}$, *i.e.*, in a series of mutiplications and divisions the order in which the operations are conducted does not affect the result.

To show graphically that $30 \div 3 \div 2 = 30 \div 2 \div 3$ or, in general, $\mathbf{a} \div \mathbf{b} \div \mathbf{c} = \mathbf{a} \div \mathbf{c} \div \mathbf{b}$.

ABCD is a rectangle containing 30 squares as in (Fig. 68), EG and FH divide the rectangle into 3 equal

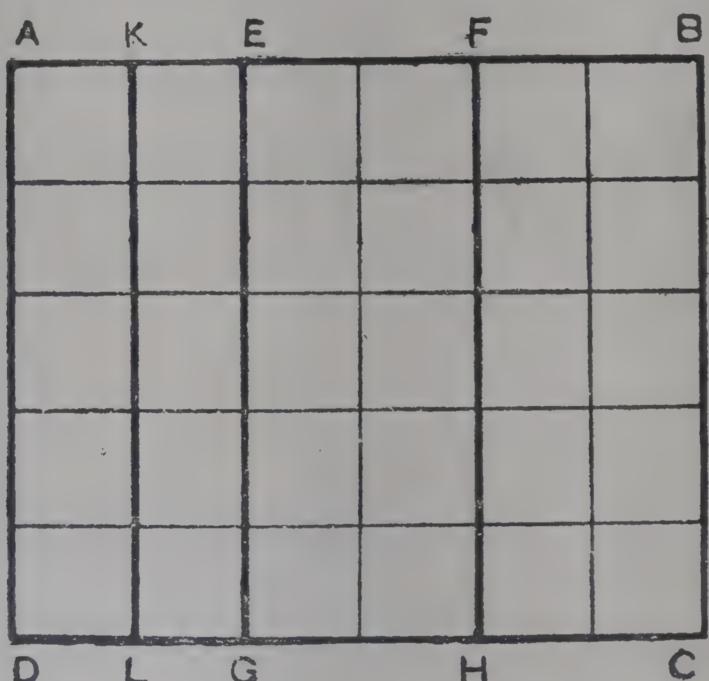


FIG. 68.

parts. Thus AEGD represents $30 \div 3$. Again KL divides AEGD into two equal parts and therefore AKLD represents $30 \div 3 \div 2$.

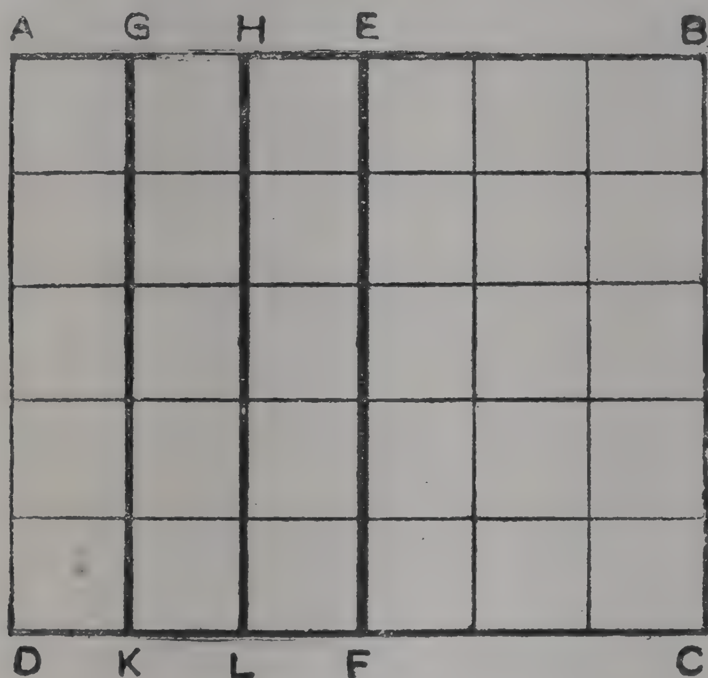


FIG. 69.

Similarly, dividing the rectangle first into two equa parts by EF as in (Fig. 69) and dividing the figure AEFD into 3 equals parts by HL and GK we get the rectangle AGKD to represent $30 \div 2 \div 3$.

Since the two rectangles AKLD (Fig. 68) and AGKD (Fig. 69) are equal.

$\therefore$ $30 \div 3 \div 2 = 30 \div 2 \div 3$: which shows that in performing a series of operations of divisions the result is not affected by the order in which the operations are conducted.

To show graphically that $12 \div (3.2) = 12 \div 3 \div 2$ or in general $\mathbf{a} \div (\mathbf{bc}) = \mathbf{a} \div \mathbf{b} \div \mathbf{c}$.

Here 3. 2 within brackets means that 3 and 2 must be

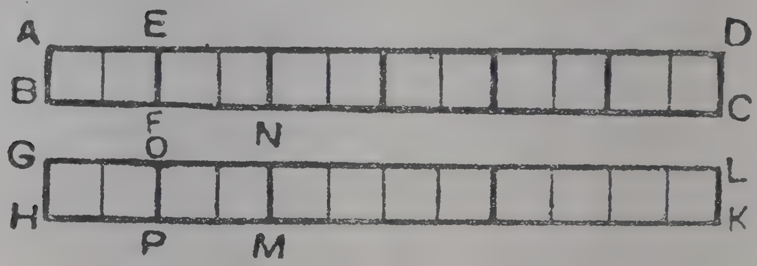


FIG. 70.

multiplied together before any other operation is performed. So we have to prove that $12 \div 6 = 12 \div 3 \div 2$. In (Fig. 70) ABCD represents 12, ABFE represents $12 \div 6$.

Again GHKL represents 12

GHMN.............. $12 \div 3$

GHPO................ $12 \div 3 \div 2$.

Now ABFE and GHPO are equal

$\therefore \; 12 \div (3 \times 2) = 12 \div 3 \div 2$.

Note.—These general results are arithmetically intelligible to us at present only if the division be exact.

The arithmetical proofs of these in such cases are easy and are left as exercise to the student.

Exercise—Practical and Graphical.

1. Show (1) $40 \div 5 \div 2 = 40 \div 2 \div 5$; (2) $48 \div 6 \div 2 = 48 \div 2 \div 6$; (3) $54 \div 9 \div 3 = 54 \div 3 \div 9$.

2. Prove that $12 \times (3 \times 2) = 12 \times 3 \times 2$ or in general that $\mathbf{a(b.c) = a.b.c}$.

3. Show graphically that $12 \div (4 \div 2) = 12 \div 4 \times 2$ or in general that $\mathbf{a \div (b \div c) = a \div b \times c}$.

4. Show graphically that $5 \times (4 \div 2) = 5 \times 4 \div 2$ or in general that

$$\mathbf{a\,(b \div c) = a \times b \div c}.$$

§ 63. Some shortened methods of Multiplication and Division.

Example 1. Multiply 256 by 25

$$256 \times 25 = 256 \times (100 \div 4) = 256 \times 100 \div 4$$
$$= 25600 \div 4 = 6400,$$

i.e., to multiply a number by 25, affix 2 ciphers to the number and divide the result by 4. Similarly to multiply a number by 5, affix 1 zero and divide by 2.

Example 2. Multiply 8756 by 125

$$8756 \times 125 = 8756 \times 1000 \div 8$$
$$= 8756000 \div 8$$
$$= \mathbf{1094500},$$

i. e., to multiply a number by 125, affix 3 ciphers and divide the result by 8.

Similarly to multiply a number by 625, add 4 ciphers and divide the result by 16 and so on.

Example 3. Multiply 2564 by 75

$$2564 \times 75 = 2564 \times 3 \times 25$$
$$= 769200 \div 4$$
$$= 192300.$$

Example 4. Multiply 2589 by 375

$$2589 \times 375$$
$$= 2589 \times 3 \times 1000 \div 8$$
$$= 7767000 \div 8 = 970875.$$

Example 5. Divide 1876 by 25

$$1876 \div 25 = 1876 \times 4 \div 100 = 7504 \div 100$$

and 75 is the quotient.

To divide a number by 25, multiply the number by 4 and strike off the last two figures; the rest will be the quotient.

Example 6. Divide 18765 by 125

$$18765 \div 125 = 18765 \times 8 \div 1000$$
$$= 150120 - 1000$$

and 150 is the quotient.

To divide a number by 125, multiply the number by 8 and strike off the last three figures, the rest will be the quotient.

Similarly to divide a number by 625, multiply by 16 and strike off 4 figures, the rest will be the quotient.

**Example* 7.* Divide 1487653 by 99

[100 = 99 + 1 and

1487600 = 14876 × 99 + 14876 units]

Now 1487653

= 1487600 + 53
= 14876 (99's) + 14876 units + 53 units
= 14876 (99's) + 14800 + 76 + 53
= 14876 (99's) + 148 (99's) + 148 + 76 + 53
= 14876 (99's) + 148 (99's) + 100 + 48 + 76 + 53
= 14876 (99's) + 148 (99's) + 1 (99's) + 1 + 48 + 76 + 53
= (14876 + 148 + 1) (99's) + 1 + 48 + 76 + 53
= 15025 (99's) + 178
= 15026 (99's) + 79

∴ when 1487653 is divided by 99, the quotient is 15026 and 79 is the remainder.

The work is easily effected as shown in the margin. By carefully noticing the work in the margin and working similar questions the student can easily deduce the rule for working such questions, *i. e.*, for dividing by 999, &c.

1487653 by 99	14876	53
	148	76
	1	48
		1
	15025	178
	15026	79

Exercise VII (d).

1. Multiply 18,625 by (*a*) 25 (*b*) 75 (*c*) 125 (*d*) 375.
2. Divide 68,734 by (*a*) 25 (*b*) 125.
3. Divide 83,7656 by (*a*) 99 (*b*) 999 (*c*) 9999, and check your answer according to the method of Art. 63.
4. Multiply 86,7345 by 99 and divide the product by 999.
5. Multiply 68,764 by 625 and 875.

***§ 64. Tests of Divisibility.** If we examine the multiplication table for 2, we find that the products end in 0, 2, 4, 6, or 8 and **therefore any number ending in** 0, 2, 4, 6, or 8 **must be divisible by 2.**

Numbers which when divided by 2, leave no remainder are said to be **even**; whereas those which when divided by 2, leave a remainder are said to be **odd**.

To test whether a number is divisible by 4 or 25, we have to examine the last two digits of the number, *e. g.*, take the number 684, it equals $600 + 84$. Now any multiple of 100 like 600 is always divisible by 4 or by 25. If the last two digits like 84 in this case be divisible by 4 or 25 the whole number will be divisible by 4 or 25.

Similarly to test whether **a number is divisible by 8 or 125,** we have to examine the last 3 digits, *e.g.*, take the number 2834; it equals $2000 + 834$; any multiple of a thousand is divisible by 8 or by 125. The last 3 figures alone like 834 in this case should be tested; therefore a number is divisible by 8 or by 125 if the last 3 digits are divisible by 8 or by 125.

To find when a number is divisible by 9 or 3.

Take any number 857.

$857 = 800 + 50 + 7$

Also $800 = 8 \times 100 = 8(99 + 1) = 8 \times 99 + 8$,

i.e, a multiple of $9 + 8$.

$50 = 5 \times 10 = 5(9 + 1) = 5 \times 9 + 5$

$=$ a multiple of $9 + 5$.

$7 = 7$

$\therefore$ $800 + 50 + 7$ or $857 =$ a multiple of $9 + 8 + 5 + 7$

$=$ a multiple of $9 +$ the sum of its digits.

Similarly every number can be shown equal to a multiple of $9 +$ the sum of its digits; and may be symbolically written $9m + s$ where m is an integer and s is the sum of the digits.

$\therefore$ A number will be divisible by 9 if the sum of its digits be divisible by 9. In this case $8 + 5 + 7$ or 20 is not divisible by 9 and the number is not divisible by 9. Take

again the number 783; it is divisible by 9 because the sum of the digits is 18 and is divisible by 9. The same test applies to 3 for it is a factor of 9.

Example 1. Find the remainder when 8246 is divided (1) by 9 (2) by 3.

When a number is divided by 9 or 3, the remainder is the same as when the sum of its digits is divided by 9 or 3. The sum of the digits = 20. This when divided by 9 or by 3 leaves a remainder 2.

Check by actual division as follows:

$$9 \,|\, \underline{8246} \qquad\qquad 3 \,|\, \underline{8246}$$
$$\;\;916—2 \qquad\qquad\quad 2748—2$$

A number is divisible by 6, when it is divisible by 2 and also by 3, *i.e.*, when it is an even number and when the sum of its digits is divisible by 3.

A number is divisible by 5 if it ends in 5 or 0.

To find when a number is divisible by 11.

$10 = 11 - 1$

$100 = 99 + 1 =$ a multiple of $11 + 1$.

$1000 = 1001 - 1 =$ a multiple of $11 - 1$.

$10000 = 9999 + 1 =$ a multiple of $11 + 1$.

$100000 = 100001 - 1 =$ a multiple of $11 - 1$; and so on.

Now take any number 834,256.

$6 = 6$

$50 = 5 \times 10 = 5\,(11 - 1) =$ a multiple of $11 - 5$.

$200 = 2 \times 100 = 2\,(99 + 1) =$ a multiple of $11 + 2$.

$4000 = 4 \times 1000 = 4\,(1001 - 1) =$ a multiple of $11 - 4$.

$30000 = 3 \times 10000 = 3\,(9999 + 1) =$ a multiple of $11 + 3$.

$800000 = 8 \times 100000 = 8\,(100001 - 1) =$ a multiple of $11 - 8$.

$\therefore$ 834,256 = a multiple of $11 + 6 - 5 + 2 - 4 + 3 - 8$

= a multiple of $11 + (6 + 2 + 3) - (5 + 4 + 8)$

= a multiple of 11 + (the sum of the digits in the 1st, 3rd and 5th, or odd places reckoning from the right) —(the sum of the digits in the 2nd, 4th and 6th, or even places).

Similarly any number = a multiple of 11 + (the sum of the digits in the odd places)—(the sum of the digits in the even places) and is therefore divisible by 11 if the difference between the sum of the digits in the odd places and that in the even places is zero or divisible by 11.

In the case of 834,256 the difference is 6 which is not divisible by 11, therefore the number is not divisible by 11.

Take again the number 354,123; here the difference is zero and the number is divisible by 11.

**Example* 2.—Find the remainder when 8764 is divided by 11.

$$\left.\begin{array}{r}\text{Now the sum of the digits in the even places } 8 + 6 = 14\\ \text{the sum of the digits in the odd places } 7 + 4 = 11\end{array}\right\}$$

$\therefore$ the number = a multiple of 11—3. The required remainder is 11 — 3 or 8.

§ **65. Casting out the nines.** For checking the results of multiplication and division the method of **casting out the nines** must be employed. It is as follows:—

Consider the number 345. 345 = a multiple of 9 + sum of the digits = a multiple of 9 + 12, or simpler, a multiple of 9 + 3. Thus by removing multiples of 9 we find the remainder to be 3. This is called *casting out the nines* and the remainder is called the *nine remainder.*

Similarly 463 = some multiple of $9 + (1 + 6 + 3)$ = some multiple of 9 + 4, and 4 is the nine remainder.

In the number 4,927,918, the nines may be cast out thus: remove the two nines first, thus, 4~~9~~27~~9~~18; then proceed with the sum of the digits, taking care to cast out a nine when the sum exceeds 9. Thus $4 + 2 + 7 = 13 = 9 + 4$. Cast out 9; then $4+1+8=13=9+4$. Cast out 9. The result is 4.

By casting out nines as shown above you see that 345 = (a multiple of $9 + 3$) and 463 = (a multiple of $9 + 4$) $\therefore$ their product must be equal to some multiple of $9 + 12$, *i.e.*, may be expressed as $9a + 3$ where $9a$ is a multiple of 9.

To test the correctness of the product you have got by multiplying them, cast out the nines in the product; if the nine remainder is 3 the product *may be correct*; if not, it is *certainly* wrong.

Notice that the answer *may be* correct; for the proof fails to detect errors in the product (*a*) if the order of figures in the product be changed; as 453 for 345; or (*b*) if the digits be so changed that their sum may differ from the sum of the digits of the given number by multiples of 9; as 642 for 345; 795 for 345.

For *checking* divisions; cast out the nines from the quotient and the divisor; multiply the results, cast out again; add the result to that obtained by casting out from the remainder; cast out again. The result must be what is obtained by casting out from the dividend.

Exercise.—Oral.

1. In what digit must a number end to be divisible by 5?

2. In what digits must a number end to be divisible by 25? State all the numbers from 1 to 100 divisible by 25.

3. In what digits must a number end to be divisible by 125? State all numbers divisible by 125 from 1 to 1000.

4. What is the least number which must be taken from 60 so that the remainder may be divisible by 8; 7; 9?

5. What is the least number which must be added to 53 so that the sum may be divisible by 8; 6; 7; 9?

Exercise VII (e).

1. Show that 644, 1,024, 3,848, 26,284, 132,864, 13,284, are exactly divisible by 4; which of these are divisible by 8 and which by 9?

2. Find the remainder when 86,345 is divided (1) by 4, (2) by 25.

3. Examine if 1,876,875 is divisible by 125. What is the remainder when 2,376,425 is divided by 125?

4. Find the remainder when the difference between 2875 and 1328 is divided by (1) 4, (2) 25; also when the sum is divided by (3) 9, (4) 25.

5. In 176, * 34 a digit is omitted in the place marked by the asterisk. What should be that digit so that the number may be divisible by 9?

6. In 1,7*a*2,34*b*, two digits are *a* and *b*. The number leaves a remainder 23 when divided by 25 and a remainder 1 when divided by 3. Find *a* and *b*.

7. In example 6 if the number is divisible by 4 and 9, find the possible values of *a* and *b*.

8. Supply the missing figures in the following example on divison worked by the Italian method.

```
         1 * * = Quotient.
197) * * * * *
       9 6 7
       1 7 9 5
           2 2 = Remainder.
```

9. A number is divided by another number by the method of factors thus:

```
7 |____________
8 |____________ remainder 2,
   1361 remainder 7.
```

Find the dividend.

10. A number is divided by another number by the method of factors thus

$$\begin{array}{l|l} a & \ldots\ldots \\ \hline b & \ldots\ldots \quad \text{remainder } 4 \\ \hline & \ldots\ldots\ldots\ldots\ldots \text{remainder } 7, \end{array}$$

If the total remainder is 60. What is a ?

11. In the number 78,32a, 856 what must be the value of a in order that it may be divisible by 11 ?

12. Multiply the following, and test the accuracy of your work by casting out the nines :—

(*a*) 3684 × 4789. (*b*) 4673 × 1001.

(*c*) 1782 × 964. (*d*) 2384 × 4384.

***§ 66. Division correct to some significant figures.** Frequently in division, we are not concerned with the remainder and we want the quotient only to some significant figures (say, to the nearest unit or ten).

Example.—The area of the Tanjore District is 3710 sq. miles and the population is 2,245,009. Find the population per square mile.

Here we have to divide 2,245,009 by 3710 and find the quotient correct to a unit. Actual division gives 605 as the quotient and 459 as the remainder. Since 459 is less than one half of 3170, the required answer correct to a unit is 605.

Exercise VII (f).

1. In a house of 12 rooms each room has 4 windows and each window has 14 panes. How many panes are there in all ?

2. A book contains 396 pages ; on every page there are 40 lines of print, and every line contains on an average 10 words. How many words are there in the book ?

3. In a garden there are 52 rows of plantain trees, each row containing 57,48 rows of mango trees each row having 37, and 68 rows of tamarind trees each row containing 39. Find the total number of trees in the garden.

4. If, in a class, there are 39 boys, and if each boy has to buy 12 note-books, each note-book containing 108 pages; what is the total number of pages in the note-books of all the boys?

5. Find the value of 280 tram cars of which 132 are worth Rs. 2,525 each and the rest Rs. 1,725 each.

6. Out of every 48 months, 28 contain 31 days each, 16 contain 30 days and 1 contains 29, the rest 28. Find the total number of days in a period of 48 months.

7. A man had Rs. 10,250 in the Madras Bank at the beginning of a certain year. He withdrew Rs. 869, 963, 1041. 1033, on four occasions respectively in the second year, and Rs. 80, five times in the third year, and also deposited Rs. 70, eight times in the same year. What has he left in the bank at the beginning of the fourth year?

8. On the eve of a certain battle an army consisted of 100,000 officers and men; after the fight it was found that the number killed were 12,384 men and 648 officers, that the number wounded were 8,926 men and 650 officers and that there were 300 men and 10 officers missing. How many men in all were there in the army after the battle?

9. How often could 529 be subtracted from 83,765 and what would be the final remainder?

10. A battalion 1,088 strong contains 4 companies of equal strength. Each company is divided into 4 equal sections, each section into 2 squads. How many men are there in each company, section, and squad?

11. A terraced house 43 ft. 6 inches broad is supported on 6 round pillars placed at equal intervals, the diameter of each being 1 ft. 6 inches. Two of the pillars are at the two ends of the house. What is the distance between the two consecutive pillars from centre to centre?

12. The lengthwise wall of a hall is 28 ft. It is proposed to hang pictures from one end of the wall to the other end in a straight row without leaving any interspace between the pictures. There are already 3 pictures whose frames are each 4 ft. wide. How many more pictures with frames each 2 ft. wide must be purchased for the purpose?

13. A man deals 208 cards to 11 people. How many will get 18 and how many 19 ?

14. A train starting from Madras at 6 A.M., reaches Arkonam (42 miles from Madras) at 7-45 A.M. At what rate per hour does it travel ?

15. The income of a railway company is Rs. 63,83,297 in a certain year and the expenses amount to Rs. 53,75,147. If the remainder is divided equally among the holders of 201,630 shares, what does the owner of each share get ? (answer correct to a pie).

16. A town receives every week two consignments each containing 5684 rice bags of 49 measures each. If each inhabitant consumes 4 measures every week with the exception of 872 people who take only 1 measure of rice every week, what is the population of the town ?

17. Three heaps of bricks containing 5840, 3670, and 10389 are to be cleared away in cart loads of 480 bricks each. How many cart loads with all the three heaps together make, and how many will be outstanding ?

18. It has been found that the total income arising from running a number of special trains from Madras to Tiruttani for a festival is Rs. 9,600. Each special carries 20 full carriages of 6 compartments each accommodating 10 persons. If each person bought a return ticket for 1 rupee, how many special trains were run by the company ?

19. The cost of a pair of solid bangles of sovereign gold is Rs. 477. The cost of making is Rs. 12. If a sovereign is worth Rs. 15, what is the weight of each bangle ?

20. The distance between the sun and the earth is 93 millions of miles. If light travels at the rate of 186,000 miles per second, what time will a ray of light take to reach the earth from the sun ?

21. A gun is fired at a distance of 1760 yards from an observer. If sound travels at the rate of 1,100 ft. per second, what time lapses between his seeing the flash and hearing the report ?

22. A sum of Rs. 4,350 was spent by the manager of a school in making reversible benches, each accommodating 4 pupils and

costing Rs. 15, for all the classes in the school, providing for a maximum strength of 40 in each class; how many classes were there in the school ?

23. The total collections of a certain electric tram car company are Rs. 38,304 per week. The company owns a number of cars each car running 18 times a day and earning Rs. 2 in each course of running. Find the number of cars running every day.

* **24**. The equator, which contains 360 degrees, is approximately 24,850 miles long. How many miles are there in a degree at the equator ?

* **25**. The total charges of the maintenance of the Medical College, Madras, in the year 1909-10 were Rs. 86,180 of which Rs. 31,474 was met from fees. If the strength of the College was 325, calculate the net cost of training each student.

* **26**. The number of members in the Triplicane Co-operative Stores on 31st December 1909 was 1,539, and the total sales in the 2nd half-year of 1910 amounted to Rs. 2,35,904 (correct to a rupee). Find the amount of sales per member (correct to a rupee).

* **27**. The number of shares in the same society for the 2nd half-year 1910 was 2,729 and the total dividend declared to be divided among the shareholders was Rs. 1607. Calculate the dividend which each share gets correct to a pie.

* **28**. The population of Travancore, according to the Census of 1901, was 29,52,157 and its area is 7,129 square miles. Calculate the population per sq. mile correct to a unit.

* **29**. The quantity of salt of all kinds, passed into consumption in Travancore in 1085, was 9,23,540 maunds. Find the consumption per head of population correct to a lb., assuming the population to be that given in question 28, and taking a maund to be approximately equal to 82 lbs.

* **30**. The total population of the Madras Presidency is 38,199,162; out of this 4,275,178 live in towns. Find how many out of every 1,000 of the total population live in town.

CHAPTER VIII.

COMPOUND QUANTITIES.

Tables of Money.

INDIAN.

12 Pies	make	1 Anna	(*a.*)
16 Annas	,,	1 Rupee	(*Re.*)
15 Rupees (*Rs.*)	,,	1 Pound or Sovereign	(£)

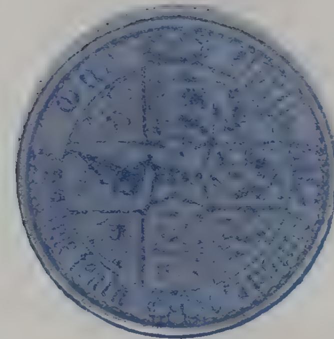
Note 1.—The currency consists in gold, of sovereigns only at present ; in silver, of rupees, half and quarter rupees, and two-anna pieces ; in nickel, of one anna pieces ; and in bronze, of quarter-anna pieces and pies. The copper half-anna pieces are no longer minted.

Note 2.—The sums of money recognised by the Government at the present day for purposes of account are represented by sovereigns and fractions or multiples of the Government Rupee, which is the standard unit of all monetary transactions in India. Sovereigns are legal tender to any amount whatever. Rupees and half-rupees are legal tender for all sums, if they have not lost more than 2 per cent. in weight. The silver coinage is of the standard of 11 parts pure silver and 1 part copper. The nominal or par value of the Government Rupee is 1*s.* 4*d.*, and it weighs 1 tola or 180 grains. The diameters of the silver coins are fixed departmentally at $\frac{3}{4}$ of an inch for the quarter-rupee, $\frac{19}{20}$ of an inch for the half-rupee, and $1\frac{1}{5}$ of an inch for the rupee. The diameter of a quarter-anna piece is 1 inch. The pie is intrinsically worth $\frac{4}{85}d.$ and weighs $33\frac{1}{3}$ grs.

ENGLISH.

4 Farthings (*q.*)	make	1 Penny	(*d.*)
12 Pence	,,	1 Shilling	(*s.*)
20 Shillings	,,	1 Pound or Sovereign	(£)

1 Sovereign = 25'2 Francs.



1 Florin.

½ Sovereign.

1 Shilling.

1 Crown.

1 Penny.

The current coins are:—

Gold :—Sovereign, half-sovereign.

Silver :—Crown or five-shilling piece : four-shilling piece ; half-crown ; florin = 2*s*. ; shilling ; six pence, three pence.

Bronze :—Penny ; half-penny ; farthing.

[*Vide* opposite page for the exact representation of some of these coins as regards form, size, colour, &c.]

Other coins are also mentioned in England, *viz*., a groat = 4*d*. ; a mark = 13*s*. 4*d*. ; a guinea = £1 1*s*. ; a moidore = £1 7*s*.

Note 1.—The English pound is generally called a pound *sterling* which distinguishes it from the weight called a *pound*, and from foreign coins having the same name.

Note 2.—*Pure gold* is gold without the mixture of any other metal. *Alloy* is any base metal mixed with gold (or with any other finer metal). The term *carat* is often used to express the fineness of gold, and, in this sense, is equal to *a twenty-fourth* part. Any quantity of gold is supposed to be divided into 24 equal parts or *carats**; if 22 parts be pure gold and the rest alloy, it is said to be 22 carats *fine*. *Standard* gold is 22 carats fine, *i.e.*, the standard of gold coin is 22 parts of pure gold and 2 parts of alloy (copper) melted together. As 1869 sovereigns are coined from 40 pounds Troy of standard gold, the sovereign weighs $123\frac{1}{4}$ grs.

The silver coinage is of the standard of 37 parts of pure silver and 3 parts of copper. As 66 shillings are coined from a pound Troy of standard silver, the shilling weighs $87\frac{3}{11}$ grs.

As 48 pennies are coined from 1 lb. Avoir. of bronze (95 parts pure copper, 4 parts tin and 1 part zinc), the penny weighs $\frac{1}{3}$ of an oz. Avoir.

FOREIGN.

United States.	1 dollar ($) = 100 cents = 4*s*. 2*d*. nearly.
France.	1 franc = 100 centimes = $9\frac{3}{4}d$.
	£1 = 25 francs and 22 centimes.
Germany.	1 German mark = 1*s*. nearly.
Japan.	1 yen = Re. 1-8-0 nearly.

*The *carat* when employed with reference to the weight of diamonds, stands for $3\frac{1}{5}$ grs.

Measures of Weight.

INDIAN,

(1)—*Madras Government Weights.*

				lbs.	oz.	drs.
3 Tolas......	make	1 Palam	=	0	1	$3\frac{131}{175}$.
8 Palams...	,,	1 Seer	=	0	9	$13\frac{173}{175}$.
40 Palams or 5 Seers	,,	1 Viss	=	3	1	$5\frac{33}{35}$.
8 Viss......	,,	1 Madras Maund	=	24	10	$15\frac{19}{35}$.
20 Madras Maunds...	make	1 Bâram or Candy	=	493	11	$6\frac{6}{7}$.

This Table is used by the Government of Madras for departmental use in all matters other than those having an Imperial bearing.

(2)—*European Mercantile Weights.*

1 Palam = $1\frac{1}{4}$ oz. Avoir.

40 Palams...make 1 Viss = $3\frac{1}{8}$ lbs. ,,

8 Viss...... ,, 1 Maund = 25 lbs. ,,

20 Maunds... ,, 1 Candy or Bâram = 500 lbs.

This Table is that of the Madras Government slightly modified by the English merchants to facilitate conversion into the Avoirdupois of English commerce. It is used by all English merchants and tradesmen, and is recognised and enforced by the Madras Collectorate. Superior native dealers also use it in all the districts.

(3) *Imperial Weights.*

1 Imperial Tola = the weight of 1 Re. = 180 grs.

80 Imperial Tolas make 1 Imperial Seer.

40 Imperial Seers ,, I Imperial Maund = $82\frac{2}{7}$ lbs. Avoir.

This Table is used in the Madras Presidency for purposes of the railway departments, salt administration, and general statistics.

ENGLISH.

Troy or Goldsmiths' Weight.

[This Table is used in weighing gold, silver, jewels and precious stones. It is also used in ascertaining the strength of liquors and in philosophical experiments.]

480 grains (gr.).........make 1 ounce (oz.)

12 ounces................ ,, 1 Pound (lb.)

AVOIRDUPOIS OR GROCERS' WEIGHT.

			M.	Viss.	Pal.
16 Ounces (= 7,000 grs.)...	Make	1 Pound (lb,)	= 0	0	$12\frac{26}{27}$
14 Pounds......................	,,	1 Stone (st.)	= 0	4	$21\frac{18}{27}$
28 Pounds......................	,,	1 Quarter (qr.)	= 0	9	$2\frac{20}{27}$
4 Quarters (112 lbs.)......	,,	I Hundredweight (cwt.)	= 4	4	$11\frac{23}{27}$
20 Hundredweights.........	,,	Ton	= 90	5	$73\frac{1}{27}$

Note 1.—lb. stands for *libra*, Latin for *pound*; the *c* in *cwt.* stands for *centum*, Latin for 100.

Note 2.—In calculations not requiring a very high degree o accuracy, **1 lb. may be taken to be equal to 13 palams**.

FRENCH.

The unit is a *gram* and is the weight of 1 cubic centimetre of pure water at 4°C.

10 milligrams...(mg.)............	make	1 centigram (cg.)
10 centigrams......................	,,	1 decigram (dg.)
10 decigrams......................	,,	1 **gram** (g.)
10 grams..............................	,,	1 decagram (Dg.)
10 decagrams......................	,,	1 hectogram (Hg.)
10 hectograms....................	,,	1 kilogram (Kg.)

1 gram = 15·432 grains

1 kilogram = nearly $2\frac{1}{5}$ lbs.

N.B.—The prefixes, deca-, hecto-, and kilo-, mean 10, 100, and 1000 respectively; and deci-, centi-, and milli-, mean $\frac{1}{10}$ $\frac{1}{100}$ and $\frac{1}{1000}$ respectively. For multiples, the prefixes are Greek, and for sub-multiples, they are Latin.

OLD APOTHECARIES' WEIGHT.

[NOW USED FOR PREPARING MEDICINES].

				Pal.	Tola.
20 Grains.........	make	1 Scruple	(sc. or ℈) =	0	$0\frac{1}{9}$
3 Scruples.........	,,	1 Dram	(dr. or ʒ) =	0	$0\frac{1}{3}$
8 Drams.........	,,	1 Ounce	(oz. or ℥) =	0	$2\frac{2}{3}$
12 Ounces.......,	,,	1 Pound	(lb. or ℔.) =	10	2

NEW APOTHECARIES' WEIGHT

[USED FOR BUYING DRUGS AND MEDICINES.]

$437\frac{1}{2}$ Grains	make	1 Ounce.
16 Ounces	,,	1 Pound.

Note.—In this Table the pound and the ounce Avoirdupois are expressed in grains instead of in drams. It will thus be seen that apothecaries buy their drugs by Avoirdupois weight.

[APOTHECARIES' LIQUID MEASURE.]

60 Minims	make	1 Fluid dram.
8 Fluid drams	,,	1 Fluid ounce.
20 Fluid ounces	,,	1 Pint.

Note 1.—See Tables of Capacity. The Table is founded on the facts that 'a pint of pure water weighs a pound and a quarter,' and that the weight of a fluid ounce of distilled water is thus an *ounce* Avoirdupois.

Note 2.—Learn also that 1 teaspoonful = 1 fluid dram ; 1 dessert-spoonful = $2\frac{1}{2}$ fluid drams ; and 1 tablespoonful = 4 fluid drams.

Measures of Length.

ENGLISH.

12 Inches...(in)	make	1 Foot	(ft.)
3 Feet	,,	1 Yard	(yd.)
220 yards	,,	1 Furlong	(fur.)
8 Furlongs (1,760 yds.)	,,	1 Mile	(mi.)

Note.—$69\frac{1}{3}$ miles = 1 degree ; $\frac{1}{60}$ of a degree = 1 geographical mile : 3 geographical miles = 1 nautical league.

6 feet = 1 fathom and 1 cable = 100 fathoms.

In measuring the heights of horses we use the measures ; palm, hand and span = 3 in., 4 in., and 9 in. respectively.

Note 1.—The names of the measures of length are derived mainly from the *members* of the human body (as *foot, palm, cubit*,) and from the *spaces* included in their ordinary motions (as

span, pace, fathom). *Mile* is literally *mille passus*, a thousand paces. *Furlong* denotes the *length* of a *furrow*. *Yard* = A.S. *geard*, a rod. *League* (from A.S. *lugen*, to see) expresses the distance the eye of a man, when standing upright, can *see* on a level plain.

LAND SURVEY MEASURE.

7·92 inches	= 1 link.
100 links (22 yds.)	= 1 Gunter's chain.
10 chains	= 1 furlong.

N.B.—"The chain devised by Mr. Gunter, and called after his name is 22 yds. long and consists of one hundred links, so that 80 chains are equal to one mile.

MADRAS.

9 Inches........................	make	1 Span.
2 Spans........................	,,	1 Cubit or Moozhum (முழம்).
2 Cubits........................	,,	1 Yard.

FRENCH.

10 millimetres (mm.)	make	1 centimetre (cm.)
10 centimetres.........	,,	1 decimetre (dm.)
10 decimetres..........	,,	1 **metre** (m.)
10 metres..............	,,	1 decametre (Dm.)
10 decametres.........	,,	1 hectometre (Hm.)
10 hectometres........	,,	1 kilometre (Km.).

1 metre = 39·37 inches nearly.
1 centimetre = nearly $\frac{2}{5}$ inch.
1 kilometre = nearly $\frac{5}{8}$ mile.

Measures of Surface.

ENGLISH.

144 Square inches (sq. in.)	make	1 Square foot (1 sq. ft.)
9 Square feet	,,	1 Square yard (1 sq. yd.)
4,840 Square yds.	,,	1 Acre (1 ac.)
640 Acres	,,	1 Square mile.

1 sq. pole = $30\frac{1}{4}$ sq. yds. ; 1 rood = 40 sq. poles ; 1 acre = 4 roods.

N.B.—It may be noted that the area of a square field whose side is $69\frac{4}{7}$ yards is very nearly equal to an acre.

LAND SURVEY MEASURE.

$62 \cdot 7264 = (7 \cdot 92)^2$ Square inches	make	1 Square link.
10,000 Square links...........	„	1 Square chain.
10 Square chains	„	1 Acre
(100 Cents......................	„	1 Acre.)

MADRAS.

2,400 Square feet.........	make	1 Manie (மனை) or Ground.
24 Grounds............	„	1 Cawni = 6,400 Sq. yds.
484 Cawnies...........	„	1 Sq. mile = 640 Acres.

The Indian Cawni = $\frac{3}{4}$ Acre nearly.

144 Square feet......................	make	1 Kuli.
100 Kulis............................	„	1 Mah = 1,600 Sq. yds.
20 Mahs............................	„	1 Veli = 32,000 Sq. yds.

FRENCH.

The French units of area are derived from the units of length; thus 10^2 or 100 square centimetres = 1 square decimetre; 10^2 or 100 square decimetres = 1 square metre; and so on.

The unit used for measuring lands is 1 sq. decametre which is called an *are*.

Thus 1 centiare = $\frac{1}{100}$ are

1 hectare = 100 ares (= $2\frac{1}{2}$ acres nearly).

1 are = $119 \cdot 5992$ sq. yds.

Measures of Solidity.

ENGLISH.

1,728 Cubic inches............	make	1 Cubic foot.
27 Cubic feet...............	„	1 Cubic yard.

FRENCH.

The measures are derived from linear measure: Thus

10^3 or 1000 cubic millimetres make 1 cubic centimetre.

10^3 or 1000 cubic centimetres make 1 cubic decimetre and so on.

1 cubic inch = $16 \cdot 3870$ cubic centimetres.

Measures of Capacity.

MADRAS.

8 Ollocks	make	1 Padi or Measure	=	62·5 fluid oz.
			=	100 c. in.
8 Measures	,,	1 Marakkal	=	800 c. in.
5 Marakkals	,,	1 Parah	=	4,000 c. in.
12 Marakkals	,,	1 Kalam	=	9,600 c. in.
400 Marakkals or 80 Parahs	,,	1 Garce	=	320,000 c. in.

The Marakkal is to the Imperial Gallon as 10,000 to 3,466.

1 Gallon = nearly 22 Ollocks; 1 Marakkal = nearly 3 Gallons.

Note.—The cubic contents of the Marakkals in use at the different commercial centres in the Presidency vary from 750 to 850 cubic inches. The diameter of a Marakkal of recognised size is 10·3 inches, and its depth 9·6 inches, while the recognised standard of the contents in rice is 1,064 tolas.

ENGLISH.

MEASURES OF LIQUIDS AND OF ALL DRY GOODS.

4 Gills	make	1 Pint (pt.)	
2 Pints	,,	1 Quart (qt.)	
4 Quarts	,,	1 Gallon (gal.)	
2 Gallons	,,	1 Peck	For dry goods only.
4 Pecks	,,	1 Bushel	For dry goods only.
8 Bushels	,,	1 Quarter	For dry goods only.
5 Quarters	,,	1 Load	For dry goods only.

Note 1.—1 Pipe = 2 hogsheads (hhd.) of wine = 126 gallons.
1 Butt = 2 hogsheads of beer = 108 gallons.

Note 2.—The weight of a pint of pure water is given by the doggerel :—

"*A pint of pure water*
Weighs a pound and a quarter."

Note 3.—The *Standard Gallon* which contains 277·274 cubic inches is equal to the volume of 10 lbs. Avoir. of distilled water at 62° Fahr., the barometer being at 30 inches; we can therefore exactly compute the *Standard Gallon* from which all other measures may be ascertained.

FRENCH.

The unit here is a *litre*. The litre is the volume of one kilogram of pure water at its maximum density. It was supposed that 1 litre was equal to 1 cubic decimetre; but it is slightly above it.

1 gallon = 4·545963 litres.

Table of Time.

ENGLISH.

60 Seconds (sec.)	make	1 Minute (min.)
60 Minutes	,,	1 Hour (hr.)
24 Hours	,,	1 Day.
7 Days	,,	1 Week (wk.)

Note 1.—52 weeks are regarded as equal to one year.

Note 2.—A year is divided into 12 Calendar months, the length of each of which is remembered by the following lines :—

Thirty days hath September,
April, June and November;
February hath twenty-eight alone,
And all the rest have thirty-one;
But leap-year coming once in four,
February then hath one day more.

Note 3.—As the *civil* or *common* year cannot contain a fraction of a day, it has been made to consist of 365 days, 365 being the integer nearest to 365·242218, the number of days in the *solar* year; but to make the deficiency good, it has been enacted that every fourth year (Leap year) shall consist of 366 days, but that in every four hundred years, those that complete a century, the number expressing which is not divisible by 4 shall not be leap years. Thus 1600 and 2000 are leap-years, whereas 1700, 1800, and 1900 are *not* leap-years.

Note 4.— The explanation of the irregularity in the lengths of the several months is given as follows in the 'Book of Days'.

"The idea of Julius Cæsar was that the months should consist of 31 and 30 days alternately; and this was effected in the leap-year, consisting, as it did, twelve times thirty with six over. In ordinary years consisting of one day less, his arrangement gave 29 days to Februaries.

Afterwards his successor Augustus had the eighth of the series called after himself (August)* and from *vanity* broke up the regularity of Cæsar's arrangement by taking another day from February to add to his own month, that it might not be shorter than July (the month of *Julius* Cæsar); a change which led to a shift of October and December for September and November as months of 31 days. In this arrangement the year has since stood in all Christian countries."—R. CHAMBERS.

INDIAN.

60 Vinadis................	make	1 Naliga = 24 min.
$7\frac{1}{2}$ Naligas................	,,	1 Jamam = 3 hrs.
8 Jamams or 60 Naligas	,,	1 Day.

Number.

12 Articles of any kind	make	1 Dozen.
12 Dozen................	,,	1 Gross.
20 Articles of any kind	,,	1 Score.

Paper.

24 Sheets of paper..............	make	1 Quire.
20 Quires..........................	,,	1 Ream.
10 Reams..........................	,,	1 Bale.

Angular Measure.

(Used in Astronomical and Geographical calculations.)

60 Seconds (60″)...........	make	1 Minute (′)
60 Minutes..................	,,	1 Degree (°)
90 Degrees..................	,,	1 Right angle.

* The original name of the month was *Sextilis*, which he changed into *August*, as he was born in that month.

§ 67. Reduction :

INDIAN MONEY.

Example 1.—Express Rs 2-5 as. 9 p. in pies.

We must first convert rupees into annas and then annas into pies. Now

Re. 1 = 16 annas.

∴ Rs. 2-5 as. = $2 \times 16 + 5$ or 37 annas.

Again 1 a. = 12 p. 37 as. = 37×12 p. = 444 pies.

∴ Rs. 2-5 as. 9 p. = 37 as. 9 p. = 444 + 9 or 453 p.

The working is shown in the margin.

Contracted thus

Rs. 2-5-9
16
———
37 as.
12
———
453 p.

(multiplication and addition both combined).

Example 2.—Express 3843 pies as a compound quantity, *i.e.*, in rupees, annas, pies.

Here pies must first be converted into annas and then annas to rupees : 12 pies = 1 anna ; therefore to find the number of annas in 3843 pies, find how many groups of twelve pies there are in 3843 pies, *i.e.*, divide 3843 by 12. You get 320 as. with 3 p. oustanding.

∵ 16 as. = Re. 1 ; divide 320 as. by 16. You get rupees.

The working is shown in the margin.

12 | 3843 p.
16 | 320 as. 3 p.
Rs. 20 0 a. 3 p.

Exercise—Oral.

1. How many pies are there in (*a*) 6 as., (*b*) 8 as., (*c*) 6 as. 3 p., (*d*) 8 as. 4 p. ?

2. How many annas are there in (*a*) Rs. 3., (*b*) Rs. 5., (*c*) Rs. 8 4 as., (*d*) Rs. 9 8 as. ?

3. How many annas and pies are there in (*a*) 62 p., (*b*) 49 p., (*c*) 83 p. ?

4. How many rupees and annas are there in (*a*) 53 as., (*b*) 67 as., (*c*) 88 as., (*d*) 93 as. ?

5. How many pencils at two pies each can be bought for 2 as. 6 p. ; for 4 as. 6 p. ; for 5 as. 8 p. ?

6. How many post cards at quarter anna each can be bought for 3 as. 6 p. ; 5 as. 9 p. ?

Exercise—Practical.

1. Draw up a table showing the value in pies of each number of annas from 1 to 16.

2. Draw up a table showing the value in annas and pies of 20 pies, 30 pies, 40 pies,......120 pies.

3. Draw a straight line AB. On this line take points C, D, E, F, G, etc., so that AC, AD, AE, AF, AG,......may respectively represent 12 p., 24 p., 36 p., 48 p., 60 p.......(Take a distance of 5 mm. to represent 12 p.). Put at the points A, C, D, E, F....... 0, 12, 24, 36, 48......respectively.

4. At C draw a perpendicular CP equal to 1 cm. Join AP and produce it. Similarly draw perpendiculars at D, E, F, G...... meeting the line AP produced in Q, R, S, T... ; measure DQ, ER, FS, GT...

5. Suppose that CP represents 1 anna, what will DQ, ER, FS, GT, &c., represent ? Mark off along the perpendiculars the number of annas represented by them.

6. Use the figure to find the number of annas equal to 84 pies, 96 pies, 108 pies. Use the same figure to give the number of pies equal to 7 as., 9 as., 10 as.

7. Similarly draw a figure to help us to read off annas equal to 4 rupees, 6 rupees and 8 rupees and also rupees equal to 64 as., 96 as., 112 as., 144 as.

8. Draw another figure to help us to read off rupees equal to 4, 5 and 8 sovereigns and also sovereigns equal to 45 rupees, 105 rupees and 135 rupees.

Exercise VIII (a).

1. Reduce the following sums to pies :—

   (*a*) Rs. 4-13 as. 6 p.

   (*b*) Rs. 49-12 as. 2 p.

   (*c*) Rs. 596-13 as. 10 p.

2. How many half-rupees are there in (*a*) Rs. 5,083, (*b*) Rs 3,084? How many quarter-rupees in (*a*) Rs. 6,040-8 as., (*b*) Rs. 4,038-12 as.; and how many two-anna pieces in (*a*) Rs. 3,640-4 as., (*b*) Rs. 3,483-10 as. ?

3. Reduce each of the following sums to annas and quarter annas :—(*a*) Rs. 354 ; (*b*) Rs. 846-3 as. ; (*c*) Rs. 1043-8 as. 6 p.

4. Convert the following sums into sovereigns :—(*a*) Rs. 17,630; (*b*) Rs. 82,445.

5. How many yards of cloth can be purchased for Rs. 2-4 as. at 3 annas a yard ?

6. How many oranges can be bought for Rs. 19-15-6 at 6 pies each ?

7. For every mile of railway journey a person has to pay 3 pies. What distance does he travel if he buys a ticket for Rs. 13-5-6 ?

8. A copy of a certain book costs 12 annas. How many copies can be purchased for Rs. 53-4-0 ?

ENGLISH MONEY.

1. Reduce to pence (*a*) 2*s*., (*b*) 3*s*. and 4*d*., (*c*) 5*s*. and 6*d*.

2. Reduce to shillings (*a*) £2-3*s*., (*b*) £3-4*s*.

3. Reduce to shillings and pence (*a*) 58*d*., (*b*) 64*d*., (*c*) 93*d*.

4. Reduce to pounds and shillings (*a*) 30*s*., (*b*) 34*s*., (*c*) 48*s*.

Exercise VIII (b).

Reduce

1. £43-7*s*. 6*d*. to pence.

2. £128-18*s*. 12*d*. to two-pence.

3. £169-19*s*. to (*a*) florins, (*b*) crowns, (*c*) half-florins, (*d*) half-crowns.

4. 16,328*d*. to £ *s*. *d*.

5. 6,843 half-crowns to £ *s*. *d*.

6. 8,934 guineas to £ *s*. *d*.

WEIGHTS AND MEASURES

(INDIAN).

Exercise—Oral.

1. Reduce to tolas (*a*) 5 plms; (*b*) 5 plms. and 1 tola ; (*c*) 18 plms. and 2 tolas.

2. Reduce to plms. (*a*) 5 seers; (*b*) 7 seers and 5 plms; (*c*) 9 seers and 3 plms.

3. Reduce to seers (*a*) 3 viss ; (*b*) 5 viss and 3 seers.

4. Reduce to viss (*a*) 9 mds.; (*b*) 10 mds. and 4 viss.

5. Reduce to tolas (*a*) 8 seers ; (*b*) 9 seers and 3 plms.

6. Reduce to plms. (*a*) 5 viss;(*b*) 6 viss and 3 seers ; (*c*) 8 mds.; (*d*) 9 mds. and 5 viss.

7. Reduce to viss (*a*) 9 can.; (*b*) 10 can. and 2 mds.

8. How many palams and tolas are there in the following numbers of tolas :—4, 23, 39, 48 ?

9. How many viss and palams are there in the following numbers of palams :—43, 86, 129, 263 ?

10. How many mds. and viss are there in the following numbers of viss :—23, 28, 34, 63, 78 ?

Exercise VIII (c).

Reduce

1. 38 mds. 9 viss 4 seers 8 plms. to palams.
2. 25 can. 9 mds. 3 seers to seers.
3. 18 can. 16 mds. 39 plms. to palams.
4. 190 can. 19 mds. 7 viss to seers.
5. 263 can. 18 mds. to viss.

(ENGLISH).

Exercise—Oral.

1. Reduce to ounces 3 lbs. ; 8 lbs. ; 4 lb. 9 ounces.
2. Reduce to hundredweights 1 ton ; 4 ton ; 5 ton, 18 cwt.
3. Reduce to ounces (*a*) 6 lb. 3 oz.; (*b*) 7 lb. 2 oz.
4. Reduce to pounds (*a*) 3 qr. 8 lb.; (*b*) 9 qr. 13 lb.
5. How many pounds and ounces are there in the following numbers of ounces :—33, 48, 67, 79 ?

6. How may cwts. and quarters are there in the following numbers of quarters :—23, 38, 47, 63, 89 ?

7. How many tons and cwts. are there in the following numbers of cwts. :—57, 78, 115, 163, 174 ?

8. How many cwts. and lbs. are there in the following numbers of pounds:—114, 116, 227, 338, 453, 664?

Exercise VIII (d).

Reduce to lbs. :—

1. 15 ton 11 cwt. 3 qr. 57 lb.

2. 29 ton 18 cwt. 26 lb.

3. 1896 cwt. 3 qr. 3 lb.

LENGTH AND TIME.

Exercise—VIII (e).

1. Reduce 3 miles 4 fur. 170 yds. 2 ft. to feet.

2. Reduce 14 miles 2 fur. 160 yds. to feet.

3. Reduce 38263 yds. to miles, &c.

4. Reduce 661893 feet to miles, &c.

5. Find the number of seconds in an hour.

6. The second hand of a watch makes one revolution in a minute. How many revolutions does it make (*a*) in 1 hour (*b*) in a day?

7. Reduce 189765 days to years (reckoning 365 days for a year.)

8. A wheel whose circumference is 5 ft. makes 1 complete revolution in 2 seconds. What time will it take to travel a mile?

9. A train travels at the rate of 22 ft. a second. How many miles will she travel in 1 hour?

10. The length of a field is 4684 links each 3 inches long. Express the length of the field in yds. and feet.

11. The highest mountain in the world is said to be 29,002 ft. in height. Express this height in miles, yards and feet.

12. A motor car runs at the rate of 44 ft. per second. How many miles does it run in an hour?

13. A bicycle wheel 60 inches in circumference makes 4381 revolutions. What is the distance passed over?

14. A balloon ascends at the rate of 40 ft. in a second. How many miles does it go up in 15 minutes.

MEASURES OF CAPACITY, &c.

Exercise VIII (f).

1. Reduce 5 pipes 3 gallons to pints.

2. Reduce 6 pipes 10 gallons to quarts.

3. Reduce 6548 pints to gallons, &c.

4. Reduce 6470 ollocks (*a*) to measures, &c. (*b*) to marakkals, &c.

5. Reduce 1874 measures to kalams, &c.

6. How many ollocks are there in 892 kalams of paddy?

7. How many quires of paper are there in 20 reams?

8. A box of steel nibs contains a gross. How many are there in 5 such boxes.

9. What is the cost of 18 dozen of oranges, at 5 pies an orange?

§ 68. Compound Addition and Subtraction.

Example 1. Add together;

RS.	A.	P.
563	14	4
38	13	8
289	12	11
1363	0	9
2255	9	8

The total of pies column is 32 p. = 2 as. 8 p. So we put down 8 under pies and carry 2 to the column of annas. The total of annas column is 41 as. = 2 Rs. 9 as. So we put down 9 under annas and carry 2 to the column of rupees. The column of rupees is totalled as in ordinary numbers.

Note.—The student should always check his answer:

Here the work is checked by adding from bottom to top as well as from top to bottom.

Example 2. Subtract 178 can. 8 mds. 5 viss 36 p. from 308 can. 3 mds. 2 viss and 18 p.

can.	mds.	viss.	pal.
308	3	2	18
178	8	5	36
129	14	4	22

Proceed as in complementary addition thus:—

36 plm. and 22 plm. make 1 viss and 18 plm. So we set down 22 palams in the difference and carry 1 viss. Again 1 and 5, *i.e.*, 6 viss and 4 viss make 1 md. and 2 viss so set down 4 viss in the difference and carry 1 md. Again 1 and 8, *i e.*, 9 mds. and 14 mds. make 1 can. and 3 mds., so set down 14 mds. in the difference and carry 1 can. The column of can. is dealt with as in ordinary numbers.

The answer is 129 can. 14 mds. 4 viss and 22 plm.

Check the answer by testing whether the difference and the subtrahend added together give the minuend.

Example 3. Subtract the sum of £8 7*s*. 8*d*., £13 9*s*. 10*d*., and £186 5*s*. 8*d*. from £485 8*s*. 6*d*.

£	*s*.	*d*.
485	8	6
8	7	8
13	9	10
186	5	8
277	5	4

Mental process.

(*d*. column). 18, 26*d*. = 2*s*. 2*d*. and **4d**. gives 2*s*. 6*d*. Set down 4*d*. and carry 2*s*.

(*s*. column). 7, 16, 23*s*. = £1 3*s*.; and **5s**. = £1 8*s*. Set down 5*s*. and carry £1.

The £'s column is dealt with as in ordinary numbers.

Checking.—If you add all the lines except the top line the result must be equal to the top line.

Exercise VIII (g).

Add the following quantities vertically and horizontally and find the total in each case:—

	1.			2.			3.		
	RS.	AS.	P.	RS.	AS.	P.	RS.	AS.	P.
4.	163	8	4	32	8	9	189	3	11
5.	38	9	3	632	7	3	1,632	6	3
6.	628	5	6	1,382	9	4	283	8	11
7.	343	10	11	692	7	11	965	3	9
8.	728	9	8	1,386	15	10	2,483	1	5
9.	....			....			....		

	10.			11.			12.		
	£	s.	d.	£	s.	d.	£	s.	d.
13.	873	2	9	1,387	10	11	1,684	2	11
14.	1,368	4	3	23,463	9	10	386	5	9
15.	897	5	11	387	14	8	1,438	2	3
16.	1,346	7	8	467	8	10	684	9	11
17.	2,864	13	10	98	3	5	2,382	5	6
18.	....			....			....		

	19.			20.			21.		
	Can.	md.	vis.	Can.	md.	vis	Can.	md.	vis.
22.	12	16	7	13	15	5	163	8	3
23.	180	3	2	89	18	4	1,731	16	4
24.	193	18	6	163	17	6	2,364	18	3
25.	185	17	3	248	13	3	1,768	15	6
26.	16	5	4	138	9	2	2,564	13	2
27.	....			....			....		

	28			29			30			
	Tons.	cwt.	lbs.	Tons.	cwt.	lbs.	Tons.	cwt.	lbs.	oz
31.	16	19	27	138	12	18	168	4	26	15
32.	153	18	25	567	3	26	83	10	21	14
33.	384	7	20	938	11	18	173	8	16	12
34.	163	15	13	634	13	24	689	13	21	11
35.	189	14	16	1,068	12	26	123	8	10	6
36.	....			....			....			

37.

	gal.	qt.	pt.
40.	8	3	1
41.	10	2	0
42.	9	3	1
43.	6	3	1
44.	....	....	....

38.

bush.	pk.	gal.
14	3	1
18	2	0
10	0	1
15	3	1
16	2	1
....	....	....

39.

qrs.	bush.	pk.	gal.
18	7	0	1
23	5	2	1
43	4	1	1
12	1	3	1
11	2	3	1
....	....	....	....

45.

Kalms.	Mark.	Meas.
163	11	7
843	10	6
365	8	3
249	7	4

46.

Kg.	Hg.	Dg.	g.
168	9	3	7
482	5	2	6
999	0	1	2
1,064	8	7	6

47.

G.	dg.	cg.	mg.
10	3	8	6
21	4	9	7
32	2	8	7
4	6	7	8

Fill in the blanks in the following sums on subtraction:—

48.

RS.	AS.	P.
20,373	8	4
4,989	10	7
....	....	....

49.

£	s.	d.
1,874	3	6
938	15	7
....	....	....

50.

£	s.	d.
11,872	0	0
......	15	9
9,681	...	...

51.

RS.	AS.	P.
1,688	15	3
......	...	...
728	14	9

52.

Can.	md.	vis.	sr.	plms.
490	3	2	4	7
279	7	5	3	6
....	....	....	....	....

53.

Can	md.	vis.	sr.	plm.
367	17	6	4	3
....	....	....	....	....
268	18	7	3	7

54.

Tons	cwt.	qr.	lbs.	oz.
32	3	2	21	14
26	...	...	...	...
...	16	3	27	15

55.

Tons	cwt.	qr.	lbs.	oz.
534	2	1	10	12
...	6	2	18	...
283	...	...	...	8

56.

Bush.	pk.	gal.
7	3	7
3	8	...
...	...	3

57.

Bush.	pk.	gal.
5	2	6
...	...	...
3	1	4

58.

Kg.	Hg.	Dg.	g.
168	2	1	3
89	7	8	6

59.

G.	dg.	cg.	mg.
83	0	0	8
79	9	3	9

Add together the last four lines and subtract the sum from the first line.

60.

RS.	AS.	P.
4,000	0	0
1,316	15	3
893	11	4
96	12	8
206	10	4

61.

£	*s.*	*d.*
1,600	0	0
163	4	9
283	10	8
365	9	8
483	1	8

§ 69. Compound Multiplication.

Example 1.—Multiply Rs. 18 11 as. 9 p. by 7.

	RS.	A.	P.
	18	11	9
			7
Rs.	131	2	3

Explanation. 7 × 9 p. = 63 p. = 5 as. 3 p. Set down 3 in the pies column and carry 5 to the anna column; 7 × 11 as. + 5 as. = 82 as. or Rs. 5, 2 as. Set down 2 as. and carry 5 Rs.; 7 × 18 Rs. + 5 Rs. = 131 Rs.

The student is recommended to verify his answer by writing Rs. 18-11-9 seven times and adding.

Example 2.—Multiply Rs. 863-5-4 by 56.

Now $56 = 7 \times 8$. So we multiply first by 7 and then by thus:—

	RS.	AS.	P.	
	863	5	4	
			7	
Rs.	6,043	5	4	= product by 7.
			8	
Rs.	48,346	10	8	= product by 56.

Example 3.—Multiply Rs. 361-5-8 by 157.

157 has no factors. But $157 = 12 \times 13 + 1$. $\therefore$ Multiplication by 157 may be effected as follows:—

RS.	AS.	P.	
361	5	8	
		12	
4,336	4	0	= product by 12
		13	
56,371	4	0	= product by 156.
361	5	8	 1
56,732	9	8	product by 157.

Let us check the result by directly multiplying Rs. 361-5-8 by 157.

RS.	AS.	P.
361	5	8
		157
56,732	9	8

Explanation. 8 p. $\times$ 157 = 1256 p. = 104 as. 8 p. Again 5×157 as. + 104 as. = 889 as. or Rs. 55-9 as. Again Rs. 361×157 + Rs. 55 = Rs. 56,677 + Rs. 55 = Rs. 56732.

The work may also be checked by changing the order of the factors 12 and 13.

Exercise VIII (h).

Multiply :—

1. Rs. 82-7-11 by (*a*) 7 (*b*) 9 (*c*) 15.
2. Rs. 173-14-10 by (*a*) 31 (*b*) 37 (*c*) 43.
3. Rs. 16,815-15-11 by (*a*) 182 (*b*) 263 (*c*) 587.
4. £732 11*s*. 10*d*. by (*a*) 648 (*b*) 831 (*c*) 999.
5. £13,042 19*s*. 6*d*. by (*a*) 5,048 (*b*) 6,079 (*c*) 803.
6. 11 mds. 5 vis. 39 plm. by (*a*) 365 (*b*) 899 (*c*) 1.029.
7. 159 can. 8 mds. 3 viss 5 seers by (*a*) 263 (*b*) 309 (*c*) 408.
8. 112 tons 9 cwt. 23 lbs. by (*a*) 65 (*b*) 96 (*c*) 87.
9. 1,260 tons 15 cwt. 3 qrs. 24 lbs. by (*a*) 463 (*b*) 869 (*c*) 728.
10. 6,534 miles 4 fur. 210 yds. by (*a*) 123 (*b*) 148 (*c*) 763.
11. 68 Km. 4 Hm. 6 Dm. 8 m. by (*a*) 32 (*b*) 68 (*c*) 91.
12. 68 m. 4 dm. 5 cm. 8 mm. by (*a*) 643 (*b*) 782 (*c*) 934.
13. 15 kal. 10 mar. 8 mea. by (*a*) 72 (*b*) 83 (*c*) 47.
14. 5 qr. 7 bush. 2 pk. by (*a*) 32 (*b*) 43 (*c*) 59.
15. 6 bush. 3 pk. 4 gal. by (*a*) 128 (*b*) 257 (*c*) 349.
16. 68 Kg. 9 Hg. 8 Dg. 7 g. by (*a*) 76 (*b*) 842 (*c*) 969.
17. 843 reams 19 quires 23 sheets by (*a*) 72 (*b*) 96 (*c*) 108.

§ 70. Compound Division.

Ex. 1. Divide £283 7*s*. 8*d*. by 12.

Explanation. 12 goes into £283 twenty-three times as in ordinary division and 7 is the remainder, *i.e.*, there are 12 sets of £23 each in £283 and £7 outstanding. These £7 give 140*s*. which together with 7*s*. make 147*s*.; and 12 goes into 147 twelve times and 3 is the remainder. These 3*s*. are equal to 36*d*. which together with 8*d*. make 44*d*. and 12 goes into 44 three times, and 8*d*. is the remainder.

	£.	s.	d.	
12	283	7	8	
	23	12	3	—8*d*. remainder.

Thus the quotient is £23 12 3, and 8*d*. is the remainder.

Ex. 2. Divide Rs. 873-5-4 by 28.

The division is effected by factors thus :—

	Rs.	A.	P.	
7	1,873	5	4	
4	267	9	10	—6 pies left
	66	14	5	—2 pies left (from each of the seven groups)

∴ the total remainder = $7 \times 2 + 6$ or 20 pies : we may check the result by long division as follows :—

```
        Rs.   A.  P.
    28)1,873  5   4(66-14-5
       168
        193
        168
         25 Rs.
         16
  28)405 as. (Multiplication and Addition combined).
     28
     125
     112
      13 as.
      12
  28)160 p. (       ,,          ,,           ,,      )
     140
      20
```

Ex. **3**. Divide 5 can. 3 mds. 4 viss by 3 viss 30 plm.

The question means how many packages containing 3 viss and 30 plm. each, can be made out of 5 can. 3 mds. 4 viss. We reduce each quantity to the lowest denomination namely palams and divide :

```
      5 can. 3 mds. 4 viss.             3 viss and 30 plms.
     20                                40
    103 mds.                          150 plms.
      8
    828 viss
     40
 33,120 plms.
```

We now divide 33,120 plms. by 150 plms. thus :—

$15\cancel{0}\ |\ 3312\cancel{0}$

220—120 plms.

The answer is 220 with a remainder of 120 plms.

Exercise VIII (i).

Divide, giving the remainder, if any—

1. Rs. 583 5 a. 4 p. by 21 ; 27 ; 28.

2. Rs. 15,837 11 a. 10 p. by 63 ; 147 ; 192.

3. £893-17-6 by 18 ; 19 ; 21.

4. £26,954-19-10 by 36 ; 58 ; 62.

5. 516 can. 3 mds. 4 viss by 63 ; 78 ; 93.
6. 1,726 can. 6 mds. 7 viss by 638 ; 789 ; ?63.
7. 684 ton 12 cwt. 3 qr. 21 lbs. by 28 ; 63 ; 98.
8. 81,236 ton 19 cwt. 84 lbs. by 128 ; 246.
9. 181 Kg. 7 Hg. 4 Dg. 3 g. by 23 ; 48 ; 76.
10. 1,284 miles 3 fur. 208 yds. by 29 ; 37 ; 43.
11. 1080 qr. 7 bush. 3 pt. by 29 ; 38 ; 46.
12. 11,080 qr. 3 bush. 2 pt. by 123 ; 551 ; 758.
13. 1,080 Kal. 5 marks. 7 meas. by 68 ; 87 ; 106.
14. 849 Km. 6 Hm. 5 Dm. by 25 ; 52 ; 78.
15. 6,849 Km. 3 Hm. 4 Dm. 6 m. by 125 ; 653.
16. 3 bush. 1 pk. 3 gal. 1 qt. by 2 qt. 1 pt.
17. 41 miles 6 fur. 103 yds. by 85 yds. 2 ft, 3 in.
18. 21 cwt. 3 qr. 21 lbs. by 7 lb. 5 oz.
19. 15 lbs. 11 oz. 432 gr. by 1 oz. 180 grains.

Exercise—Problems VIII (j).

1. Find the total of the following collections :—18 sovereigns, 34 half-rupees, 68 quarter-rupees, 128 two-anna pieces, 32 annas, 64 half-annas, and 198 quarter annas.

2. Each copy of the 'Madras Times' is sold for 2 annas. Find the price of 285 copies of the same newspaper.

3. A gentleman has Rs. 300 in his pocket and pays bills to the amount of Rs. 6-8-1, Rs. 12-7-8, Rs. 26-4-11, Rs. 38-15-10 and Rs. 53-7-0 What money has he left ?

4. At a public meeting the following sums were received as donations for a certain memorial fund :—Rs. 650; Rs. 830 ; Rs. 1,500 ; Rs. 2,525 ; and also the following minor sums were taken—193 quarter rupees ; 845 two-anna pieces. Find the total collection.

5. In a certain business the receipts and payments during a certain week were—

			Receipts.		*Payments.*
Monday	...	Rs.	1,546- 8-11	Rs.	6,083- 9-8
Tuesday	...	,,	9,378- 6- 3	,,	5,491-10-4
Wednesday	...	,,	12,673- 8- 9	,,	8.324- 1-8
Thursday	...	,,	15,984-10-11	,,	9,476-12-2
Friday	...	,,	16.586-12- 3	,,	8,756- 8-9
Saturday	...	,,	24,897- 5- 4	,,	19.358- 3-3

By how much do the receipts exceed the payments ?

6. A bale of piece-goods weighs 1834 lbs. Express this as a compound quantity.

7. The earth's equatorial diameter is 41,841,792 feet. Express this as a compound quantity in miles and yards.

8. A contractor undertakes to construct a building and employs 3 maistries, 25 masons, 42 blacksmiths, 132 coolies. Each maistry is paid Rs. 2 per day, each mason, Re. 1 2 as., each blacksmith Re. 1 4 as. and each cooly receives 5 as. What are the total daily wages paid by the contractor? If the work lasts for 56 weeks, what is the total sum spent on wages?

9. A offers to pay Rs. 750 for a horse while B offers Rs. 825; which is the better offer and by how much?

10. A house was sold for £835 15*s*. which had been bought 2 years ago for 820 guineas. What was the gain or the loss?

11. A clerk gives in exchange for a Rs 50 note 1 sovereign, 12 half-rupees, 20 quarter-rupees, 54 two-anna pieces and the rest nickel coins. How many of these last were given?

12. A certain land-owner pledged his lands to the extent of Rs. 12,650. He was able to return on three occasions Rs. 2,500, Rs. 1,575, Rs. 875 repsectively. How much has he yet to pay?

13. The correct weight of a certain article was 2 ton 1 cwt. 3 qr. 27 lb. but in a hurry it was given to be 2 ton 1 cwt. 2 qr. 19 lbs. What was the amount of the error, and in whose favour the buyer's or the seller's?

14. A sidereal day is 23 hr. 56 min. 4 sec. long. Find the difference between a sidereal day and an ordinary day.

15. A map is drawn so that the actual distance between any two places is 3500 times as great as it is on the map. Find the actual distance between A and B in miles and yards if it is represented by 27 inches on the map.

16. How many spoons can be made out of 450 lb. of silver, if each spoon contains 462 gr. of silver?

17. From A to B is 250 chains. A chain is equal to 22 yds. Express the distance as a compound quantity in miles and yards.

18. If a bag of chillies weighs 1 md. 6 viss and 28 plm., find the weight of 950 bags.

19. A man starts from A at 10-42 A.M. and goes to B at 5-16 P.M. on the same day. If the distance between A and B is 28 miles, find at what rate (*a*) per hour, (*b*) per minute, he travels.

20. If 250 boxes of sugar weigh 28 ton 14 cwt., find, to the nearest pound, the average amount of sugar in each box, given that each empty box weighs 10 lb.

21. The annual premium for insuring a house for Rs. 15,600 i Rs. 150. How many such houses must be insured in order that the insurance company may not lose if one house be detroyed ?

22. A tramway company carries in a year 12,895 passengers at $1\frac{1}{2}$ as. each, 612,483 at $1\frac{1}{4}$ as. each, 1,612,354 at 1 anna each, 8,72,357 at 9 pies each, 3,70,824 at 6 pies each and 1,88,834 at 3 pies each. How much money does it realise during the year ?

23. A train is made up of an engine weighing 53 tons, a tender of 25 tons, 7 carriages each weighing 10 tons 2 cwt. and 2 brake-vans of 9 tons each. How many carriages should be detached before the train can pass over a bridge which can support only 150 tons ?

24. A man in India wishes to send his son studying at Cambridge University £200 a year in equal monthly instalments. How many rupees should he send monthly if £1 = Rs. 15 ?

25. The railway line from A to B is 3 miles long and the average gradient is 1 in 90. Find, to the nearest foot, the difference in level between the two places.

26. The sun rises at 5-58 A.M. and sets at 6-30 P.M. Find the middle of the day.

27. In a certain year the number of farthings coined at the mint was 6,873,000 and in the next year it was 5,684,000. Find the difference in value in (£ *s. d.*) of the farthings coined in those 2 years.

28. Among how many persons can the sum of Rs. 10,000 be distributed at the rate of Rs. 29-9-11 to each ? And if the remainder be distributed equally among 26 other persons, how much will each of them receive ?

29. Divide Rs. 68 between A and B so that A may have Rs. 13 more than B.

30. At 5 pies in the rupee a man has to pay an income-tax of Rs. 25-8-9. What is his income ?

31. Find the income-tax on Rs. 850 at the rate of 5 pies in the rupee.

32. A person buys a certain number of mangoes at 12 as. a dozen and sells them at 16 as. a dozen. He gains Rs. 16-4 on the whole. How many mangoes were bought ?

33. A circular track is 4 fur. 178 yds. round. What distance will a cyclist have described when he has gone round the track 8 times ?

34. If telegraph posts stand at intervals of 80 metres, how many kilometres apart are the 1st and the 31st posts ?

35. A train travels at the rate of 50 miles an hour. How many yards does it travel in a minuute and how many feet in a second ?

36. A carriage wheel is 6 ft. 4 inches in circumference ; how many complete revolutions does it make in travelling 6 mi. 1,436 yds. ?

37. From a tank containing 3640 gallons, water is drawn off by a pipe which discharges 1 qt. in a second. The discharge tap is turned on at 11 A.M. on Monday. When will the tank be empty ?

38. How many days are there between the following dates, one day only of those named being counted ?

(1) March 23, 1896 and September 15, 1895.

(2) August 24, 1895 and July 16, 1896.

39. A year contains $365\frac{1}{4}$ days and a lunar month $29\frac{1}{2}$ days. How many lunar months are there in 19 years ?

40. 24th October 1910 was a Monday ; what day of the week was July 21st, 1908 and what day of the week will be 30th March, 1912 ?

CHAPTER IX.

AVERAGES, UNITARY METHOD AND MISCELLANEOUS EXAMPLES.

AVERAGES.

§ **71.** Suppose that 3 beggars are given 1, 4 and 7 pies respectively. Then the 3 beggars receive on the whole 1 + 4 + 7 or 12 pies. If the same sum, *viz.*, 12 p. were distributed equally among the three beggars, each of them would get 12 ÷ 3 or 4 p. This is expressed by saying that the beggars were paid *on an average* 4 p. each. Also four pies is said to be the **average** of 1 p., 4 p. and 7 p.

If we have a number of quantities in a group then the sum of the quantities divided by the number of quantities is called the **average** of the given quantities.

Exercise—Oral.

Find the average of—

1. 20 and 40. **2.** 13, 17, 16. **3.** 14, 16, 18, 20.

4. Rs. 8 and Rs. 10. **5.** Rs. 20, 40, 60.

6. Four mangoes are bought at 8 pies each and 6 mangoes are bought at 3 pies each. What is the average price of a mango?

7. Two pencils are bought at 6 pies each and 4 pencils at 9 pies each; what is the average price of each pencil?

Note.—It is evident that the average of a set of numbers must lie between the greatest and the least. This will be useful in checking our answers.

Example 1.—The profits of a merchant amount to Rs. 1,200 in the 1st year; to Rs. 1,550 in the 2nd year; and to Rs. 3,400 in the 3rd year. Find his average annual profits for the three years.

The total profits for the three years are

Rs. 1, 200+1,550+3,400 or Rs. 6,150.

∴ dividing this sum into 3 equal parts, we get

Rs. $\frac{6150}{3}$ or Rs. 2050 as the average.

In calculating the average of a series of numbers or results that do not much vary, the work can be simplified as in the following example :—

Example 2.—Find the average age of a class of 10 boys whose ages are—

Yrs.	Months.	Excess over or Defect from 16 yrs. 6 mos.
16	8	+ 2 mos.
16	9	+ 3
15	8	− 10
16	11	+ 5
17	10	+ 16
16	3	− 3
16	6	0
18	7	+ 25
14	1	− 29
16	7	+ 1
		Total + 10 months.

The average age may be seen to be about 16 yrs, 6 months. Indicate the excess of each age over, or its defect from 16 yrs. 6 months, by + and — respectively as shown above. The total of these differences is + 10 months. The average difference from 16 yrs. 6 months is $\therefore \frac{+10}{10}$ or + 1 month.

$\therefore$ the average age is 16 yrs. 6 months + 1 month or 16 yrs. 7 months.

N.B.—If the average difference from 16 yrs. 6 months were negative, we should subtract it from 16 yrs. 6 months.

The result may be checked by working out according to the usual method.

Example 3.—The average profits of a contractor for the first three months in a certain year are Rs. 6,500. The profits in the first month amount to Rs. 7,580, in the second month to Rs. 9,875. Find the profits in the third month.

the average for 3 months = Rs. 6,500

the total profits in the 3 months

= Rs. 6,500 × 3 or Rs. 19,500.

But the total profits in the first 2 months

= Rs. 7,580 + Rs. 9,875 = Rs. 17,455.

∴ the profits in the third month

= Rs. 19,500 — Rs. 17,455 = Rs. 2,045.

Exercise IX (a).

In all the following questions the answers are to be found to the nearest unit :—

1. There are 6 bags of rice containing 52, 50, 45, 40, 38, 39 measures respectively. Find the average quantity of rice contained in a bag.

2. There are 4 heaps of bricks; the first contains 680, the second 845, the third 635, and the fourth 520. Find the average number of bricks in a heap.

3. In a school there are six sections in the Fourth Form; the first is 38 strong, the second 37, the 3rd 35, the 4th 34, the 5th 32 and the 6th 40. Find the average number of boys in a section.

4. The population of 8 Municipal divisions of the City of Madras for the year 1904 is given in the accompanying table. Find the average population of a division in that year :—

Divisions of the City of Madras.	Population.
I	84,156
II	89,375
III	61,963
IV	23,223
V	86,020
VI	23,843
VII	93,854
VIII	46,912

5. The number of passengers carried on a certain line of Railway in the 9 years 1901 to 1909 is given in the following table:—Find the average number of passengers carried annually during that period.

Years.	Number of passengers carried.
1901	3,468,650
1902	5,687,930
1903	6,748,562
1904	6,343,050
1905	7,646,348
1906	8,323,640
1907	9,125,809
1908	10,348,725
1909	11,246,080

6. The expenditure on Public Instruction from Provincial Revenues in British India in the three years 1905-06 to 1907-08 is as follows:—

Year.	Expenditure.
1905-06	Rs. 1,67,10,066
1906-07	Rs. 1,85,49,258
1907-08	Rs. 1,99,07,681

Find the average annual expenditure during the period from that source.

7. The following table gives the number of male and female scholars in public and private institutions in the Madras Presidency in the official years 1898-99 to 1907-08. Find the average number of males and females under instruction during these years.

Year.	Number of male scholars.	Number of female scholars.
1898-99	715,763	124,284
1899-00	733,923	129,060
1900-01	722,151	128,078
1901-02	731,047	130,432
1902-03	752,031	133,188
1903-04	784,621	139,139
1904-05	821,205	150,037
1905-06	819,527	157,439
1906 07	842.412	164,706
1907-08	884,848	172,322

8. In the previous example construct a table showing the total number of male and female scholars for each year and find the average total number of male and female scholars during the period. Check your result by finding the average in another way.

9. The average mean temperature at a certain place in degrees Fahrenheit for each month is as follows:—January 68°, February 69°, March 71°, April 73°, May 84°, June 85°, July 84°, August 75°, September 68°, October 64°, November 62°. If the average mean temperature for the 12 months is known to be 73°, find the average mean temperature during December.

10. The number of people visiting the Museum at Madras during the 51st week in a certain year was—

Monday, 98; Tuesday, 103; Wednesday, 96; Thursday, 84; Saturday, 120; Sunday, 135. Find the average number of visitors in a day during that week; if this brought the total for the 51 weeks to 32,136, find whether the number during the given week was above or below the weekly average for the first 50 weeks.

11. Find the average of the following weights:—

64·327 g., 64·879 g. and 68·378 g. (work out by expressing each weight as a compound quantity and check your result). If each weight were increased by 2 g., what would be the average?

12. There are 4 sections of the VI Form in a certain school. The sections are 39, 38, 37 and 36 strong respectively; and the average ages are respectively 17 yrs, 16 yrs. 5 months., 16 yrs. 8 months and 17 yrs. 3 months. Find the average age of the boys of the VI Form.

13. The heights of 100 men are taken, 21 men measure 5 ft. 5 in.; 22 men, 5 ft. 6 in.; 24 men, 5 ft. 7 in.; and the rest 5 ft. 9 inches. Find the average height.

14. A town contains 60 streets, each street having 140 houses. A man wished to find roughly the population of the town and counted the number of persons in ten houses chosen at random. They were 10, 11, 10, 3, 12, 9, 9, 3, 8, 4. Taking the average of these as the number of men in a house, he calculated the population of the town. What was his result?

15. A person owns 6 houses. The average price of each house is Rs. 4,890. If he bought another house for Rs. 13,650, how would the average be affected?

16. A shepherd has 25 cows of which 4 cows give each on an average 72 measures 5 ollocks of milk in a month; 8 cows give each on an average 60 measures 9 ollocks in a month; and the rest each on an average 50 measures 7 ollocks a month. What is his monthly income if each measure is sold at 5 as.?

17. In a school, the total number of boys present in a certain week on each working day of the week both in the forenoon and in the afternoon is given below:

Day of the week.	Forenoon.	Afternoon.
Monday	523	510
Tuesdsy	535	529
Wednesday	550	568
Thursday	549	548
Friday	547	538

Find (1) the average morning attendance for the week.

(2) the average evening attendance for the same week.

18. A cricketer made the following scores in successive innings: 108, 160, 50, 20, 3, 0, 6, 57 and 63. What was his average?

19. A sum of Rs. 174-12 as. was distributed among a certain number of persons. One half of the number received Re. 1-3-6 each, and the other half Re. 1-2-6. Find the number of persons.

20. A school with 78 boys and 72 girls on the books met 432 times in a year. If each boy loses one meeting in 9 and each girl one in 8, find the average attendance of each pupil?

21. A merchant received 5 monthly consignments of iron bars of the following weights :—

1st Month	...	2 ton 8 cwt. 3 qr. 27 lb.
2nd ,,	...	3 ton 4 cwt. 1 qr. 21 lb.
3rd ,,	...	3 ton 2 cwt. 2 qr. 18 lb.
4th ,,	...	4 ton 11 cwt. 1 qr. 9 lb.
5th ,,	...	8 ton 2 cwt. 2 qr. 7 lb.

What was the average supply per month?

22. The full pension granted to an officer retiring from Government service is half his average salary for the last 3 years of his service. Determine the pension a retiring officer gets, if his salary per mensem during the last 3 years of his service was Rs. 350 for 9 months, Rs. 400 for 11 months, Rs. 425 for 10 months and Rs. 500 for 6 months.

23. Fill up the blanks in the following table giving the marks obtained by a boy in 5 different terms of his school course :—

Term.	Marks in English	Mathematics.	Vernacular.	History.	Geography.
A	28	40	...	80	90
B	35	45	37	...	75
C	46	50	38	65	...
D	63	...	39	73	33
E	23	60	41	82	42
Average for a term	...	46	36	70	56

24. The income from the passenger traffic of a railway company in a certain year was Rs. 78,43,650. Every day in that year there were 16 up trains and 15 down trains running on the line.

Reckoning 365 days for the year, find the average income derived from each train?

UNITARY METHOD.

Example 1.—If 6 cows cost Rs. 750, find the cost of 8 cows.

6 cows cost Rs. 750.

$\therefore$ 1 cow costs Rs. 750 $\div$ 6 or Rs. 125.

$\therefore$ 8 cows cost 8 $\times$ Rs. 125 = Rs. 1,000.

In this method we first find the value of 1 cow and then that of 8 cows, *i.e.*, if we are given the value of a number of things, we find the value of one thing and then that of the required number of things. This is called the **Unitary Method**.

Example 2.—If 6 iron bars weigh 3 qr. 6 lb., find the weight of 9 iron bars.

6 iron bars weigh 3 qr. 6 lb.

$\therefore$ 1 iron bar weighs $\dfrac{\text{3 qr. 6 lb.}}{6}$ or 0 qr. 15 lb.

$\therefore$ 9 iron bars weigh 15 lb. $\times$ 9 or 4 qr. 23 lb.

Note.—Th question may be easily worked thus—

6 bars weigh 3 qr. 6 lb.

$\therefore$ 3 bars weigh $\dfrac{\text{3 qr. 6 lb.}}{2}$ or 1 qr. 17 lb.

$\therefore$ 9 bars weigh 3 qr. 6 lb. + 1 qr. 17 lb., *i.e.*, 4 qr. 23 lb.

Thus it will be seen that it is not always necessary first to find the value of *one thing*, but we may find the value of a convenient group of things.

Exercise—Oral.

1. If 2 pencils cost 8 pies, what is the cost of 4 pencils, 5 pencils?

2. If 3 penknives cost Re. 1-8-0, what is the cost of 5 penknives?

3. If a maund of ghee costs Rs. 14, what is the value of a viss?

4. If a maund of sugar costs Rs. 5, what is the value of a viss?

5. If a dozen plantain fruits cost 2 annas, what is the cost of 100 such fruits?

Exercise IX (b).

1. If 10 yards of cloth cost Rs. 2, what is the cost of 1 yard correct to the nearest pie? What is the cost of 15 yards?

2. If 25 chairs cost Rs. 78 2 as., what is the cost of (1) 6 chairs, (2) 100 chairs?

3. If a ton of firewood costs Rs. 12, what is the cost of (1) a load of 56 lbs., (2) of 6 such loads? And if a cooly is to be paid 4 pies for carrying each load, what will a person buying 10 loads have to spend on the whole?

4. If 6 ton 6 cwt. of coal cost Rs. 990-8-0, what do (1) 7 cwt., (2) 10 tons cost?

5. If 12 yards of Assam silk cost Rs. 48-4-0, how many yards can be bought for Rs. 100-8-4?

6. What is the cost of 38,604 pencils at Rs. 4-8-0 per gross?

7. If each third class ticket from Madras to Rameswaram costs Rs. 5-2-0, what is the cost of 15 tickets?

8. If 100 rupees' weight of silver is worth Rs. 85, what is the cost of 35 rupees' weight of silver?

9. If a man earns Rs. 30-8-0 in 32 days, in what time will he earn Rs. 42 14-3?

10. If 8 cwt. 2 qr. 23 lb. of tea cost £55 6*s*., how much will 5 cwt. 3 qr. 6 lb. cost?

11. If a man walks 36 miles in 9 hours, what time will he take to walk 15 miles?

12. A train travels 152 miles in 9 hours 10 minutes. What time will it take to travel 363 miles at the same rate?

13. A man's wages are at the rate of Rs. 1,055-3-4 per year of 322 working days. What should be paid for 23 days' work?

14. If 46 bushels of corn cost £9 4*s*. 10½*d*., how many quarters can be bought for £1 13*s*. 6*d*.? (Answer to the nearest quarter).

15. A servant is engaged on a salary of Rs. 8 per mensem. He works from the 1st to the 20th of April both days inclusive. What should he get for his wages?

16. If 100 kilogrammes are equivalent to 220 lbs., how many kilogrammes are equal to a ton?

17. A steamer moves at the rate of 360 knots per day; how many hours will she take to describe 260 knots?

18. If a ship of 1,728 tons displacement costs £6,000, find the value of a ship of 5,280 tons displacement?

19. If a piece of copper wire 8,000 ft. long weighs 361 lbs., what will be the weight of a similar piece of wire 2,155 ft. long?

20. A person travelling from Madras to Arkonam, is charged Re. 1-5 3 for his luggage which weighs 12 maunds 5 seers; what will be the charge for the luggage of another passenger proceeding from Madras to Arkonam, which weighs 2 maunds and 30 seers?

21. If a cargo of 8,000 tons could be sent for £6,720. find the weight (to the nearest ton) of a cargo whose freight is £1,524?

22. In producing coke from coal, 1,630 lb. of coke are obtained from a ton of coal; how much coke can be obtained from 3,640 lb. of coal?

23. On a map, an inch represents 8 miles. What is the actual distance between two places which are 18·9 inches distant from one another on the map?

24. If Re. 1 = 1*s*. 4*d*., how many rupees are equivalent to £260 9*s*. 4*d*.?

25. If a ship consumes 60 tons of coal in 30 hours, how many tons does she consume in 23 hours at the same rate?

26. A man whose income is Rs. 250 pays an income-tax of Rs. 8-8-2. What does another man pay whose income is Rs. 165?

27. A clock loses 2 minutes in a week. If it is set right on Sunday at noon, by how much will it be slow on the next Sunday at noon?

After how many weeks will it show (1) 11-30 o'clock, (2) 11 o'clock, (3) 10-30 o'clock, at noon?

28. If 12 candles burnt successively, *i e*., one after another, last for 72 hours, how many candles are required to burn for 100 hours?

29. If 24 Ditmar Austrian lamps consume a gallon of kerosene oil for a whole night, how many quarts of the same oil do you require for 5 such lamps if they burn the whole night as before?

30. If the cost of constructing a railway 10 mi. 2 fur. 88 yds. be Rs. 10,876,560, what will be the cost of constructing a railway 92 mi. 0 fur. 20 yds. long?

MISCELLANEOUS EXAMPLES.

Example 1.—The sum of 2 numbers is 5060; their difference is 80. Find the numbers.

The greater number + the smaller number = 5060
The greater number — the smaller number = 80
$\therefore$ twice the greater number = 5060+80=5140.
The greater number = 2570.
$\therefore$ the smaller number = 2570 − 80 = 2490.

Or thus :—If x be the greater number and y the smaller number then

$$\begin{aligned} x+y &= 5060 \\ \text{and } x-y &= 80 \\ \text{adding } 2x &= 5140 \\ \therefore x &= 2570 \\ \therefore y &= 2570-80 \\ &= 2490. \end{aligned}$$

Example 2.—Divide Rs 460 among A, B and C giving B Rs. 20 more than C, and A Rs. 30 more than B.

B's share = C's share + Rs. 20
A's share = B's + Rs. 30 = C's share + Rs. 20 + Rs. 30
= C's share + Rs. 50
$\therefore$ A's+B's+C's = 3 C's+50+20 = 3 C's+Rs. 70.
$\therefore$ C's+Rs. 70 = Rs. 460.
$\therefore$ 3 C's = Rs. 460—Rs. 70 = Rs. 390.
$\therefore$ C's = Rs. 130.
$\therefore$ A's = Rs. 130 + Rs. 50 or Rs. 130 + Rs. 20 = Rs. 180.
$\therefore$ B's = C's + Rs. 20 = Rs. 130 + Rs. 20 = Rs. 150.

The answer is checked by testing whether the sum of all the three share equals Rs. 460.

Example 3.—Divide Rs. 200 between A, B, and C so that A may have Rs. 20 more than B and B twice as much as C.

Let the shares of A, B, C be Rs. a, Rs. b, Rs. c respectively, then

$a=b+20$
$b=2c$
$\therefore a=2c+20$
$\therefore a+b+c=2c+20+2c+c=5c+20$

But by the question the total sum to be divided is Rs. 200

i.e., $a+b+c=200$

$\therefore\ 5c+20=200$

$\therefore\quad 5c=180$

$\therefore\quad c=36$

$\therefore\ b=2c=72$ and $a=b+20=92$

$\therefore$ A gets Rs. 92, B Rs. 72, and C Rs. 36.

Example 4.—A grocer mixes 11 lbs. of tea at Rs. 3-2-0 per pound with 15 lbs. at Re. 1-6-0 per pound. He sells the whole at Rs. 2-4-0 per pound. What is his total gain ?

The cost of 11 lbs of tea at Rs. 3-2-0 per pound
= Rs. 3-2-0 × 11 = Rs. 34-6-0

Similarly the cost of 15 lbs at Re. 1-6-0 = Re. 1-6-0 × 15
= Rs. 20-10-0

$\therefore$ the total cost of 11+15 or 26 lbs.
= Rs 34-6-0 + Rs 20-10-0 = Rs. 55.

He sells 1 lb. for Rs. 2-4-0

$\therefore$,, 26 lbs. for Rs. 58-8-0

$\therefore$, the gain = Rs. 58-8-0 − Rs. 55 = Rs. 3-8-0

Or thus :—On each lb. of tea of the first sort he loses Rs. 3-2-0 —Rs. 2-4 0, *i.e.*, 14 as. On each lb. of the second sort he gains Rs. 2-4-0-Re. 1-6-0, *i.e.*, 14 as. ; and there are 11 lbs. of the 1st sort and 15 lbs. of the 2nd ; the net result is $\therefore$ a gain of 4 times 14 as. or Rs. 3-8-0.

Example 5.—A certain number of rupees, twice as many half-rupees and thrice as many quarter-rupees amount to Rs. 275. Find the number of *coins* of each sort.

If there is 1 rupee, there must be 2 half-rupees and 3 quarter-rupees. The total amount on this supposition = Re. 1 + Re. 1 + 12 as.

$\therefore$ if Rs. 2-12 or 44 as. be the total amount, the number of rupee coins is 1.

If Rs. 275 or 4400 as. be the amount, the nnmber of rupee coins is 4400 ÷ 44 or 100.

i.e., there are 100 rupees, 200 (half-rupees) and 300 (quarter rupees).

The symbolic method of working the question is left as an exercise to the student.

Example 6.—The forewheel of a carriage whose circumference is 11 ft. makes in 1 mile 120 revolutions more than the hind-wheel. Find the circumference of the hind-wheel.

The number of revolutions made by the fore-wheel in one mile $= 1760 \times 3 \div 11 = 480$.

$\therefore$ the hind-wheel makes $480 - 120$ or 360 revolutions in going over the same distance.

$\therefore$ the circumference of the hind-wheel

$$= \frac{1 \text{ mile}}{360} = \frac{1760 \times 3}{360} \text{ ft.} = 14 \text{ ft. } 8 \text{ inches.}$$

Example 7.—I add 40 to a given number and subtract 80 from it. I multiply the remainder by 4 and divide the result by 16. The result is 80. Find the given number.

In the final division the Divisor is 16 and the Quotient 80.
$\therefore$ the Dividend $= 1280$.

This by the question is got by multiplying the remainder (mentioned in the question) by 4.

$\therefore$ that remainder must be $\frac{1280}{4}$ or 320.

If the original number is N, by the question, $N + 40 - 80$ is this remainder and is equal to 320.
i.e., $N - 40 = 320$. $\therefore N = 360$.

Exercise IX (c).

1. Find the sum of all prime numbers between 70 and 100. (A prime number is one which has no factor other than unity).

2. Find the sum of 5 consecutive numbers beginning with 156825.

3. Show that out of any three consecutive numbers the sum of the first and the third is twice the middle number.

4. The ages of the members of a family are 48, 34, 26, 18, 9, 3. How old were the others when the youngest was born?

5. How much must be added to the product of 85 and 37 to give the product of 85 and 39?

6. The product of two numbers is 64941912 and one of them is 12978. Find the other number.

7 Find the least whole number above 6000 which is divisible by 73.

8. The continued product of 2, 5, 7, 9, 13 and a certain number is 1711710. What is that number ?

9. Split [illegible]87325 into two numbers of which one is greater than the other by 1525.

10. Supply the missing figures in the following :—

(a) 3864 * * divided by 346 leaves a remainder 2.

```
(b)  5 4 .          (c)  1 2 3 8
       6 8                 2 . 3 .
    --------           ------------
    . . 8 .            . . . .
     4 3 . .            . . 3 .
    --------              . . . .
    . . 1 . .              4 9 . .
                       ------------
                       . 6 4 1 8 . .
```

11. In a train containing 620 passengers, the total number of first and third class passengers was 605, that of the second and third class 615. How many passengers of each class were there ?

12. At a game of cricket, A, B and C together score 287 runs. A and B together score 221 runs, while B scores 68 more than C. Find the number scored by each.

13. A father is twice as old as his son and their combined ages are 60 What will be their ages 4 years hence ?

14. The value of a house is equal to 8 imes that of its furniture which is worth 24 times as much as the matting. If the house alone be worth Rs. 1,920 find the value of the furniture and the matting.

15. Five cows and 8 sheep cost Rs. 140 but the cows cost Rs. 60 more than the sheep. Find the price of a cow and a sheep respectively.

16. Two men bought in common 300 mangoes at 7 pies each ; and one paid the seller Rs. 3-10-4 more than the other. How many fruits should each take ?

17. Two boys having an equal share in a box containing pencils agreeed to divide them thus : one to take 60 pencils, the other to take 80 pencils and give the first annas ten. Find the price of a pencil.

18. An equal number of men, women and boys earn altogether Rs. 136-8 ; a man earns 12 as., a woman 7 as. and a boy 5 as. How many of each set are there ?

19. A certain number of men, twice as many women and three times as many boys earned in a week of 5 days £15-10*s*. each man earned 3*s*., each woman 1*s*. 8*d*. and each boy 1*s* 4*d*. a day. How many were there of each set and how many on the whole ?

20. If a certain number of sovereigns, three times as many crowns and 7 times as many florins amount to £24·10*s*., how many coins are there of each sort ?

21. A bag contains half annas, quarter annas, and two-anna pieces ; the amounts expressed by the different coins are equal. If there are 650 coins in the bag, find the number of coins of each sort.

22. A mixture is made of 20 gallons of spirit at 1*s*. 2*d*. a gallon, 16 gallons at £1-5-8 a gallon and 14 gallons at £1-1 ; it is sold at 9*s*. 6*d*. per gallon. What is the gain or loss ?

23. A grocer bought sugar for Rs. 45 ; 160 seers at 9 p. a seer, and the remainder at 8 as. a viss. He mixed the two and sold the whole at 9 as. a viss. Find his gain.

24. A dealer bought a flock of 400 sheep : 240 at Rs. 3-8 a head and the rest at Rs. 2-8 a head. He sold the whole and found he gained Rs. 180. What was the selling price of a sheep on an average ?

25. A railway engine runs at the rate of 24 miles per hour, and the circumference of its wheel is 20 ft. Find how many revolutions it will make in 1 hr. 20 minutes.

26. The driving wheel of a locomotive is 264 inches in circumference and makes 80 revolutions per minute. At what rate per hour does the engine travel ?

27. The fore-wheel of a carriage whose circumference is 10 ft. makes in a distance of 1 mile 132 revolutions more than the hind-wheel. Find the circumference of the hind-wheel.

28. A father left by will £3,322 to be divided among his 3 sons A, B and C ; A was to have £75 more than B and C £109 more than B. Find the share of each son.

29. From 321,785 subtract any smaller number consisting of the same 6 digits in a different order and divide the remainder by 9.

30. 198 men are employed on a certain piece of work and each of them receives 8 as. per day. If 22 of them fall ill and their daily work and wages are divided among the others, what will be the daily wages of each man doing the work?

31. A sum of Rs. 2,040 is to be distributed between 10 men, 32 women and 48 children. If each man's share is equal to that of 2 women and if all the women have twice as much as all the children, how much will the several individuals receive? (Answer to be given to the nearest pie.)

32. If 11 ton 10 cwt. of coal cost £138, what is the price per cwt.? Find the cost of 75 ton 5 cwt. of coal.

33. In a certain year 1,670,900,000 of anna and 251,800,000 of $\frac{1}{2}$ anna and 648,700,000 of $\frac{1}{4}$ anna postage stamps were issued. Find their value in rupees.

34. A bill-collector received from one party Rs. 15-8-4; from another party Rs. 315-1-8; from a third Rs. 90-8-9. On returning home he found in his purse Rs. 463-12-7. With how much money in the purse had he started before he made the collections?

35. What sum will remain when 4 bills amounting to £6-18*s*.-4*d*., £14-19*s*.-7*d*., £3-16*s*.-2*d*., and £12-17*s*.-2*d*. respectively have been paid out of £47.

36. Make out a bill for the following :—

25 quires of paper (post) at 4 as. 10 p. a quire; 1800 envelopes at Rs. 1-4-6 a hundred; 13 gross of pen-holders at Rs. 2-6-4 a dozen; 33 copy slips at as. 2-9 each; 4 dozen account-books at Rs. 2-9-8 each book.

37. The following articles were purchased and the grocer was paid £200. What should the purchaser claim from him as balance? 1 cwt. of candles at 6*d*. a lb.; 648 lb. of coffee at 1*s*. 10*d*. a lb.; 683 yards of cloth at 1*s*. 9*d*. a yd.; 629 caps at 2*s*. 8*d*. each; 2 gross of copy-books at 5*d*. each copy-book.

38. At a public examination of 72 candidates the following detailed expenses were exactly covered by the examination fees

paid by the candidates. Find the fee paid by each candidate: 6 yards of cloth at 3 as. 11 p. per yard, 3 sticks of sealing wax at 1 a. 10 p. per stick, 10 papers of pins at 3 as. 7 p. per paper, 6½ dozen bottles of ink at 7 pies per bottle, 1 gross steel pens at 7 as. 3 p. a dozen, 7 reams of paper at Re. 1·14·6 per ream, Examiner's fee Rs. 130, Miscellaneous charges Rs. 21-9·7.

39. A has 280 cows, each worth Rs. 125 and B has 60 horses, each worth Rs. 650. Supposing they exchange their property, which man should give the other money, and how much?

40. A safe and the money it contains are worth Rs. 51,180-4-6 and the money in the safe is worth 500 times the value of the safe. Find the value of the latter.

41. 165 subscribers contributed in equal shares a sum of Rs. 15,625 towards an enterprise which was afterwards given up. The money was returned to the subscribers, after Rs. 515-10-0 was paid for expenses out of the collections. Find how much each subscriber got back.

42. A clerk whose salary is Rs. 35-8-0 per month is fined 0-8-0 for each day he is late. If in four months he received Rs. 130-8-0, how often was he late?

43. A, B and C go on a tour and agree to share the expenses equally. Each starts with Rs. 100, and at the end A has Rs. 38-10-8, B has Rs. 28-6-4, and C has Rs. 5-9-8. How must hey settle accounts?

44. In a distribution of alms each child had 3 pies, each woman twice as much, and each man three times as much. The whole sum distributed was Rs. 180. How mauy were there in all supposing that there were twice as many men and thrice as many women as boys?

45. A gives to B 100 gallons of beer worth 20*s*. 6*d*. a gallon return for 40 sovereigns and 140 yards of silk. What is the [illegible] of the silk per yard?

REVISION PAPERS. I Series

1

1. Express in words 18324576 according to both the Indian and English systems.

2. Divide 87,634 by 128 using factors. Explain the method by which you find the remainder.

3. Draw a rectangle 9·5 cm. by 11·3 cm. Find by measurement the distance from one corner to the opposite corner. Find also the sum of the four sides of this rectangle.

4. $N \times 30 + 48 = 1548$. What is N ?

5. Reduce £12 16*s*. 8*d*. to pence and check your answer by changing the answer back to pounds, shillings and pence.

6. Add together 847·3, 687·98 and 438·357, and subtract the result from 3000. Check your final result by testing whether the first three numbers and the result added together give 3000.

7. A man's salary is *a* rupees per month. He spends *b* rupees a month and sends the rest to the Savings Bank. What is the sum at his credit in the Savings Bank at the end of 4 months ?

8. If every tram-car has a conductor and a motor-man who are paid Rs. 20 and Rs. 15 per mensem, what are the total annual expenses under this item of a company which runs 72 cars ?

2.

1. Explain the difference between
$80-40-35$ and $80-(40-35)$.
Simplify $87+60\times 45-34\times 3$.

2. Multiply 8625 by 64 and hence find the product of 8625 and 6464.

3. Divide 64832 by 25 (1) by using factors; (2) by multiplying the number by 4 and dividing the result by 100. Account for the difference in the remainder in the two cases.

4. A ———————————————————————— B

Draw on tracing paper a line AB equal in length to the line given here. Find the middle point of AB by folding. Measure the length of AC where C is the middle point. Check your result by measuring AB and taking half of it.

5. a rupees + 105 quarter annas + 230 quarter rupees = Rs. 119-2-3. Find the value of a.

6. The working below gives the division of £96·5·8 by 143 using factors. Write down the remainder, and check the quotient and the remainder by changing the order of the factors.

	£	s.	d.	
11)	96	5	8	
13)	8	15	0	— remainder 8d.
	3	10	5	— remainder 7d.

7. The sum of the lengths of 2 rods is equal to 5m. 7 cm. 8 mm. and their difference equals 1 m. 3 cm. 4 mm.; find the lengths of the two rods. Check your result by working out the question in decimals of a metre.

8. Twenty horses were bought for Rs. 650 each, 35 at Rs. 570, 65 at Rs. 470 each. What was the average price of each horse?

3

1. Write the digits of 64327 in any other order so that the number so formed may be less than the given number. Show that the difference between the two numbers is divisible by 9.

2. Fill up the 4th and the 5th columns in the following table :—

Month and Date.	Sunrise.	Sunset.	Length of day.	Middle of day.
Jan. 1	6—28	5—30	...	...
,, 10	6—27	5—33	...	...
,, 20	6—26	5—34	...	...
,, 30	6—22	5—38	...	...
Feb. 10	6—17	5—43	...	...
,, 20	6—15	5—45	...	...
,, 28	6—11	5—49	...	...

3. The lengths of 4 straight lines are 13·5 cm.; 12·3 cm.; 11·2 cm.; and 7 cm. Construct a square having a side equal to the average of these lengths and find by measurement the length of the diagonal of that square.

4. "If the price of a maund is given in rupees, multiply the number of rupees by 2 and the result will be the price of a viss in

annas." Apply this rule to find the price of a viss of sugar if a maund costs Rs. 3. How do you account for the rule?

5. (*a*) Find the value of a^2-1 when a = 10, 15, 16.
(*b*) Find the value of a^3+1 when a = 20, 25.

6. Mars makes a revolution round the sun in 1 year, 10 months, 16 days, 19 hours (1 month = 30 days.) How many revolutions will it make in 188 years?

7. A person bought 16 sovereigns at Rs. 15-2-6 each to make a gold belt and paid Rs. 15 as making charges. After a few years he was obliged to sell it at the rate of Rs. 5-14 per pagoda weight. Assuming the weight of 4 sovereigns equals that of 9 pagodas, find how much he lost.

8. A watch which gains 1 minute a day was set right at noon on Monday the 16th Jauuary 1911. What time did it show at noon on the 16th of March 1911?

4.

1. A train runs at the rate of 30 miles per hour. Express the velocity of the train in feet per second.

2. Multiply 287654 by 105735 in 3 lines, and check your answer by the method of casting out the nines.

3. Find the factors of 1456, and use these factors in dividing 873546 by 1456 and check your result by testing whether the relation 'Dividend = Divisor × Quotient + Remainder' is satisfied.

4. A man walks 40 yards in an eastern direction, then turning towards the north walks 30 yds., he then turns west and walks 80 yds and finally returns to the starting point. Draw a figure (on squared paper) showing his path. taking 1 inch to represent 10 yards; and find by measurement how many yards he has travelled on the whole and in what direction he walked last to return to the starting point.

5. A person buys x books at the rate of a rupees each, and y slates at the rate of b annas each. Express the total sum paid in annas.

6. Multiply 9 g. 85 mg. by 260 and give the answer as a compound quantity in kilogrammes, grammes, etc.

7. If the cost of 1,000 bricks is Rs. 5-10, find the cost of 950 bricks.

8. In a class of 40, in a certain examination in Arithmetic 10 get 28 marks; 20 get 25 marks, and the rest get 42, 26, 18, 13, 17, 9, 8, 5, 3, 0. Find the average marks of the class.

5

1. A train consisting of 12 carriages of 6 compartments each, each compartment accommodating 10 persons, starts with its full complement of passengers. At the second station it drops 38 and takes in 25. At the next station it drops 47 and takes in 32. Represent the changes symbolically and find the number in the train when it arrives at the fourth station.

2. Simplify, with as little multiplication as possible, $773 \times 529 - 769 \times 529$.

3. A ladder placed with one extremity against a wall and the other resting on the ground at a distance of 12 ft. from it is found to reach a point 16 ft. from the ground. Draw a plan taking an inch to represent 10 ft., and find by measurement the length of the ladder.

✓ 4. $87559 = a \times 9 + b$ where b is less than 9, what is a and what is b?

5. How many times round a rectangular garden 500 ft. long and 140 ft. broad will make a walk of 5 miles?

6. If n be a number, express in terms of n (1) the number next above n, (2) the number next below n; and if the sum of these three numbers is 330, find the number.

7. Make out a bill for 185 needles at 5 for 2 pies; 9 dozen buttons at Rs. 4-8 as. per gross; 30 yds. of cloth at Rs. 10 a piece (of 40 yds.) and 5 oz. of thread at Rs. 3 per pound.

8. A woman buys 100 eggs at 4 for an anna, and 180 eggs at 6 for two annas and sells the whole at 10 for 3 as. Does she gain or lose and by how much?

6

1. Show, by a mere examination, why the following results are wrong:

(a) $1876 \times 7625 = 16248930$.
(b) $87 \times 35 = 2345$.

2. Given that when 300 is divided by 21 the quotient is 14 and the remainder 6, and that when 543 is divided by 21 the quotient is 25 and the remainder is 18, find the quotient and the remainder when 843 is divided by 21.

3. Find graphically with the help of your protractor (1) $45^\circ+30^\circ$; (2) $45^\circ-30^\circ$. Check your result by measuring the angles.

4. Express the following numbers in the form $a \times 10^m$;
(*a*) 7836000, (*b*) 643890000. (*c*) 180000000.

5. A gramophone together with 5 rolls of music costs Rs. 105, the same gramophone with 20 rolls of music costs Rs. 135. How much should be paid for the gramophone with 30 rolls of music?

6. A fleet that can steam 10 miles an hour on the average has to relieve a blockaded port 8640 miles away. The port can hold out only for 30 days. Will the fleet be in time to relieve the port? If not, how should it steam so as to be just in time for succour?

7. A jewel was valued by five appraisers for Rs. 1,250, Rs. 1,365, Rs. 1,500, Rs. 1,240 and Rs. 1,650, respectively. If the owner of the jewel had originally purchased it for Rs. 50 more than the average of these estimates, how much did he gain by selling it to the last appraiser?

8. In a purse there are a certain number of sovereigns, twice as many crowns, five times as many shillings. The total amount of all the coins is £105. Find the number of each sort.

7

1. I go shopping with a fifty-rupee note, and make the following purchases: 12 shirts at Rs. 1-2-0 each; 4 pairs of socks at Rs. 2-4-0 per pair; 2 ties at 8 as. each; 2 pairs of boots at Rs. 3-12-0 each pair. How much will be left in my purse?

2. Divide 168438 by 2240 using factors. Select as factors the divisors that would occur in reducing 168438 lbs. to tons. cwt., qr. lb.; and calculate the remainder in the division.

3. Describe two concentric circles whose radii are 9·8 cm. and 10·5 cm. Draw any diameter cutting the circles in A, B, C, D. Calculate AB and CD. Check your result by measurement.

4. What is the difference between $a \times b + c$ and $a (b + c)$? Find their values when $a = 21$, $b = 13$ and $c = 7$. Show graphically on squared paper that $10^2 = 7^2 + 3^2 + 2.7.3$.

5. A retail dealer buys penknives at Rs. 48 per gross and sells them at Rs. 7 per dozen and makes a profit of Rs. 72. How many knives should he have purchased and sold ?

6. Three people A, B and C starting on a three days' trip agree to divide their expenses equally. A pays the first day's charges which amount to £3-17-8; B the second day's, *viz.*, £5-3-0 and C the third day's, *viz*,, £4-6-10. Show how they can settle accounts most simply.

7. How many revolutions of a wheel of a locomotive running at the rate of 25 miles an hour make in 5 minutes if the circumference of the wheel is 250 inches ?

8. At a certain place the readings of Fahrenheit's thermometer at noon of each day were as follows :—Sunday 68°, Monday 69°, Tuesday 66°, Wednesday 72°, Thursday 74°, Friday 70° and Saturday 78°. What was the average reading of the thermometer at noon for the week ?

8

1. Find all the prime factors of 10989, and check your result by finding the continued product of those factors.

2. The population of a country on January 1st, 1909, was 3,168,000 and on January 1st, 1910, 3,248,000. If the increase were the same every year, what would be the population on January 1st, 1912 ?

3. Divide 23,561 by 299. Add the quotient to 23,561 and divide the result by 300. Show that the remainders are the same and the quotient too the same.

4. If 1 ounce of standard gold is worth £3-17*s*. $10\frac{1}{2}$*d*. show that with 40 lbs. (1 lb. = 12 oz. Troy) of gold you can make 1,869 sovereigns.

5. Take a point O. At O draw successively 12 angles each equal to 30°. With O as centre and a radius equal to 1·5 describe a circle cutting the arms. Join the points of intersection in succession, measure the sides and the angles of the figure so formed.

6. If (a) (b) (c) represent the successive digits in a number of three figures, express the value of the number in terms of a, b, c.

7. The number of spectators at a circus performance was 5,672. The sum of 4 as. was charged for admission and an additional sum of 4 as. each was charged for reserved seats. If the total receipts were Rs. 1,620, how many persons occupied reserved seats ?

8. A owes a debt of Rs. 5,625. His property however, is worth only Rs. 3,845-11-4. Find, to the nearest pie, how much he can pay in the rupee.

9

1. Below are given certain barometer readings in inches. Find the change between each reading and the next, writing + for a rise and — for a fall : Nov. 30th 3 A.M., 29·23 ; 5 A.M., 29·30 ; 8 A.M., 29·40 ; 10 A.M., 29·27 ; 2 P.M., 29·18 ; 4 P.M., 28·65.

2. A person bought a house for Rs. 2,700. The cost of the sale-deed was Rs. 30 and the registration charges amounted to Rs. 10. He spent Rs. 1,500 to improve it. For how much should he sell it to gain Rs. 500 on the whole ?

3. A boy multiplies 843 by a number and obtains 92289 as the product. If both the twos are wrong, but the other figures are right, find the multiplier and also the correct product.

4. Show graphically that $7 \times (5 + 3) = 7 \times 5 + 7 \times 3$.

5. Two straight lines AO, OB, 2·5 and 3·2 cm. long respectively intersect at O at an angle of 63°. From A and B draw AD and BD ⊥ to AO and BO respectively meeting at D. Find the angles BDA. Change the lengths of AO, OB and repeat the experiment 3 times. What do you notice about the angle BDA ?

6. A railway company opens a trial station at a place and before beginning to establish a regular station there, wants to ascertain what would be the average monthly receipts. The monthty collections in a certain term are as follows :—

		Rs	A.	P.
January	...	153	8	6
February	...	158	3	9
March	...	162	4	6
April	...	181	8	9
May	...	193	12	6
June	...	342	10	9

Calculate the average monthly receipts for the term.

7. A bicyclist rides up a hill 3 miles long in 24 minutes and then down a hill of the same slope 4 miles long in 16 minutes. How many minutes will it take him to ride back ?

✓ 8. If 14 lbs. of tea costing 1*s*. 8*d*. per lb. is mixed with 16 lbs. at 1*s*. 10*d*. per lb., what is the total cost of the mixture ?

10

1. Divide six millions eight hundred and ninety-one thousand seven hundred and fifty-five by two hundred and twenty-four, by factors. Express the result in words and explain the process by which the remainder is obtained.

2. Make out a bill for the following :—

2 md. of sugar at 5 *a*. 6 *p*. per viss ; 3 md. of arecanut at 15 *a*. per viss ; 5 md. 6 viss of ghee at Rs. 2-2-0 per viss ; 2 md. 4 viss of coffee at Re. 1-10-0 per viss.

3. Take a sum of money (Rs. A. P.) less than Rs. 12. Under this write the sum obtained by interchanging the rupees and pies. Subtract the smaller of these from the greater. Interchange the rupees and pies in this difference and add the two together.

e.g.,

Rs.	A.	P.
8	3	7
7	3	8
0	15	11
11	15	0
12	14	11

Try this with different sums. Do you always get the same answer ? Try the same operations with (1) yds. ft. inches ; (2) £ *s*. *d*.

4. How many revolutions does (1) the minute hand, (2) the hour hand of a clock make in a week ?

5. A rectangle is 280 yds. long and 210 yds. broad. A man walking at 70 yds. per minute starts from one corner of the field and goes to the opposite corner. Which will be the shorter cut : (1)

going along the sides or (2) walking diagonally? What time can be saved by taking the shorter cut?

6. If a hectolitre of wine costs £1-5, how many litres should one get for £1 at the same rate?

7. A man walks for 2 hours at the rate of x miles an hour and cycles for 3 hours at the rate of y miles an hour. What is the total distance travelled?

8. The following table gives the number of successful candidates at the Matriculation, F.A. and B.A. Examinations of a certain College for the 5 years 1905 to 1909. Fill in the blanks in the table:—

Year.	Matric.	F.A.	B.A.
1905	42	83	120
1906	39	84	115
1907	38	...	113
1908	37	78	146
1909	40	96	158
Average ...	...	84	...

CHAPTER X.

MULTIPLICATION AND DIVISION OF DECIMALS.

§ 72. Multiplication and Division by ten and powers of ten.

If we multiply 8 mm. by 10 we get 80 mm. or 8 cm.; similarly if we multiply 7 m. by 10 we get 70 metres or 7 Dm.; or if we multiply 9 dm. 7 cm. by 10 we get 9 m. 7 dm. 0 cm.

The effect of multiplying any metric compound quantity by 10 is that every denomination in it is put one grade higher, *i.e.*, is converted into the next higher denomination. Similarly it may be seen that the effect of dividing a metric compound quantity is that every denomination in it is put one grade lower, *i.e.*, is converted into next lower denomination.

Exercise—Oral.

Express as a compound quantity—

1. 8 m. 3 dm. 7 cm. × 10.
2. 9 m. 2 dm. 1 cm. × 10.
3. 9 Km. 8 Dm. 9 dm. × 10.
4. 7 g. 8 dg. 7 cg. 9 mg. multiplied by 10.
5. 7 dg. 8 cg. multipliad by 10.
6. What is the value of 8 cm. 9 dm. × 10, and of 8 cm. 9 dm. × 10 × 10. Can you give the answer at once without multiplying twice. What is the effect of multiplying by 100?
7. What is the value of—
   (1) 8 Dm. 9 m. 7 cm. × 100?
   (2) 9 m. 8 dm. 7 cm. 5 mm. × 100?

8. What is the value of—

(a) 7 m. 8 cm. 9 mm. ÷ 10 ?

(b) 8 Dg. 7 dg. 5 cg. ÷ 10 ?

(c) 9 Kg. 8 Hg. 7 g. ÷ 10 ?

9. What is the value of 6 m. 8 dm. 9 cm. ÷ 10 and 6 m. 8 dm. 9 cm. ÷ 10 ÷ 10. Can you give the answer without two divisions ? What is the effect of dividing by 100 ?

Example 1.—Find the value of 3·83 m. × 10.

3·83 m. × 10 = 3 m. 8 dm. 3 cm. × 10 = 3 Dm. 8 m. 3 dm. = 38·3 m. (expressing in metres).

Example 2.—Find the value of 3·765 m. × 100.

3·765 m. × 100 = 3 m. 7 dm. 6 cm. 5 mm. × 100 = 3 Hm. 7 Dm. 6 m. 5 dm. = 376·5 m. (expressing in (metres).

From the above examples it will be seen that multiplying a decimal by 10 and 100 is effected by shifting the decimal point one place and two places to the right respectively. The rule may be extended to the multiplication by any power of 10.

Exercise—Oral.

1. Multiply each of the following by (a) 10 ; (b) 100 ; (c) 1,000 ; (d) 10,000 (1) 8·325 m. (2) 65·389 Kg. (3) 98·037 in. (4) £48·365 (5) 687·345 miles.

2. Find the value of $83{\cdot}75 \times 10$; $98{\cdot}3659 \times 1{,}000$; $96{\cdot}584 \times (10)^5$: $106{\cdot}32 \times (10)^6$.

Example 3—Divide 83·75 m. by 10.

83·75 m. ÷ 10 = 8 Dm. 3 m. 7 dm. 5 cm. ÷ 10 = 8 m. 3 dm. 7 cm. 5 mm. = 8·375 m, (expressing as decimals of a metres).

Example 4—Divide 76·782 m. by 100.

376·7 m. ÷ 100 = 3 Hm. 7 Dm. 6 m. 7 dm. ÷ 100 = 3 m. 7 dm. 6 cm. 7 mm. = 3·767 m. (expressing as decimals of a metre).

Thus the division of decimals by 10 and 100 is effected by shifting the decimal point one place and two places to the left respectively. The rule may be extended to the division by any power of 10.

Exercise—Oral.

1. Divide each of the following by (*a*) 10, (*b*) 100, (*c*) 1,000. (1) 468·7 m. (2) 368·95 g., (3) 643·89 Kg. (4) £853,96, (5) 182·35 miles.

2. Find the value of 8·73 ÷ 10; 96·358 ÷ 100; 8168·93 ÷ 1000; 6·8548 ÷ $(10)^4$; 1238·4569 ÷ $(10)^6$.

§ 73. Multiplication by an integer.

Example 1.—What is the total length of 6 rods, each being 8 m. 7 dm. 9 cm. 8 mm. long?

Here we have to multiply 8 m. 7 dm. 9 cm. 8 mm. by 6 and the work is shown below. The same may be looked upon as multiplying 8·798 m. by 6 and the corresponding processes are written opposite to each other.

m.	dm.	cm.	mm.	(or expressed in metres)
8	7	9	8	8·798 m.
			6	6
52	7	8	8	52·788 m.

Since multiplication is only a shortened form of addition, the method shown on the right is only a shortened form of the addition shown in the margin.

Thus the procedure is the same as in the multiplication of ordinary numbers. Only the decimal point is marked in the product as we pass it.

8·798
8·798
8·798
8·798
8·798
8·798
―――
52·788

Example 2.—What is the weight of 80 stones, each stone weighing 12·356 cwt. The required weight = 12·356 × 80 cwts.

= 12·356 × 10 × 8 cwts. = 123·56 × 8 cwts. = 988·48 cwts.

Exercise—Oral

Multiply

1. 4·6 by 2; 3; 4; 5.

2. 8·9 by 6; 7; 8; 9,

3. 0·5 by 2; 4; 5; 6.

4. The length of a pencil is 5·8 inches. What is the length of 4 such pencils?

5. If a metre be taken as 39·4 in. long, find the length of 7 metres.

6. If a Kilogramme be taken to weigh 2·2 lb., what is the weight of 8 Kilogrammes?

7. Multiply 8·76 by 60; 600; 6000.

8. Multiply 5·13 by 9; 90; 9000.

Exercise X (a).

Multiply (1) as compound quantities, (2) as decimals.

1. 8 g. 3 dg. 7 cg. 9 mg. by 6; 7; 8; 9.

2. 3 m. 9 dm. 4 cm. 5 m. by 8; 9; 2; 3.

3. 8 fur. 6 ch. 38 links by 3; 5; 7; 8.

4. Multiply 86·356 by (1) 7, (2) 63, (3) 72, (4) 64, (5) 50, (6) 60, (7) 700, (8) 800 (using factors).

5. Multiply 0·865 by 3, 9, 600, 900.

6. Find the height of a pile of 126 boards, the thickness of each board being 1·2 cm.

7. Find the height of a pile of 90 bricks, if each brick is 1·12 in. thick.

8. Find the cost of 700 grammes of gold at Rs. 2·36 per gramme.

9. The weight of one cubic centimetre of mercury is 13·6 grs. What is the weight of 1 litre (1000 cubic centimetres) of mercury?

10. A tram-car runs at the rate of 15·85 metres in 1 second. At how many kilometres per hour is it running?

11. Light travels at the rate of $1·86 \times (10)^5$ miles per second. Find the distance of the sun from the earth if a ray of light takes 8 minutes to reach the earth from the sun.

§ 74. Multiplication by a single figure (decimal).

If a golden wire of a certain thickness and one inch in length costs Rs. 5, a similar wire 3 inches long would cost Rs. 5×3, a wire 4 inches long would cost Rs. 5×4. Similarly the cost of a similar wire $\cdot3$ inches in length would be written as Rs. $5 \times \cdot3$. We say the cost would be *written as* Rs. $5 \times \cdot3$ because $5 \times \cdot3$ is an operation which is meaningless to us, for it is, according to our definition of multiplication, a shortened form of saying 'write down 5, $\cdot3$ times and add;' which conveys no meaning, since $\cdot3$ is a fraction. We can however find the cost of $\cdot3$ in. of the wire and thus get a meaning for the operation $5 \times \cdot3$.

$\cdot3$ inches means three-tenths of an inch. Since an inch costs Rs. 5, a tenth of an inch costs Rs. $5 \div 10$ and 3 tenths or $\cdot3$ in. of the wire costs Rs. $5 \div 10 \times 3$.

Thus $5 \times \cdot3$ must be taken to mean $5 \div 10 \times 3$.

Again $5 \div 10$ is $\cdot5$ (according to Art. 72). $\therefore 5 \div 10 \times 3 = \cdot5 \times 3 = \cdot5 + \cdot5 + \cdot5 = 1\cdot5 = \cdot3 \times 5$, *i.e.*, with this meaning given to $5 \times \cdot3$ it is seen that $5 \times \cdot3 = \cdot3 \times 5$. Similarly we can show that $7 \times \cdot4 = \cdot4 \times 7$, &c., *i.e.*, the relation $a \times b = b \times a$ which has been proved for integers is found to be true when either a or b is a decimal fraction.

Also we have $5 \times \cdot3 = 1\cdot5 = 15 \div 10 = 5 \times 3 \div 10$. This shows incidentally that $5 \div 10 \times 3 = 5 \times 3 \div 10$ (a result given in general terms in Art. 62) both being equal to $5 \times \cdot3$.

Further we can similarly show that $3 \times \cdot1$ means $3 \div 10$ or $\cdot3$. $\therefore 5 \times \cdot3 = 5 \times 3 \times \cdot1$, and also that $5 \times \cdot03 = 5 \times 3 \div 100$, &c.

Example 1. Multiply 74·68 by ·3.

$74\cdot68 \times \cdot3 = 74\cdot68 \times 3 \div 10 = 224\cdot04 \div 10 = 22\cdot404.$

The work may be arranged thus :

$$\begin{array}{r} 74{\cdot}68 \\ 0{\cdot}3 \\ \hline 22{\cdot}404 \end{array}$$

Example 2. Multiply 85·689 by 0·07.

85·689 × ·07 = 85·689 × 7 hundredths = 85·689 × 7 ÷ 100 = 599·823 ÷ 100 = 5·99823.

The work may be arranged thus :

$$\begin{array}{r} 85{\cdot}689 \\ 0{\cdot}07 \\ \hline 5{\cdot}99823 \end{array}$$

Note.—In Example 1 the number of decimal places in the product is 3, *i.e.*, 2 + 1, *i.e.*, the number of decimal places in the multiplicand + the number of decimal places in the multiplier; in Example 2 the number of decimal places in the product is 5, *i.e.*, 3 + 2, *i.e.*, the number in the multiplicand + the number in the multiplier. *When two decimals are multiplied the number of decimal figures in the product is equal to the sum of the numbers of the decimal figures in the multiplicand and multiplier.*

This fact is useful in checking the position of the decimal point in the product.

Exercise—Oral.

Multiply

1. 3·5 by ·04. **2.** 4·8 by 0·9.

3. 0·06 by ·6. **4.** 0·38 by ·08.

5. 13·4 by 0·03. **6.** 16·8 by 0·04.

7. 0·003 by 0·03. **8.** 0·005 by ·08.

9. 6·83 by 0·6 **10.** 3·85 by 0·8

11. Given that 18·3 × ·6 = 10·98, write down the product of 1·83 by ·06.

12. Given that ·008 by ·07 = ·00056, write down the product of ·8 × ·007.

§ **75. Rough Checks**. The product of two numbers is unaltered if either is multiplied by some power of 10 and the other divided by the same power of 10.

We have shown above that (5 ÷ 10) × 3 = 5 × (3 ÷ 10). Similarly the fact given here may be established.

Example. 35×45
$= 35 \times (4{\cdot}5 \times 10)$
$= 35 \times 10 \times 4{\cdot}5 = 350 \times 4{\cdot}5$ .... (*a*)
$= 350 \times ({\cdot}45 \times 10) = 3500 \times {\cdot}45$ .... (*b*)
Also $35 \times 45 = (3{\cdot}5 \times 10) \times 45 = 3{\cdot}5 \times 450$ .... (*c*)
$= {\cdot}35 \times 4500$ .... (*d*)

Exercise—Oral

Fill in the blanks in the following equalities :—

1. $700 \times 90 = 7000 \times (\quad)$.
2. $2600 \times 970 = (\quad) \times 97$.
3. $94 \times 5 = (\ldots\ldots) \times 50$.
4. $0{\cdot}266 \times 4{\cdot}73 = (\quad) \times {\cdot}473$.
5. $86{\cdot}75 \times {\cdot}005 = (\ldots\ldots\ldots) \times {\cdot}5$.

Example—Find roughly the value of $18{\cdot}63 \times 0{\cdot}835$. The simplest method is to alter the position of the decimal point in the multiplier, without the product being affected, so that there is only one figure in its integral part.

$18{\cdot}63 \times 0{\cdot}835 = 1{\cdot}863 \times 8{\cdot}35 = 2 \times 8$ or 16 roughly and the product is a little more than $18 \times {\cdot}8$ or $14{\cdot}4$.

Find roughly the value of the following :—

1. $2{\cdot}07 \times 4{\cdot}2$.
2. $0{\cdot}063 \times 3{\cdot}123$.
3. ${\cdot}089 \times 1600$.
4. $(1{\cdot}26)^2$.
5. ${\cdot}65 \times {\cdot}298$.
6. $243{\cdot}01 \times 653{\cdot}7$.
7. $483 \times 0{\cdot}015$.
8. $({\cdot}57)^2$
9. $157{\cdot}3 \times 4{\cdot}892$.
10. $4680 \times {\cdot}072$.
11. Given that $819 \times 173 = 141687$, find the value of—
(1) $81{\cdot}9 \times 0{\cdot}173$. (2) $8{\cdot}19 \times 0{\cdot}173$. (3) ${\cdot}819 \times 17{\cdot}3$.

§ 76. Multiplication : General case.

Example 1 :—To multiply $37{\cdot}82$ by $5{\cdot}6$.

$5{\cdot}6$ means $5 + {\cdot}6$ $\therefore$ $37{\cdot}82 \times 5{\cdot}6$

$$= 37{\cdot}82 \times 5 + 37{\cdot}82 \times {\cdot}6$$
$$= 189{\cdot}10 + 22{\cdot}692 = 211{\cdot}792.$$

The work may be arranged thus :

37·82			
5·6			
189·10	product corresponding to		5
22·692	,,	,,	·6
211·792	,,	,,	5·6

Example 2:—To multiply 43·065 by 34·73.

$$34{\cdot}73 = 30 + 4 + {\cdot}7 + {\cdot}03.$$

$$\begin{aligned} \therefore\ 43{\cdot}065 \times 34{\cdot}73 &= 43{\cdot}065 \times 30 \\ &\quad + 43{\cdot}065 \times {\cdot}4 \\ &\quad + 43{\cdot}065 \times {\cdot}7 \\ &\quad + 43{\cdot}065 \times {\cdot}03 \\ &= 1291{\cdot}95 \\ &\quad + 172{\cdot}260 \\ &\quad + 30{\cdot}1455 \\ &\quad + 1{\cdot}29195 \\ &= 1495{\cdot}64745 \end{aligned}$$

$$\begin{array}{r} 43{\cdot}065 \\ 34{\cdot}73 \\ \hline 1291{\cdot}95 \\ 172{\cdot}260 \\ 30{\cdot}1455 \\ 1{\cdot}29195 \\ \hline 1495{\cdot}64745 \end{array}$$

The work may be conveniently arranged as shown on the left, where it should be noted that (1) the multiplier is written below the multiplicand so that its units figure comes below the last figure of the multiplicand; (2) the partial product corresponding to any figure of the multiplier is set down so that its last figure comes directly (as it should) below that figure of the multiplier.

Explanation.—We have seen in (Art. 73) that, in multiplying a decimal by units (an integer of one figure) the number of decimal places in the product is the same as in the multiplicand. Therefore if the units figure be placed under the last figure of the multiplicand, the last figure of the partial product corresponding to it will come directly under the last figure of the multiplicand, *i.e.*, just below the units figure of the multiplier; therefore the last figure of the partial product corresponding to any other figure of the multiplier will also come directly under that figure as in ordinary multiplication, (*vide* Art. 52).

Alternative method. Alter the position of the decimal point in the multiplicand and the multiplier so that the product is not affected and that there is only one significant figure in the integral part of the multiplier; and find the product as in the last para. Thus to multiply—

43·065 by 34·73

is the same as to multiply

430·65 by 3·473

and the process may be arranged as in the last para. thus:

This method is specially useful for finding a rough estimate of the product and the student can never go wrong in fixing the decimal point and in writing down the partial products.

In the above example the answer is roughly 430 × 3 or 1290, *i.e.*, consists of 4 figures in the integral part.

∴ the decimal point in the product must come after 4 significant figures from the left.

```
430·65
  3·473
---------
1291·95
 172·260
  30·1455
   1·29195
---------
1495·64745.
```

N. B.—Note that the decimal point in the various partial products is in the same vertical line with the decimal point of the multiplicand.

Definition—A decimal such as 3·473 consisting of only one digit in the integral part may be said to be in the *standard form.*

Exercise X (b).

I. Multiply (using rough checks before beginning to work)—

(*a*) 23·68 by (1) 12; (2) 12·8; (3) 12·86.

(*b*) 123·98 by (1) 0·04; (2) 10·68.

(*c*) 70·02 by (1) 1600; (2) 16·08; (*c*) ·1608.

(*d*) 81·352 by (1) ·678; (2) 19·85.

(*e*) ·0003814 by ·0456.

II. Find the value of—

(*a*) (·6823 + ·753) × ·0098.

(*b*) (·7385 — ·0083) × ·168.

(*c*) 23·86 × (·635 — ·0635).

III. (*a*) Find the continued product of ·023, ·345 and ·063.

(*b*) Find the value of $(·634)^2$, $(39·37)^3$.

§ 77. Division by one figure.

Example 1.—Divide 85·36 by 4.

The dividend is 85 units + 3 tenths + 6 hundredths.

4 goes into 85 twenty-one times leaving a remainder one. ∴, in dividing 85 units by 4 we get 21 units as the quotient and one unit left over; this one unit together with 3 tenths in the dividend makes 13 tenths which when divided by 4 gives 3 tenths as the quotient and one tenth as the remainder. This remainder one-tenth with 6 hundredths in the dividend makes 16 hundredths which when divided by 4 gives 4 hundredths as the quotient. Thus the complete quotient is 21 units + 3 tenths + 4 hundredths or 21·34.

Thus it will be noted that this division is exactly similar to the division of integers and the work is shown in the margin.

$$\begin{array}{r|l} 4 & 85{\cdot}36 \\ \hline & 21{\cdot}34 \end{array}$$

Example 2.—Divide 68·37 by 8.

In this example 68·37 is not exactly divisible by 8, so we write 68·37 in the form 68·37000 (the ciphers thus affixed after the decimal point do not alter the value of the decimal) and carry on the division as in Example 1.

$$\begin{array}{r|l} 8 & 68{\cdot}37000 \\ \hline & 8{\cdot}54625 \end{array}$$

Exercise—Oral.

Divide—

1. 6·4 by 2; 4; 8.
2. 8·1 by 3; 9.
3. 12·6 by 5.
4. 6·2 by 4; 8.
5. 2·8 by 5.
6. 7·3 by 5; 8.

Exercise X (c).

Divide—

1. 63·168 by 6; 7; 8; 12; 16.
2. 583·0038 by 16; 18.
3. 26·37 by 80; 90.
4. 26·318 by 500; 900.

§ 78. Division by integers of two or more digits.

Ex. (1) Divide 86·452 by 23.

Divide by the Italian method as in simple division and write the digits of the quotient above the dividend as each figure in the quotient is obtained.

```
     | 3·758
  23 |86·452
     |17·4
     | 1·35
     |  ·202
     |  ·018
```

The quotient is 3·758 and the remainder is ·018.

Ex. (2) Divide ·008357 by 365.

```
     | ·000022
 365 |0·008357
     |    1057
     |     327
```

The quotient is ·000022 and the remainder ·000327.

Exercise X (d).

Divide—

1. 493·389 by (1) 37 ; (2) 43 ; (3) 73 ; (4) 89.
2. 8763·4387 by (1) 113 ; (2) 129 ; (3) 149 ; (4) 173.
3. 2036·8749 by (1) 285 ; (2) 374 ; (3) 489 ; (4) 576.
4. 11238·47895 by (1) 878 ; (2) 1031 ; (3) 3048; (4) 4278.

When the divisor can be split up into factors, factors may be employed.

Ex. (3) Divide—·008397 by 450.

$450 = 10 \times 9 \times 5.$

```
 10 | ·008397
  9 | ·0008397
  5 | ·0000933
    | ·00001866
```

Exercise X (e).

Divide—

1. 83·9475 by (1) 104 ; (2) 125 ; (3) 144 ; (4) 168 ; (5) 192.
2. 93·0627 by (1) 8400 ; (2) 720 ; (3) 9600 ; (4) 2300.
3. 114·6262887 by (1) 25600 ; (2) 48400 ; (3) 56000 ; (4) 86400.
4. ·78325 by (1) 132 ; (2) 482 ; (3) 576 ; (4) 779.
5. ·0083456 by (1) 640 ; (2) 7200 ; (3) 8640 ; (4) 1080.

§ 79. Division by decimals.

Example 1.—Divide 92·88 by ·9. The question means :—find how many times ·9 or 9 tenths is contained in 92·88 units. *i.e.*, in 92·88 × 10 tenths or 928·8 tenths which is found by dividing 928·8 by 9. Thus 92·88 ÷ 9 = 928·8 ÷ 9 = 103·2.

Example 2.—Divide 89·3875 by ·8476.

The question can, as in Example 1, be shown to be equivalent to 89·3875 × 10000 ÷ ·8476 × 10000, *i.e.*, 893875 ÷ 8476.

If the divisor be a decimal the division can thus be converted into an equivalent division by an integer by multiplying both the dividend and the divisor by a number consisting of 1 followed by as many ciphers as there are decimal figures in the divisor, *i.e.*, by removing the decimal point to the right both in the dividend and in the divisor to the same number of places so that the divisor becomes an integer.

Alternative Method.—To get some rough idea of the quotient even before working the question, a more convenient method will be to reduce the divisor to the standard form, *i.e*, to make the divisor to contain one significant figure in the integral portion (the principle involved being the same as is explained in the last para.)

Example 1. Divide 82·84 by 0·8

82·84 ÷ ·8 = 828·4 ÷ 8 = 103·55.

The position of the decimal point is correct for the quotient is larger than 83 ÷ 1 or 83.

Example 2. Divide 369·1584 by 48·96

369·1584 ÷ 48·96 = 36·91584 ÷ 4·896.

The work may be arranged thus :

Here you simply divide as though both the dividend and the divisor were integers and mark the decimal point in the quotient as indicated by a rough checking.

$$\begin{array}{r|r}
 & 7{\cdot}54 \\
4{\cdot}896) & 36{\cdot}91584 \\
 & \underline{34{\cdot}272} \\
 & 2{\cdot}6438 \\
 & \underline{2{\cdot}4480} \\
 & {\cdot}19584 \\
 & {\cdot}19584
\end{array}$$

Here the answer is about 37 ÷ 5. ∴ the decimal point should be between 7 and 5. ∴ the answer is 7·54.

Rough check :—369·1584 ÷ 48·96 = 369 ÷ 49 = 7 and odd roughly.

§ 80. Division correct to a certain number of decimal places.—Frequently in the division of one decimal by another we may not require the quotient beyond a certain number of decimal places. In such cases we stop the process of division just wherever we want and correct the last figure of the quotient on the same principles as in ordinary division (*vide* Art. 66).

Example. Given that 1 metre = 39·37 in., find the number of feet in a metre to two decimal places. 1 foot = 12 in. ∴ 1 metre = 39·37 in. = (39·37 ÷ 12) feet = 3·28 ft.

Explanation. 39·37 when divided by 12 gives 3·28 as the quotient and 1 (which stands for ·01) as the remainder. So if the division be continued as explained in Example (2) Art. 27, the next figure will be less than 5, hence no correction need be made to 8, the figure in the second decimal place correct to which we want the quotient.

Exercise X. (f).

Find the quotient and check carefully the position of the decimal point in the following cases :—

1. 10·815 ÷ 3.
2. 146·88 ÷ 96.
3. 180·2 ÷ 1700.
4. 76·23 ÷ 46·2.
5. 69·006 ÷ 742.
6. 398·3383 ÷ 199.
7. 8·728572 ÷ ·00123.
8. ·03495 ÷ ·725.
9. 1085·826 ÷ 105·42
10. 52·53996 ÷ 116·6.
11. 60·51 ÷ 111.
12. 697187 ÷ 937·3.
13. ·00142335 ÷ ·18562.
14. 8192·38 ÷ ·2575.
15. How many lengths each 43·68 cm. can be cut off from a string 645·89 cm. long and what will be left over ?
16. How often is ·125 of a yard contained in 53·625 yds. ?
17. A mass of gold weighs 123·657 oz. Gold is 19 times as heavy as water. Find the weight of an equal volume of water.

18. How many cubic centimetres are there in 117·93256 cubic decimetres ?

19. A metre is equal to 39·37 inches. State how many metres there are in 116 yds.

20. How many times can a vessel holding 8·76 pints be filled from one holding 283·43 gallons and what quantity is left over ?

21. How many revolutions will the wheel of an engine make in a distance of 3 miles if its circumference is 36·2 inches ?

22. Divide 109·235 by 3·65 and then find the quotient in the division of (*a*) 10·9235 by ·0365 ; (*b*) 1092·35 by 36·5.

23. Divide 108·345 by 8·93 and hence find the value of (1) 108345 by 893 and (2) ·000108345 by ·00893.

24. If 1869 sovereigns are coined out of 40 pounds weight of standard gold, what is the weight of a sovereign in grains ? Also what is the value of an ounce (Troy) of standard gold ?

25. The gold reserve of a bank weighs 13 ton 15 cwt. 0 qr. 18·8 oz. If a sovereign weighs 123·274 grains, find the value of the reserve (1 lb. = 7000 grains.)

26. Find the height of a pile of 40 drawing boards if each drawing board is ·093 dm. thick.

27. Find the cost of 18·29 grains of gold at Re. 1·575 per gram.

28. If 10 litres of mercury cost Rs. 250·75, what is the cost of 36·5 decilitres ?

29. A constable finds that a motor-çar traverses 75 metres in 4·5 seconds. At how many kilometres per hour is it moving ?

30. Light travels with a velocity of $2·9 \times 10^8$ m. per sec. Find in kilometres the distance of a star from which light takes 3 years to reach the earth.

31. If the thickness of a penny is 2 mm., find the least number of pennies to be piled so that the height of the pile may be 1 decimetre.

32. My luggage weighs 32·75 mds. and I am allowed 2·5 mds. free Excess luggage is charged at the rate of Rs. ·175 per maund per mile. What should I pay for luggage between Madras and Arkonam, 42 miles ?

33. If 15·35 metres of chain are required to wrap round a drum 432 times, what is the circumference of the drum?

34. A train travels 109·75 miles in 3·1 hours; find to the nearest mile the average rate in miles per hour.

35. A ream of paper is 2·35 in. thick. Find the thickness of a sheet.

***36.** Out of 300 bottles of brandy, 6 bottles are taken at random and found to contain 0·125, 0·123, 0·121, 0·126, 0·124, 0·127 gal. Assuming the average contents of these 6 bottles to be the average contents of all the 300 bottles, find, to the tenth of a gallon, the total amount of brandy.

***37.** When a screw is turned round through a complete revolution its head goes forward by 0·145 of an inch (the distance the head goes forward in one revolution is called the *pitch* of the screw). How many turns must you give such a screw to move it forward through 1·056 in.?

***38.** The pitch of a screw is 0·125 in. How many turns must you give it to move it forward through 1·5 in.?

***39.** The pitch of a screw is 0·1625 in. If 8·25 turns are given to it, through what distance does it move forward?

***40.** The increase in the length of a steel rod when it is heated one degree Centigrade, is got by multiplying its length by ·000012. Find by how many inches a rod 12 ft. long increases when so heated.

CHAPTER XI.

PARALLELS, SIMPLE LOCI AND TRIANGLES.

PARALLELS.

§ **81**. In (Fig. 71) are two straight lines AB and CD. If the lines were produced, would they ever meet? If so, would they meet towards the right or towards the left? They would meet towards the right, because they get closer together when produced towards the right.*

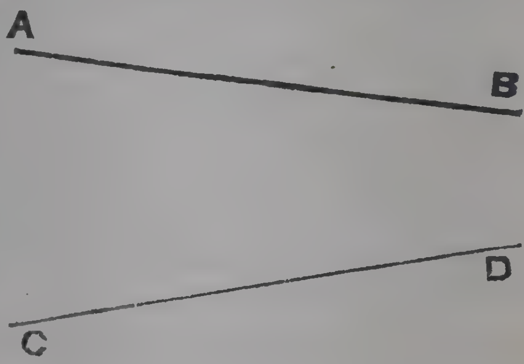


FIG. 71.

Answer the same questions with respect to the two lines in (Fig. 72). They would meet towards the left and not towards the right: because they get farther and farther apart when produced towards the right, but they get closer together towards the left.

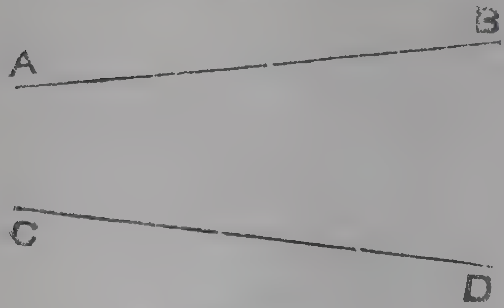


FIG. 72.

In both the cases measure the distance between the lines at different parts and see on which side the distance increases and on which side it decreases. It will be found that the lines meet on that side on which the distance decreases, *i.e.*, in other words, on the side opposite to that on which the distance increases.

* *N.B.*—The answers are meant to be elicited from the students.

Draw two straight lines which get closer together neither towards the left nor towards the right as in (Fig. 73). Would they meet if produced? No; because they do not get closer together whether produced to the right or to the left.

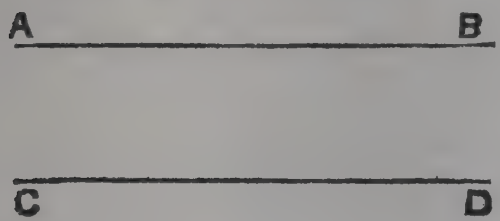


FIG. 73.

Two such straight lines which do not meet on either side if produced ever so far, are called **Parallel straight lines.**

Measure the distance between the lines in (Fig. 73) at different parts. You will find the distance to be the same everywhere. Thus **parallel straight lines have the same distance between them everywhere.**

Exercise—Oral.

1. Look about you and give examples of parallel lines.

2. Observe the edges of your slate; the edges of a bench, a desk; the two door posts of a door-way; which of these are parallel?

§ 82. Another aspect of parallel straight lines. In (Fig. 74) you have two straight lines AB, CD standing on a straight line BDE. Which is the larger angle, ABD or CDE? Measure the angles with your protractor, you will find that CDE is the greater of the two. Produce the lines BA and DC and see if they meet. You will find they meet above the line BD.

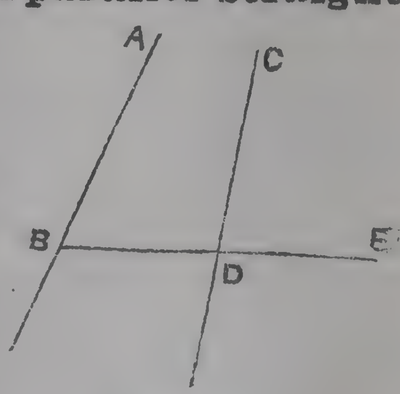


FIG. 74.

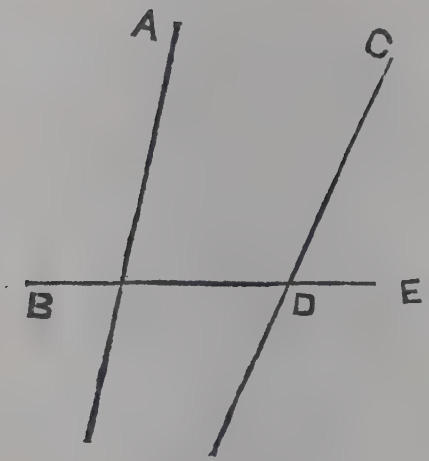


FIG. 75.

In (Fig. 75), which is the larger angle, ABD or CDE? Measure the angles with your protractor; you will find ABD to be the greater. Produce the lines AB and CD, and see if they meet. You will find they meet below the line BDE.

In (Fig. 76), which is the larger angle, ABD or CDE? Measure them and you will find they are equal; and produce the lines both above and below the line BDE and see if they meet. You will find they do not meet, *i.e.*, they are **parallel.**

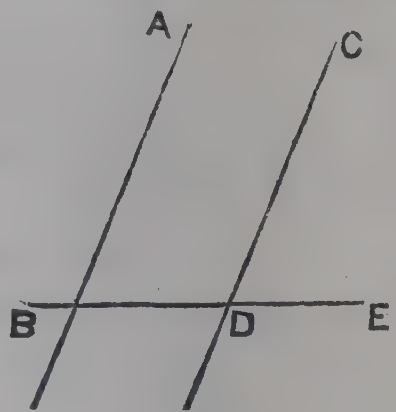


FIG. 76.

In (Figs. 74 and 75), AB and CD meet and the angles ABD, CDE are unequal; and in (Fig. 76) the lines do not meet and the angles ABD, CDE are equal.

The angles ABD, CDE may be called **corresponding angles.**

You may draw pairs of lines, as in the above cases and measure the corresponding angles formed by a straight line falling upon them; you will find the angles to be unequal in cases where the lines meet when produced, and equal in cases where the lines do not meet when produced, *i.e.*, when the lines are parallel. Thus when the **corresponding angles are equal the straight lines are parallel.**

§ 83. Another property of parallel lines.

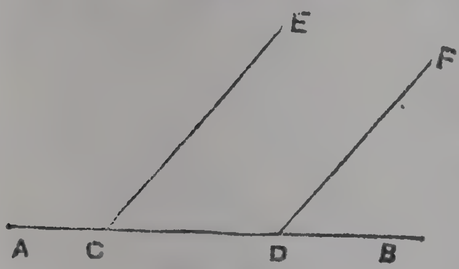


FIG. 77.

Draw a straight line AB, take points C and D on it and at C and D make with your protractor angles BCE, BDF each equal to 45° as in (Fig. 77). Then, since the corresponding angles are equal, the two lines EC and FD are parallel.

If AB points to the east what direction does DF point to? It points to the north-east. Similarly CE also points to the north-east, *i.e.*, the lines DF and CE which are parallel have the same direction.

If instead of making the angles BCE, BDF each equal to 45° they are made each equal to 60°, then also since the corresponding angles are equal, the lines CE and DF will be parallel. In this case both the lines point to the direction which is 60° north of east, *i.e.*, they have the same direction.

Note then that **parallel straight lines point to the same direction.**

Exercise.—Oral.

1. Give some examples of parallel lines.

2. Examine the parallelism of the opposite sides of a square, a rectangle.

3. Are the rails of a railway line parallel? Give a reason for your answer.

4. Given two straight lines, how do you test whether they are parallel?

Exercise—Practical.

1. AB is a straight line, C and D are two points in it. From C and D draw two straight lines CE and DF at right angles to AB. Examine the parallelism of CE and DF by applying any of the tests.

2. In the previous example take any points P_1, P_2, P_3, ... in AB and draw P_1X_1, P_2X_2, P_3X_3...... $\perp$ to AB, as in Fig. 78 each $=$ 1 inch. Examine if X_1, X_2, X_3...are collinear ; what can you say about the directions of AB and the line X_1X_2 ?

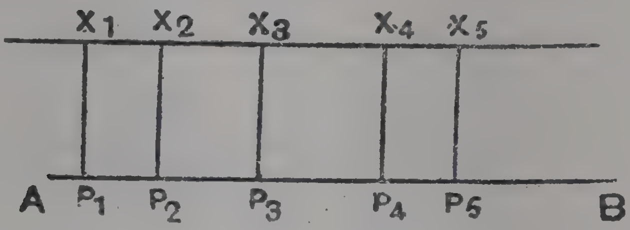


FIG. 78.

3. Construct a rectangle whose sides are 10 cm. and 8 cm. Bisect the sides and join the middle points in order. Show that the sides of the figure so formed are parallel to the diagonals. State the test you apply.

4. Describe a circle of radius 3 inches. Draw any two diameters AB and CD. Show that AC and BD are parallel.

§ **84. Drawing parallels**. Place a set square as

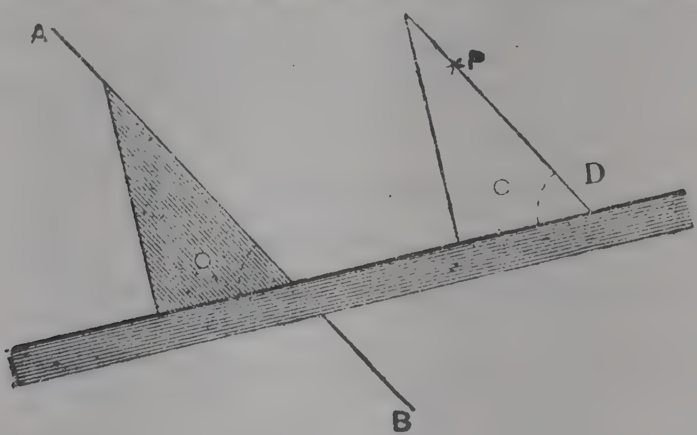


FIG. 79.

in (Fig. 79) and rule a straight line AB. Holding the set square firmly in position, place against its edge a flat ruler or straight edge as in the figure. Now hold the ruler firmly in position and slide the set square along

it to the position CD and rule the line CD. Then, what angles do the lines AB and DC make with the ruler? Certainly the angle of the set square, *i.e.*, 60° in this case. Therefore what kind of lines are AB and DC? They are parallel; because the corresponding angles are equal.

Thus by sliding the set square to different positions along the ruler firmly held, you can draw any number of parallel straight lines.

NOTE.—"$\parallel$" is the symbol for "parallel."

§ 85. To draw through a point P a straight line parallel to a given straight line AB.

Place a set square so that one of its edges lies along the given line AB (Fig. 79) and, keeping the set square in that position, place a straight edge as before against an edge of the set square. Now holding the straight edge firmly, slide the set square until the edge which lay originally along AB passes through P; and draw the line CD which as we have seen will be $\parallel$ (parallel) to AB.

(NOTE.—The straight edge should not be bevelled: if it is, the set square will slip over it.

The base of the protractor or another set square may be used instead of a flat ruler in the last exercise).

§ 86. To draw a straight line perpendicular to AB through a point P.—Place a set square with a side containing its right angle lying along AB. Place the flat ruler (or the other set square) in contact with that side of the set square which is opposite to the right angle as in (Fig. 80). Now holding the flat ruler fixed, slide the set square; then the side lying along AB will, as we have seen, move parallel to itself and the other side containing the right angle will always be perpendicular to AB. Slide the set square until this other edge passes through P.

Then draw the line CP along that edge. CP is then ⊥ to AB.

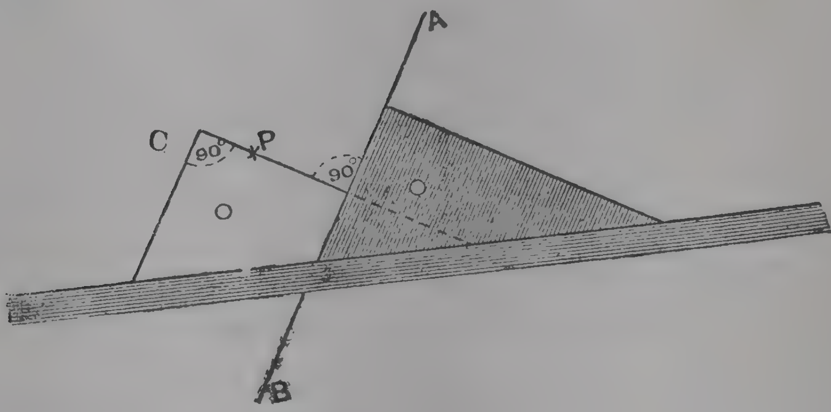


FIG. 80.

Exercise—Practical.

1. Draw any three straight lines parallel to one another.

2. Draw any number of parallel straight lines close together. This is how any part of a drawing is "shaded.")

3. Draw a straight line QR and mark a point P outside it. Through P draw a line || to QR.

4. Repeat this several times using different edges of the set square.

5. Draw a straight line AB, mark a point C, 2″ distant from AB and through C draw a straight line CD || to AB.

6. Draw a straight line AB and draw another line CD, parallel to AB and at a distance of 1″ from AB.

7. Draw two || straight lines 2″ apart. Draw another straight line at right angles to one of them. What angle does it make with the other straight line? Verify by measurement. Draw a fourth straight line parallel to the third at a distance of 2″. What figure have you made?

8. Draw a straight line AB 1·5 inches long. At B draw another straight line ⊥ to AB.

9. Draw a straight line AB one inch long and on it describe a square.

10. Draw a rectangle 3 inches long and 2 inches wide.

§ 37. Properties of parallel straight lines. Draw two lines AB and CD. Let EFGH be drawn across the lines AB, CD so as to cut them at F and G (Fig. 81). (A line drawn across and cutting other lines is called a **transversal**). Eight angles are then formed. Of these 1, 2, 7 and 8 are called *exterior angles* (because they lie *outside* the lines AB and CD), 3, 4, 5 and 6 are called *interior angles* (because they lie *inside* the lines). Again (2 and 6), (1 and 5), (7 and 3), (8 and 4) are called *corresponding angles.* (3 and 6) and (4 and 5) are called **alternate angles.**

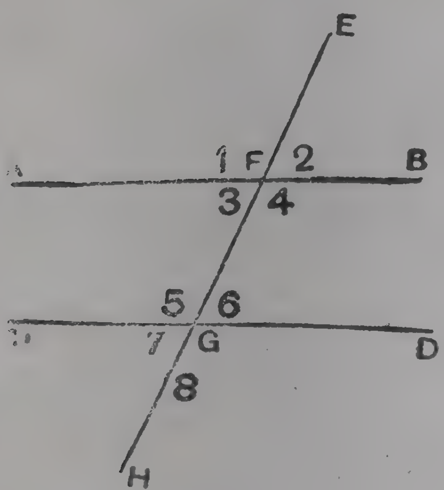


FIG. 81.

Form a similar figure with AB and CD parallel and measure the angles 4 and 5 and also 3 and 6. Repeat the experiment with two or three sets of parallel lines you will find that they are equal.

Also $\angle 3$ and $\angle 4 = 2$ right-angles but $\angle 3 = \angle 6$.

$\therefore$ $\angle 4$ and $\angle 6 = 2$ right-angles. Or measure angles 4 and 6 and find their sum. Similarly measure angles 3 and 5 and find their sum; you will get 2 right-angles as the sum in each case.

We therefore conclude that, **when a straight line falls on two parallel straight lines—**

(1) a pair of corresponding angles are equal;

(2) the alternate angles are also equal;

(3) the two interior angles on the same side of the transversal are together equal to 2 right-angles.

Exercise XI (a)—Practical

1. Draw AB and CD parallel and get a figure similar to (Fig. 81) with angle (1) equal to 32°; assuming AB and CD to be parallel, find the magnitudes of the remaining 7 angles by calculation, and check your result by measuring the angles.

2. If in the figure to Question 1, angle 5 be 60°, what is the magnitude of the angle 2?

3. AB and CD are parallel; EFGH cuts them at F and G; cut out the portion AFGC; invert the paper and place it on the portion BFGD so that G falls on F and F on G. Then observe what pairs of lines coincide. What do you infer about the angles CGF and GFB and also about AFG and FGD?

4. Similarly show by paper cutting and superposition that the corresponding angles are also equal.

5. Make an angle ABC = 75°. cut off BA = 2·4 in., BC = 1·6 in. Through A draw AD parallel to BC, through C draw CD parallel to BA.

The figure thus formed has its *opposite sides* **parallel** and is called **a parallelogram.**

6. Make a parallelogram whose adjacent sides are 7·8 cm. and 6·2 cm. the angle between them being 46°. Measure the other sides and angles.

7. Repeat this experiment with 9·8 cm., 3·4 cm. and 104°.

8. Repeat the same experiment with 9·8 cm., 3.4 cm. and 76°. Compare the opposite sides and angles; you will find them to be equal. Hence learn that **the opposite sides and angles of a parallelogram are equal.**

9. Make a parallelogram of card-board having one pair of sides 4 inches long and the other pair 2 inches long with one of the angles 60°.

10. Draw a parallelogram having sides = 8·6 cm. and 4·7 cm. and one angle = 135°. Draw its diagonals and measure their parts.

11. Repeat the experiment with the following measurements:—9·8 cm., 5·4 cm., 72°. Test any facts you noted in the previous example.

12. Draw AB = 3·4 in. long, AC at right angles to AB and 3 in. long. Join BC and from A draw AD perpendicular to BC. Measure AD and the angle BAD.

13. Draw AD 3·4 inches, make the angle BAC 45°, draw AC 4·3 in. long. From B and C draw BD, CD at right angles to AB and AC respectively to meet at D. Join AD and measure its length.

14. Draw a rectangle having sides=3·6 in. and 2·4 in. Measure its diagonals.

15. Draw a parallelogram having all its sides=2·3 in. and one angle = 56°. Measure the sides and angles of the parallelogram and also the diagonals and the angles between them.

16. Repeat the experiment making all the sides = 6·9 cm. and one angle = 126°. **A parallelogram which has all its sides equal but not its angles right-angles is called a rhombus.**

In the course of the above experiments you should have observed the following properties :—

(1) **The opposite sides and angles of a parallelogram are equal.**

(2) **The diagonals of a parallelogram bisect one another.**

(3) **The diagonals of a rectangle are equal.**

(4) **The diagonals of a square and a rhombus intersect at right-angles.**

TRIANGLES.

§ **88**. Take any three points A, B, C not in the same line and join them as in (Fig. 82). The figure thus got is called a **triangle** because it has *three angles*; and the three angular points are called the vertices of the triangle. In (Fig. 82) ABC is the triangle, and A,B,C are its vertices. The figure, you see, is enclosed or bounded by three straight lines and these are called its **sides**. In (Fig. 82) the sides are BC,

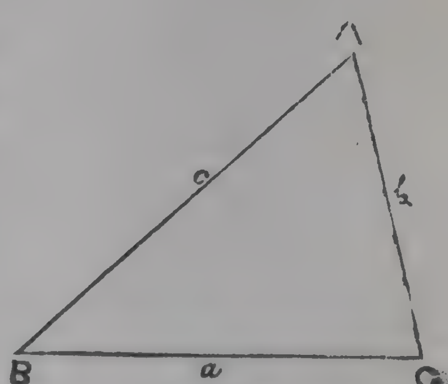


FIG. 82.

CA, AB. Its angles are generally denoted by A, B, C while a, b, c denote the sides opposite to the vertices A, B, and C respectively.

Thus in the figure

A	stands for	$\angle$BAC	a	stands for	BC
B	,,	$\angle$ABC	b	,,	CA
C	,,	$\angle$ACB	c	,,	AB

The symbol '$\triangle$' is used to denote a triangle. The triangle ABC is symbolically written $\triangle$ ABC.

Exercise—Practical.

1. Draw any triangle by taking any 3 points at random (not in the same line) and joining them. Measure its sides and angles. Fill up the following tabular form :—

Angles.	Measure in degrees.	Sides.	Measure in inches.	The sum of sides.	Measure in inches.	Difference of sides.	Measure in inches.
A B C		a b c		$b+c$ $c+a$ $a+b$		$b \backsim c$ $c \backsim a$ $a \backsim b$	
A+B+C		$a+b+c$					

2. Repeat the experiment with 5 other triangles and similarly fill up five similar tabular forms.

What do you notice about A + B + C in all these six cases? You get either 180° or very nearly 180°. You may be inclined to guess **that the three angles of a triangle are together equal to two right-angles.**

Draw a good sized triangle of any shape you like (Fig. 83). Cut it out and tear off the corners. Fit these together at a point O as in (Fig. 84) and observe the outer edges of the angles A and C. You will find that they fall in a straight line which also shows that the 3 angles of a triangle are together equal to 2 right-angles.

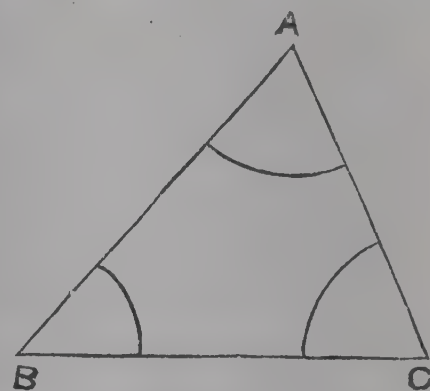


FIG. 83.

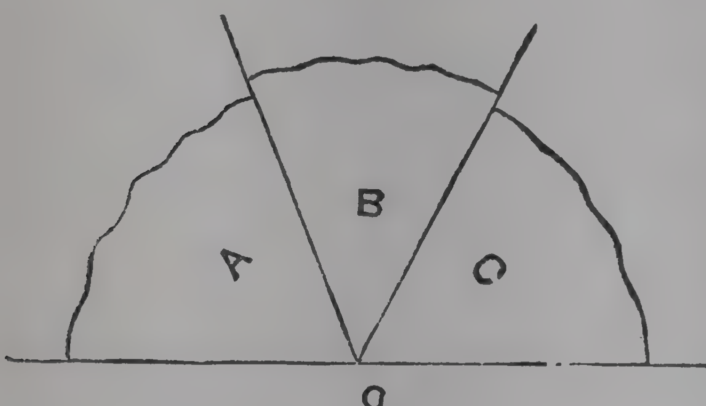


FIG. 84.

Also compare $b+c$ and a in all these cases which is the greater of the two? Similarly compare $c+a$ and b and also $a+b$ and c.

In all these cases you will find that

$b+c$ is greater than a
$c+a$,, b
and $a+b$,, c.

Hence we conclude that **any two sides of a triangle are together greater than the third.**

Next compare in all these cases $b \sim c$ and a.
,, ,, ,, $c \sim a$ and b.
,, ,, ,, $a \sim b$ and c.

You will find that $b \sim c$ is less than a.
$c \sim a$,, b.
$a \sim b$,, c.

Hence we conclude that **the difference of any two sides is less than the third.**

The symbols $>$ and $<$ are respectively used for "is greater than" and "is less than"

Thus '$b + c$ is greater than a' is written '$b + c > a$' and '$b \backsim c$ is less than a' is written '$(b \backsim c) < a$.'

Exercise—Oral.

1. In a $\triangle$ ABC, $\angle A = 30°$, B, $= 45°$, what is the value of C ?

2. In a $\triangle$, A $= 80°$, C $= 90°$, what is the value of B ?

3. Can a triangle have 3 angles of the following values, 80°, 30° and 90° If not, why not ?

4. Can the sides of a $\triangle$ be of lengths 8, 4, and 3 inches ? If not, why not ?

§ **89.** Again describe any triangle ABC and produce the side BC to D (Fig. 85); measure the angle ACD and also the angles A and B and compare ACD with A + B. Repeat the exercise with a number of triangles drawn at random. Then you will find ACD and A + B are either equal or very nearly equal.

You already know that $\angle ACB + \angle ACD =$ two right-angles, and three angles of the triangle ABC are together equal to two right angles. $\therefore$ ACD + ACB $=$ ABC + BAC + ACB. Taking away the angle ACB from both, ACD $=$ ABC + BAC. Thus whatever triangle you may draw, the angle ACD is equal to the angles ABC and BAC put together. The angle ACD is called an **exterior angle** of the triangle and the angles ABC, BAC are called **interior opposite angles** with respect to ACD. Thus we have the theorem, *viz.*, **an exterior angle of a triangle is equal to the sum of the two interior opposite angles.**

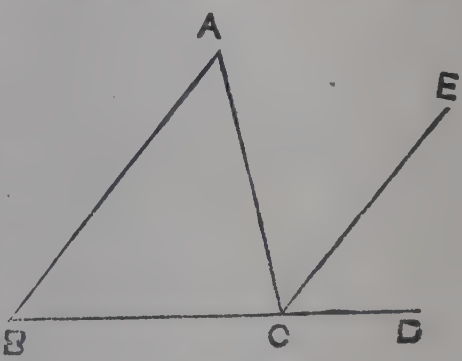


FIG. 85.

Draw through C a line CE parallel to AB. Then because the line BCD falls on the parallel lines BA and CE, you know the corresponding angles ECD and ABC are equal. Also because the line AC falls on AB and CE, the alternate angles BAC and ACE are equal. Thus we have

$$ABC = ECD$$
$$BAC = ACE$$

$$\therefore\ ABC + BAC = ECD + ACE$$
$$= ACD \ldots\ldots\ldots \qquad (1)$$

Also, adding ACB to both sides, we have ABC + BAC + ACB = ACD + ACB = two right angles......(2) (Art. 38).

Thus we prove by reasoning that (1) *an exterior angle of a triangle is equal to the two interior and opposite angles* and (2) *that the three angles of a triangle are together equal to two right angles*. In fact the effect of drawing the parallel CE is, as it were, to bring all the three angles of the triangle ABC together at C as required in Art. 88 (page 231) for establishing the result.

Also since the *three* angles of a triangle are together equal to two right-angles, **any two angles of a triangle are together less than two right-angles.**

Also since an exterior angle of a triangle is equal to the two interior and opposite angles, it is evident **that an exterior angle of a triangle is greater than any one of the interior opposite angles.**

Exercise—Practical.

1. Place a straight thin pencil along CB position *1* marked in (Fig. 86). Rotate it about C from the direction CB to CA. Through what angle has it turned now? Slide the pencil along CA from position *2* to position *3*. Observe that in so sliding

there is no rotation or change of direction. Rotate it again about A from the direction CA to BA. Through what angle has it turned now? Through what angle has it turned on the whole from position *1* to position *4*? What do you conclude?

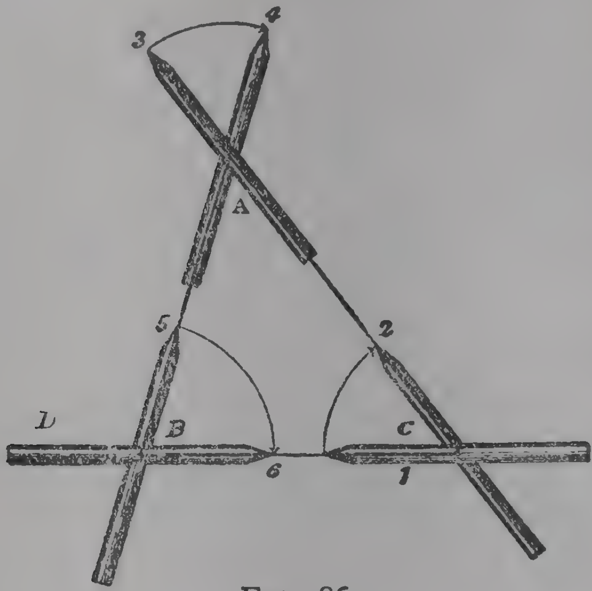


FIG. 86.

2. Show by cutting out the angles C and A and placing them over the angle ABD that $\angle ABD = \angle C + \angle A$.

3. In Exercise 1, suppose you slide the pencil along AB from *4* to *5* and then rotate it about B from the direction BA to BC., through what angle has it turned now? Through what angle on the whole from position *1* to *6*? Compare position *1* with position *6* and then say through what angle it must have turned. What do you conclude from this?

LOCUS.

§ 90. Drive a nail into a wall at a height of 5 ft. from the ground. Drive another nail at the same height, yet another nail and so on. What do you notice about these nails? They will all be found to be in a straight line. How is that straight line situated with respect to the bottom line of the wall? The two lines are at a distance 5 ft. apart and are parallel.

Now if you are told that a small nail is driven into a wall at a height of 5 ft. from the ground and asked to pull it out, where would you look for the nail? Certainly along a line 5 ft. from the ground, *i.e.*, you expect the nail to be

placed or *located* on that line, *i.e.*, that line is the place or **the locus** of the nail.

Similarly all points distant 1 in. from a line AB will be found to be on a line parallel to AB at a distance of 1 in. from AB *vide* Question 2, (Art. 83). Instead of 1 in. we may have any given distance, and generally we have that the place or **the locus** of points at a given distance from AB is a line parallel to AB at that distance, (*vide* Fig. 78).

§ **91**. Mark a point O, and mark a point distant 1·5 in. from O. Mark another point 1·5 in from O, yet another point and so on. Then these points will all be found to be on the circumference of a circle whose centre is O and whose radius is 1·5 inches, *i.e.*, the place or **the locus** of points distant 1·5 in. from O is the circumference of a circle with O as centre and 1·5 in. as radius. Here again the distance may be anything and in general we have "the locus of points at a given distance from a certain point is the circumference of a circle with that point as centre and the given distance as radius."

Definition.—The **locus** of points satisfying a given condition is the line or lines on which the points lie.

In each of the two preceding examples the points that we get satisfying the given condition are numerous, and so if one such condition alone be given about a point we cannot definitely fix the point.

Suppose a father tells his son at the time of his death that there lies a hidden treasure in his garden at a distance of 12 ft. from a banyan tree at a depth of 4 ft. below the ground; and the son, after his father's death, wants to discover the treasure; where would he search for it? Certainly along the circumference of a circle of radius 12 ft. having the banyan tree as centre. The task becomes very

laborious for the ground has to be dug along the entire circumference to a depth of 4 ft. The son then remembers, say, another direction given by the father, *viz.*, the treasure is at a distance of 10 ft. from an almond tree in the same garden. According to this direction the treasure lies on the circumference of a circle of 10 ft. radius having the almond tree as centre. As the hidden treasure has to satisfy both the directions given by his father the son infers that the hidden treasure must be on the circumference of both the circles, *i.e.*, it must be at one of the points of intersection of the two circles. Then he has to dig only at two places to a depth of 4 ft. But if the father had told him that the treasure was on a particular side of the two trees, the son could have exactly hit upon the spot.

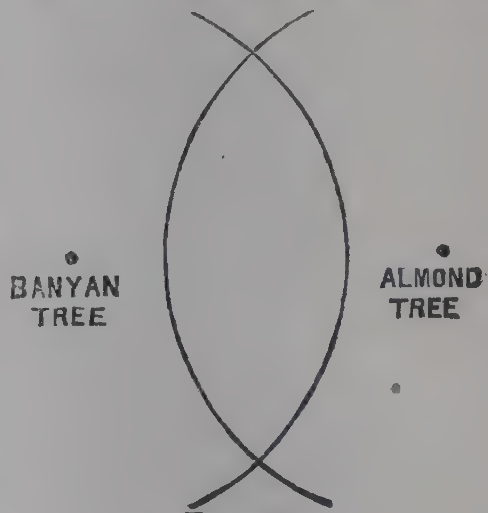


FIG. 87.

§ **92**. If, in the last example, instead of giving the distance of the treasure from the banyan tree, the father gives the direction of the treasure saying that it lies to the north-east of the tree, where would the son search for it, *i.e.*, what is the **locus** of the treasure? If in (Fig. 88) B be the banyan tree and

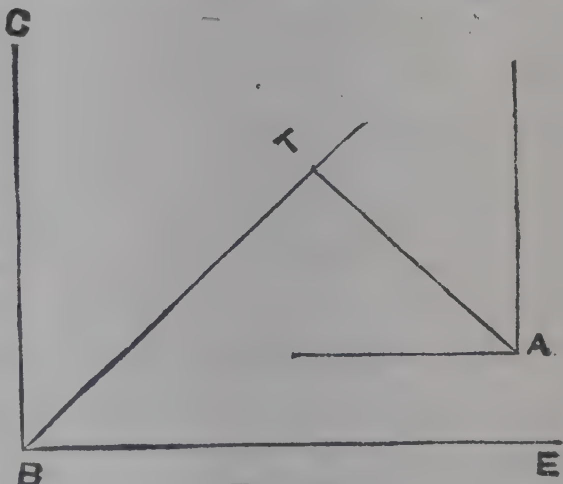


FIG. 88.

BE point to the east, and BT north-east, *i.e.*, the direction in which the treasure is said to lie, then certainly the son would search for the treasure along the line BT or BT is the locus.

If this were the only clue for the treasure, the son has to dig all along the line BT; but if, in addition, its direction from the almond tree be also given (say, to the north-west of it), then as in the above case he will get another line, *viz.*, the line drawn in the direction north-west from the almond tree A as shown in Fig. 88 along which to dig for the treasure.

Combining the two directions of the father he infers that the treasure must be at the point of intersection of the two lines BT and AT of the figure, *i.e.*, at T.

Exercise—Oral.

1. What is the locus of the pointed extremity of the minute-hand of a clock?

2. What is the locus of a man's hand as he works the handle of a common pump?

3. A man walks along a straight road so that he is always equi-distant from the two sides of the road. What is his path?

4. What is the locus of the point of contact with the ground of the wheel on which a gate-door moves as it opens?

5. What is the locus of a cow tethered to a post by a rope 10 ft. long, assuming the rope to be tight?

6. What is the locus of a point on the floor situated to the north-east of the south-east corner of this class room?

7. On the play-ground, a ball is said to lie to the south-west of the point where you stand, where will you search for the ball?

Exercise XI (b)—Practical.

1. Mark two points A and B 3 in. apart. Find a point 1·8 in. from A and 2·3 in. from B. How many solutions are there?

2. Find a point above AB 3·6 cm. distant from A and 4·8 cm. distant from B.

3. Find a point 1·5 in. distant from a line AB and 2·4 in. distant from a point C which is 2·6 in. from AB. How many solutions are there?

4. Find a point 3·8 cm. and 4·8 cm. distant from a point C. Is it possible to find such a point?

5. AB and CD intersect at O. Find a point 2·4 in. distant from AB and 1·8 in. distant from CD.

6. AB and CD are parallel 2 inches distant from one another. Is it possible to find a point 1 inch from AB and 1·5 in. from CD. If not, why not?

7. A tower T lies to the north-east when seen from a point A and to the north when seen from a point B due east of A. If the distance of AB be 200 ft., draw a diagram showing the positions of A, B, and T taking 1 cm. to represent 40 ft.

8. If, in the case of the treasure, after giving its direction from the banyan tree, its distance from the almond tree be given, will it help the son in locating the treasure? If so, show by a diagram the position of the treasure.

9. A point P is south-west of a point A and is at a distance of 3 inches from a point B due west of A. If the distance AB be 2 inches, find the position of P.

10. If, in Question 9, instead of giving the distance of P from the point B, its distance from the line AB be given to be 3 in., how do you find the point P?

Construction of triangles.

§ 93. *Case I.*—**Construct a △ given the lengths of the sides.**

If in (Art. 91) instead of the banyan tree and the almond tree, you have the points A and B and if, instead of the treasure, the point C is given to be distant 2 in. from A and 3 in. from B, how do you find the position of C? Mark

points A and B on paper; since C is given to be 2 in. from A, it lies on a circle described with A as centre and 2 in. as radius; and since it is 3 in. from B it lies on a circle with B as centre and 3 in. as radius. C is evidently the point of intersection of the circles.

NOTE.—It is not necessary to describe the two circles completely. But it is enough if parts near the points of intersection are alone shown as in (Fig. 87).

Construct a triangle ABC given AB = 3 cm., BC = 2 cm. and CA = 2·5 cm. The student is to draw the figure and supply the explanation.

Exercise XI (c).

1. Can you draw another triangle satisfying the given conditions in the above case. If so what are your reasons?

2. Draw the two triangles one above AB and the other below AB having the given dimensions. Cut out the double figure so formed and fold it about AB. What do you find? Are the triangles of the same shape and size?

3. Construct a $\triangle$ having given a, or BC = 5 cm., b or CA = 8 cm., c or AB = 2 cm. Begin by first drawing CA = 8 cm. What difficulty do you notice? Why is the construction impossible?

Examine if the condition "any two sides of a triangle are together greater than the third side" is satisfied in this case. How is it violated? Note therefore that, if any two lengths given are less than the third, no triangle can be described with the given lengths as sides. Also the two lengths form the radii of the two circles to be described and the third length the line joining the centres of the two circles, and the circles do not cut one another. Hence learn that—

(1) If the two lengths given be less than the third the circles described with the extremities of the third length as centres and radii respectively equal to the other two given lengths do not cut one another.

(2) In general, when the distance between the centres of two circles is greater than the sum of their radii the two circles do not intersect.

4. Construct a Δ given $a = 9$ cm., $b = 6$ cm., and $c = 3$ cm. Begin by first drawing the side AB = 3 cm. What difficulty now arises and give a reason for it? Why can you not draw the Δ? Examine if the condition 'any two sides of a triangle are greater than the third side' is satisfied. How is it violated?

Note therefore that (1) **a triangle can be formed only when the lengths given are such that any two are greater than the third** and incidentally you also note that (2) **two circles will cut only if the distance between the centres be greater than the difference between their radii.**

5. Construct the Δ in the following cases and note the impossible cases if there are any, giving reasons for your answer. Measure the angles and enter the measures in your figures—

(1) $a = 2{\cdot}5''$, $b = 2{\cdot}5''$, $c = 2{\cdot}5''$.
(2) $a = 2{\cdot}5''$, $b = 2''$, $c = 2''$.
(3) $a = 2{\cdot}5''$, $b = 1{\cdot}5''$. $c = 1''$.
(4) $a = 3{\cdot}2''$, $b = 2''$, $c = 1''$.
(5) $a = 4{\cdot}3$ cm., $b = 7{\cdot}3$ cm., $c = 3{\cdot}8$ cm.
(6) $a = 4$ cm., $b = 7$ cm., $c = 3$ cm.
(7) $a = 4$ cm., $b = 6{\cdot}8$ cm., $c = 3$ cm.
(8) $a = 8{\cdot}9$ cm., $b = 4{\cdot}5$ cm., $c = 8{\cdot}3$ cm.

If a triangle has all its sides equal, it is said to be **equilateral.**

If it has two sides equal it is said to be **isosceles.**

If all the three sides are unequal it is said to be **scalene.**

6. Classify the ten triangles you have drawn, under the headings 'equilateral,' 'isosceles' and 'scalene,'

7. Measure the angles of the equilateral Δ you have drawn in (1) of Exercise 5. What do you notice about the angles of such a Δ?

Draw a number of equilateral Δs having sides = 3"; 5 cm.; 6 cm., &c. Have all these triangles the same shape? Why? Hence note that **in an equilateral Δ all the three angles are equal, each being 60°.**

8. Take the isosceles △ you have drawn in (2) of Exercise 6. What do you notice about the base angles, *i.e.*, those angles which are opposite to the equal sides? Draw any base and construct two or three triangles on the base so that the sides are equal. Measure the base angles in every case and compare; you will find they are equal. Next make a tracing of an isosceles △ ABC (Fig. 89); turn the tracing over like ACB of Fig. 89 and see if you can fit it over the original △ABC. If so where do the traces of the ∠s, B and C fall? What do you conclude? Hence note **that the base angles of an isosceles triangle are equal.**

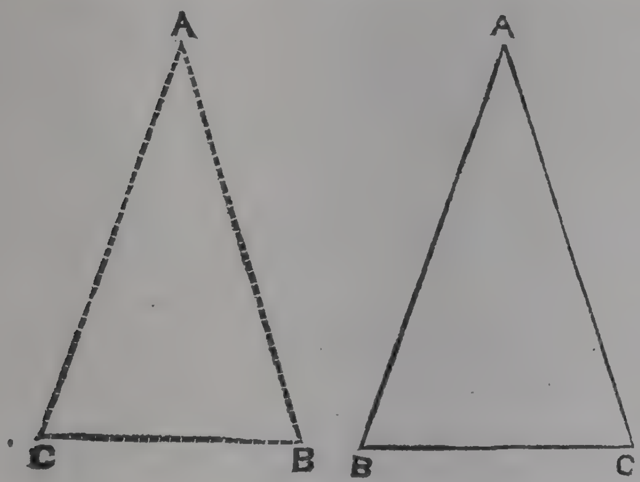


FIG. 89.

9. Take a scalene triangle you have drawn in Ex. 6 Measure the angles and the sides; and prepare the following tabular form:—

Angles.		Sides.	
Greatest.	...	Greatest.	...
Middle sized.	...	Middle sized.	...
Least.	...	Least.	..

Draw two or three triangles of any size and shape (not from measurements) and prepare similar tabular forms for these triangles.

Observe what you notice about—

(1) the greatest side and the greatest angle.

(2) the least side and the least angle.

(3) the side and the angle of middle size.

Hence learn that—

(1) that the greatest side is opposite to the greatest angle

(2) that the least side is opposite to the least angle.

(3) that if one side of a Δ is greater than another side, then the angle opposite to the former is greater than the angle opposite to the latter, and also

(4) that the greatest angle is opposite to the greatest side.

(5) that the least angle is opposite to the least side.

(6) that if one angle of a Δ is greater than another, the side opposite to the former is greater than the side opposite to the latter.

§ **94**. *Case II.*—**Construct a Δ given two sides and the included angle**, *e g.*, $b=2$ in., $c=2{\cdot}3$ in., and $\angle A=60^\circ$.

* Draw an angle $BAC=60^\circ$, cut off a length AB (or c) $=2{\cdot}3$ in. and AC (or b) $=2$ inches and join BC. Then ABC is the required triangle (measure the angles B and C and verify $A+B+C=180^\circ$).

Exercise XI (d).

1. Draw a triangle in which $b=6{\cdot}9$ cm., $c=5{\cdot}8$ cm, and $A=108^\circ$. Measure a, B and C and verify $A+B+C=180^\circ$.

2. Draw a triangle in which $c=b=8$ cm. and $A=96^\circ$. Measure B and C and also a. Note that the angles B and C are each less than 90°, *i.e., acute.*

3. Construct triangles in the following cases :—

(1) $a=1{\cdot}9''$, $b=1{\cdot}3''$, $C=40^\circ$.

(2) $c=7{\cdot}6$ cm., $a=5{\cdot}4$ cm., $B=109^\circ$.

(3) $a=6{\cdot}3$ cm., $b=5{\cdot}4$ cm., $C=86^\circ$.

(4) $b=4{\cdot}2$ cm., $a=6{\cdot}3$ cm., $C=117^\circ$.

Measure the remaining sides and angles in each case and verify $A+B+C=180^\circ$ and also,

$$a+b > c\,;\ b+c > a\,;\ c+a > b.$$

4. Draw a triangle having $a=2{\cdot}5$ in., $b=3$ in. and $C=90^\circ$. Measure B and A and find their sum. Calculate the sum of A and B from the facts that $A+B+C=180^\circ$ and $C=90^\circ$.

---

* The student is to be led to see this and the following constructions for himself as in Case I.

(1) Can a triangle have more than one right angle?

(2) Can a triangle having a right angle have also an obtuse angle? If not, why not?

If one angle of a triangle be a right angle, show that the remaining two angles must be acute.

A triangle having a right-angle is called a right-angled triangle.

5. Draw a triangle having $a = 2{\cdot}5$ in., $b = 3$ in., $C = 130°$. Measure B and A and find their sum. Check your measurement by calculation.

(1) How many acute angles has it?

(2) Show that each of the angles other than the obtuse angle is acute.

A triangle having an obtuse angle is called an obtuse-angled triangle.

6. A triangle is said to be **acute-angled only when all three of its angles are acute.** Why can you not say, as in the case of a right-angled triangle or an obtuse-angled triangle, that a triangle is acute-angled when *one* of its angles is acute?

7. How many angles are acute in each of the triangles? (1) right angled; (2) obtuse-angled; (3) acute-angled? Hence learn **that every △ must have at least two acute angles**

§ **95**. *Case III.*—**Construct a triangle given two angles and a side,** *e.g.*, $a = 1{\cdot}3$ in., $B = 40°$, $C = 30°$.

Draw a straight line $BC = 1{\cdot}3$ in. At B and C construct angles with your protractor on the same side of BC facing each other equal to 40° and 30° respectively. Produce the lines to meet at A.

State without measuring, the size of the angle A: then test your answer by measuring with the protractor.

Exercise XI (e).

1. Draw a triangle ABC in which $a = 6{\cdot}9$ cm., $B = 100°$ and $C = 45°$. Calculate the size of the angle A. Check your result by measurement.

2. Each of the angles at the base of a triangle = 53°. What is the vertical angle? Draw a △ in which $a = 2{\cdot}5''$ $B = C = 53°$. Measure b and c.

3. Draw a Δ in which $b = 7{\cdot}8$ cm., $A = 64°$ and $C = 38°$. What is the size of the angle B? Measure a and c and also the angle B.

4. Draw a triangle in which $a = 8{\cdot}3$ cm., $\angle A = 35°$, $\angle B = 60°$. (You must know the angles B and C, *i.e.*, the angles at the extremities of the side a. So find the angle C which is equal to $180° - 35° - 60°$ or $85°$).

5. Construct Δs in the following cases:—

(*a*) $a = 3{\cdot}4''$, $B = 40°$, $C = 50°$.
(*b*) $b = 4{\cdot}8$ cm., $A = 35°$, $B = 64°$.
(*c*) $c = 8{\cdot}9$ cm., $A = 42°$, $C = 68°$.
(*d*) $a = 1{\cdot}7$ in., $A = 36°$, $B = 104°$.
(*e*) $b = 2{\cdot}8$ in., $B = 74°$, $A = 28°$.

6. Draw Δs in which

(1) $a = 9{\cdot}3$ cm., $B = 120°$ and $C = 60°$.
(2) $a = 10{\cdot}4$ cm., $B = 80°$ and $C = 100°$. What difficulty arises? How do you explain the difficulty?

Note that, in these exercises and in the example worked, when you have constructed the two given angles, the third angle is determinate; which shows that the third angle is not left to your option and consequently there must be a definite relation between the three angles of a triangle. Such a relation you have already ascertained, *viz.*, $A + B + C = 2$ right-angles.

NOTE:—By assuming such a definite relation, and taking the limiting cases of triangles, the relation given here may be led up to

§ **96.** *Case IV.*—**Construct a Δ given two sides and an angle opposite to one of them,** *e.g.*, $a = 1{\cdot}4''$, $b = 1{\cdot}6''$, and $A = 36°$. Draw a line AC

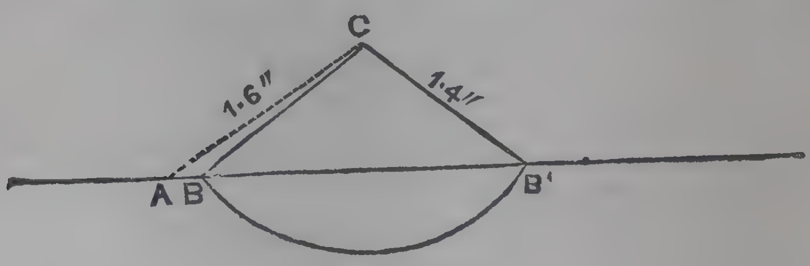


FIG. 90.

(or b) = 1·6″. At A make an angle CAB = 30° and cut off AC = 1·6″. AB is not limited because its length is not given. B must be somewhere in the line AB; also CB must be 1·4″, *i.e.*, B must be at a distance of 1·4″ from C. Describe a circle of radius 1·4″ with C as centre, cutting the line AB at B, join CB. Then ACB is the required triangle.

Note that in construction the circle cuts AB in two points and you have two triangles.

Exercise XI (f).

Construct a $\triangle$ in each of the following cases and measure the remaining sides and angles :—

1. a = 2·3 cm., b = 3·4 cm., B = 40°.
2. b = 4·3 cm., c = 7·6 cm., C = 80°.

(Note that in this case the two points are on opposite sides of AC and ∴ there is only one triangle formed with the given elements.)

3. a = 9·5 cm., b = 8·3 cm., A = 74°.
4. c = 7·8 cm., a = 9·3 cm., $\angle$C = 117°.

(In this case you do not get a triangle having the given elements. Why?)

5. a = 3·2″, b = 2·6″. A = 118°.

Exercise XI (g).

1. Construct a triangle having its sides equal to 30 ft., 40 ft., 50 ft. respectively (scale 10 ft. to 1 cm.) Measure the angles.

2. Construct two equilateral triangles having their sides respectively equal to 4·3 cm. and 6 5 cm. Measure the angles in each case. Construct half a dozen equilateral triangles, measure their angles. What inference can you draw about the angles?

3. Draw a triangle ABC having BC = 2 7 in., $\angle$B = 40°, $\angle$C = 60°; on tracing paper draw a triangle DEF having EF = 2·7 in., $\angle$E = 40°, $\angle$F = 60°. Apply $\triangle$ ABC to $\triangle$ DEF so that B falls on E, and BC falls along EF. Do the two triangles coincide? What do you infer?

4. Construct an isosceles triangle ABC having a = 3″, b = 3″, c = 4″. Take any point D in CB. Draw DE parallel to AB, meeting CA in E, measure the sides and angles of the $\triangle$ CDE.

Take D in different positions and examine the nature of the Δ CDE in all these cases.

5. Two persons A and B start from a place P and walk east and north-east at the rate of 3 and 4 miles an hour. After walking for an hour and a half A and B change their courses and walk north-east and east respectively exchanging their rates. When and where will they meet? Find the distance of the meeting point from P.

6. Construct a triangle having its sides equal to 6, 8, 10 cm. respectively. Find the middle points of the sides and join them. Into how many triangles is the whole figure divided? Arrange in a tabular form the sides and the angles of each triangle. Repeat the experiment with three more triangles drawn at random. What inference can you draw?

7. In the figure of Question 6, compare the directions of the sides of the central triangle with those of the sides of the original triangle. Repeat the experiment with three or more triangles. How many **quadrilaterals** (figures bounded by 4 sides) have you in each of those triangles? Are these figures parallelograms?

8. Construct a parallelogram having its adjacent sides 8·4 cm. and 6·8 cm. long and containing an angle = 56°. Join the middle points of the opposite sides. Into how many quadrilaterals is the whole figure divided? Are these figures parallelograms? Give reasons for your answer.

9. If a tramway crosses a railway line (where there are no curves) what kind of a figure is formed? Give reasons for your answer.

10. A cricket ball is at a distance of 40 ft. from a straight wall enclosing a field. It is at a distance of 20 ft. from the batman, who is 50 ft. from the wall. Show by means of a figure how you will find the position of the ball.

11. Draw two lines OX and OY at right angles. Find a point P 3 cm. and 2 cm. distant from OX and OY respectively. How many solutions are there?

12. A ladder 15 ft. long is placed against a vertical wall with one end resting on the ground at a distance of 10 ft. from the wall. Find by a figure (take 1 in. for 5 ft.) the height of the other end of the la der and the inclination of the ladder to the horizontal.

CHAPTER XII.

SIMPLE ALGEBRAICAL ADDITION, SUBTRACTION AND EQUATIONS.

§ **97.** The student has already seen that letters are used as if they are ordinary numbers and that they are chiefly used whenever general statements regarding numbers are to be made. When so used, the letters may have any values given to them, *i.e.*, a letter has not a single particular value as in the case of the figures 1, 2, 3,...9, *e.g.*, when we say that $a+b = b + a$, a and b may be given any values whatever and still the statement is true; but in $2+3 = 3+2$, 2 and 3 have got only the definite values 2 and 3, *i.e.* $(1+1)$ and $(1+1+1)$.

Arithmetic is the science which deals with numbers and **Algebra** is the science which deals with letters and symbols and in this chapter we shall deal largely with letters used as numbers. We shall begin with a few examples where letters are used as if they are whole numbers.

Ex. 1. Find a number less than x by y. Let us take a similar case in Arithmetic. "Find a number less than 30 by 5." The answer is $30-5$ or 25. Similarly the number less than x by y is $x-y$.

Ex. 2. A man is x years old, how old will he be y years hence? If a man is 20 years old now, after 5 years he will be $20 + 5$ or 25. Similarly if he is x years old now, after y years he will be $(x \times y)$ years old.

Ex. 3. Reduce a pounds, b shillings, c pence to pence.

$\therefore$ £1 = 240d. $\therefore$ £a = $a \times 240d$. or 240 a pence.

b shillings $= b \times 12$ or 12 b pence.

c pence $= c$ pence.

$\therefore$ a pounds $+$ b shillings $+$ c pence $= (240a+12b+c)$ pence.

Exercise—Oral.

1. What is the sum of p, q and r?

2. The quantity a is to be taken from the quantity b. How do you express this?

3. A boy earns x pies and spends y pies. How much does he save?

4. If n represents a certain number: (*a*) what is the next lower number? (*b*) what is the next higher number?

5. What is the number greater than p by q?

6. The difference between two numbers is d; the greater is u. Find the less.

7. How many lbs. are there in a ton b cwt. c qr.?

Exercise XII (a).

1. If a man earns a annas a day and a boy earns b annas a day, find the total sum earned in a day by p men and q boys. What is the total when $a = 6$; $p = 20$; $b = 3$ and $q = 12$?

2. A bookshelf has m shelves each holding a books and n shelves each holding b books. Express algebraically the total number of books, and find the number when $m = 41$, $a = 15$, $n = 5$, $b = 16$?

3. If $a = 4$, $b = 2$, $c = 1$, $x = 4$, $y = 8$, $z = 5$, find the value of

(*a*) $3a+4b$ (*b*) $3a-6c$

(*c*) $5ab+2xy$ (*d*) $4ac+2bc$

(*e*) $yz+xz+xy$ (*f*) $\frac{by}{ab}$

(*g*) $\frac{xyz}{a+b}$ (*h*) $\frac{a^3}{c^2}$

(*i*) $\frac{4y^3}{b^2c^2}$ (*j*) a^2xy^3

(*k*) a^3x^2yz (*l*) $ab^2x^2y^3$.

4. If $a = 4$, $b = 5$, $c = 6$, $d = 8$, $x = 9$, $y = 0$, find the value of

(*a*) $6a-3b-2d-4c$. (*b*) $7c-4x+a-3b$.

(*c*) $a^2+b^2+c^2+d^2$. (*d*) $ab+cx-dy+a^2x$.

5. Find the value of $3x^2-4x+2$, when $x=0$; 1; 2: 3; 5; and 7.

6. Find the value of $5x^2+6x+1$, when $x=0$; 4: 5; 8.

7. Express x miles in feet.

8. How many pence are there in a purse containing x shillings and y six-pences?

9. Two bags contain mangoes the first has a mangoes more than the second which contains b mangoes. How many mangoes are there in the first bag? How many are there altogether? If x is the total number of mangoes, what is the relation connecting a, b and x?

10. What are the 3 consecutive numbers of which (*a*) $x + 3$ is the least, (*b*) x is the greatest?

11. Give the 3 consecutive odd numbers of which (*a*) $2n + 1$ is the least. (*b*) $2n + 1$ is the greatest.

12. If $5x + 2 = 22$, what is the value of x?

13. I bought mangoes for x annas at 5 for an anna; these together with 2 bought by my father make 22. For how many annas did I buy?

14. What is the cost of—

(*a*) a lbs. of tea at x *d.* per pound?

(*b*) a lbs. of tea at x *d.* per pound and b lbs. of sugar at y *d.* per pound?

15. A train travels at the rate of x miles per hour. How many miles does it travel in 7 hours?

16. A man walks for t seconds at the rate of v feet per second, what is the distance walked? If that distance be s feet., what is the relation connecting v, s and t?

17. If I walk 28 miles in x hours, what is my rate? If my rate is 7 miles per hour, what is x?

18. It takes x hours to walk from A to B at 5 miles an hour and y hours to walk from B to A at the rate of 6 miles an hour. What is the relation connecting x and y?

19. Taking the formula $s = vt$

(1) find the value of s when $v = 88$ and $t = 12$.

(2) find the value of v when $s = 440$ and $t = 5$.

(3) find the value of t when $s = 220$ and $v = 44$.

20. A stone falls from rest and moves through s. feet in t seconds. It is found that s and t are connected by the formula $s = 16t^2$.

(1) How far does the stone fall in 1; 2; 3; 4; 5 seconds

(2) If $s = 16; 64; 144$; find the value of t^2 and thence the value of t?

§ 98. **Some definitions** —A collection of symbols denoting numbers and operations is called an **expression**, *e. g.*, $8x^2y$; $9x - 2y + 3z$; $5^2x - 3x - 2$ are called *expressions*. In the expression $5x^2 - 3x - 2$: $5x^2$, $-3x$ and -2 are called its **terms** Similarly in the expression $9x - 2y + 3z$, the terms are $9x$, $-2y$ and $+3z$.

A statement that two expressions are equal is called **an equation**. Thus "$a+b=b+a$a" is an equation; so also is "$3x = 6$." But the statement "$a+b=b+a$" is true whatever values we may give to a and b, whereas the statement '$3x=6$' is true only when x is equal to 3.

Equations of the first kind which are true for all values of the letters involved are called **identical equations** or simply **identities**; and equations of the second kind which are true only for particular values of the letters involved are called **conditional equations** or simply **equations**, *e.g.*, in the equation $x + 5=9$, 4 is the only value of x for which both sides are equal. Any other value will make $x + 5$ and 9 unequal. Hence $x+5=9$ is an *equation*. In the relation $7x+3x=10x$ the two sides are equal when $x=1$, or 2 or 3 or any number. This is a statement of equality which is true for all values of the letters concerned and is an **identity**.

Again $a\times b=b\times a$ is an *identity*; so is $(a+b)^2 = a^2 + b^2+2ab$ whereas $x+4=7$ is an *equation* and so is $3x^2=27$.

The equation $x + 5 = 9$ is said to be **satisfied** when $x = 4$ and the value 4 of the unknown quantity x for which the *equation is satisfied*, *i.e.*, for which both sides of the equation become equal, is called a **root** of the equation and when the root is found out, the equation is said to have been **solved**.*

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* At this stage the student must go through the Art. on negative quantities."

"

Exercise XII (b).

1. What values of x will satisfy the following equations :—

(*a*) $5x + 3 = 23$ (what is the value of $5x$?)
(*b*) $6x - 4 = 56$ (what is the value of $6x$?)

2. Solve the equations. *i.e*, find the roots—

(*a*) $x + 2 = 10$. (*b*) $x - 3 = 9$.
(*c*) $3 = x - 2$. (*d*) $5 = x - 3$.

3. Show that—

(*a*) 1 is a solution of the equation $x + 3 = 4x$
(*b*) 3 is a solution of $6x + 2 = 7x - 1$.
(*c*) 5 and 4 are solutions of the equation $x^2 + 20 = 9x$.

4. A man earns a rupees per month, spends b rupees per month His savings per month are x rupees. What is the equation connecting, a, b and x?

5. I go with x rupees to a shop and buy 4 books at the rate of a annas a book and 3 shirts at the rate of b annas a shirt. I return home with c annas. What is the equation connecting a, b, c and x? Find x, given $a = 8$, $b = 12$ and $c = 3$.

6. I ride on a bicycle x miles in t hours at the rate of r miles per hour. What is the equation connecting x, t and r? and find x if $t = 4$ and $r = 12$.

7. A train travels b hours with a velocity of a miles per hour and then for another c hours at the rate of d miles per hour. If x is the total distance travelled, what is the equation connecting a, b, c, d and x? and find x if $a = 20$, $b = 2$, $c = 3$, $d = 25$.

8. In a school there are p, q and r students in three different classes who pay monthly fees of a, b and c rupees respectively. If x rupees is the total monthly fee income from the three classes, what is the equation connecting p, q, r, a, b, c and x? Find x when $p = 40$, $q = 35$, $r = 30$, $a = 3$, $b = 2$, $c = 1$.

9. If I buy h horses for r rupees and x is the average price of a horse, what is the equation connecting x, h, and r? Find h if $x = 35$ and $r = 2275$.

10. I buy a cows at b rupees each and another lot of c cows at d rupees each. What is the average price of a cow?

§ **99. Addition of like terms.** When terms differ only in their numerical co-efficients, they are called **like**; otherwise, **unlike**. Thus $3a$, $7a$, $9a$, are *like* terms as well as the groups $4b$, $6b$, $9b$; $4c^2$, $8c^2$; $5a^2b$, $3a^2b$; whereas $4a$, $3b$, $9c$; $4a^2c$, $6b^2c$, $9ac^2$ are groups of *unlike* terms.

We have already seen in the chapter on multiplication that $5a + 3a = (5 + 3)\,a = 8a$.

Again $17a - 9a = 8a$; for, a gain of 17 rupees followed by a loss of 9 rupees will result in a gain of 8 rupees.

Also $-17a + 9a = -8a$; for, a loss of 17 rupees followed by a gain of 9 rupees will result in a loss of 8 rupees.

Similarly it may be shown that $-17a - 9a = -26a$.

Hence the following rules for addition of like terms :—

Rule 1. The sum of a number of like terms all of the same sign is a single like term of the same sign and its co-efficient is the sum of the several co-efficients.

Rule 2. The sum of a number of like terms not all of the same sign is a single like term, and to find the co-efficient, add the co-efficients of the positive terms; add also those of the negative terms; take the difference of these two results, and prefix the sign of the greater.

Ex.—What is the value of : $7a^2b - 8a^2b + 15a^2b - 3a^2b$?
The expression $= +22a^2b - 11a^2b = 11a^2b$.

When like quantities are thus added together the resulting expression is called their *algebraic* sum.

Thus $-5a + 3a + 7a = 5a$ and $5a$ is the algebraic sum of $-5a$, $+3a$ and $+7a$.

Exercise XII (c).

Find the sum of—

1. $3a$, $2a$, $5a$ and $6a$. 2. $7a^2b$, $3a^2b$, $5a^2b$ and $6a^2b$.
3. $8a^2b^2$, $9a^2b^2$, $-3a^2b^2$ and $-4a^2b^2$.

Find the value of—

4. $8p-9p+6p-10p$. 5. $5xy-9xy-7xy+3xy-6xy$.
6. $8pq^2+3pq^2-7pq^2+10pq^2-6pq^2$.
7. $8c^2+3c^2-4c^2+6c^2$. 8. $6xyz-3xyz+8xyz$.

Work the following questions by first framing equations :—

9. One number is 5 times as great as another. Their sum is 48. Find the numbers. Verify your answer.

10. The total of a certain number of mangoes, twice that number and 5 times that number is 56. What is the number? (Verify your answer).

11. Divide 60 into two parts so that one part is 9 times the other. (Verify your answer).

12. One number is 12 times another, their difference is 55, find the numbers. (Verify your answer).

13. Divide £240 between A, B and C so that A may have 5 times and B 6 times as much as C.

14. x pounds, x shillings and x pence together amount to £42-3-4. What is x?

15. A man is 4 times as old as his daughter, their ages differ by 36 years. Find their ages.

16. A is worth 9 times as much as B and 3 times as much as C, the combined worth is Rs. 9,165; what is each worth?

§ **100. Addition of expressions.** In Arithmetic the sum of 5 rupees, 3 annas and 4 pies is written Rs. 5 3 as. 4p. and no addition is possible unless they are reduced to the same denomination. Similarly in adding two unlike quantities a and b all that can be done is to write the sum as $a+b$; but if we know their numerical values the addition can be actually performed. Similarly the sum of $3a$, $4b$ and $5c$ is merely written $3a+4b+5c$.

Ex. (1). Find the sum of—

$$3a - 5b + 4c,\ 2a - 3b + 6c \text{ and } 3a - 5c + 6b.$$

The sum $= (3a - 5b + 4c) + (2a - 3b + 6c) + (3a - 5c + 6b)$
$= 3a + 2a + 3a - 5b - 3b + 6b + 4c + 6c - 5c.$
$= 8a - 2b + 5c.$

Note.—It has been already proved that $a + (b + c) = a + b + c$, $a + (b - c) = a + b - c$,

The following rule may be given :—

Arrange the expressions in lines so that like terms may be in the same vertical column, just as in the addition of numbers the numbers of the same order are arranged in the same vertical column, then add each column beginning with the left.

Ex. (2). Add together—

$8ab - 9ac + 6bc$, $9ac + 3bc - 2ab$, and $6ac - 5ab - 9bc + ad$

The work is arranged thus:

$$\begin{array}{rrrr} 8ab & -\,9ac & +\,6bc & \\ -\,2ab & +\,9ac & +\,3bc & \\ -\,5ab & +\,6ac & -\,9bc & +\,ad \\ \hline ab & +\,6ac & & +\,ad \end{array}$$

Exercise XII (d).

Ex. (2). Add together—

1. $2a + 4b - 5c$, $3b + 4c - 6a$, $8a - 9b + 10c + d$.
2. $4x + 5y - 6z$, $8y + 9z - 3x$, $4z - 6y - x$.
3. $8p + 3q + 4r$, $9p - 3q - 2r$, $7r - 2p - 2q$, $8r - 9p - 3q$.
4. $6ab + 3ac - bc$, $3ab - 4ac - 5bc$, $6ab - 7ac + 9bc$.
5. $8xy + 9xy - xy$, $10xz - 6yz - 19xz$, $19yz + 8xy - 6xz - 15xl$.
6. $a + 2b$, $c + 4d$, $a - c$, $b - d$.
7. $7xy - 8yz$, $9yz - 2xy$, $10xy - xz$.
8. $5a^2b + 6a^2c - 9b^2c - 8bc^2$, $4a^3b + 6ac^2 - 8b^2c - 6bc^2$, $4ab^3 - 9a^2c - 4bc^2 - 8b^2c$.
9. $1 + (a + b + c) + ab + ac + bc$, $(3a - 4b - 4c) - 2ab - 3ac - 5bc$, $(3 + 6a - 5b - 9c) - 9ab - 5ac - 6bc$.

§ 101. Subtraction of algebraical quantities.—We have seen, ($b>c$ and $a>b+c$), that

$$a + (b + c) = a + b + c$$
$$a + (b - c) = a + b - c$$
$$a - (b + c) = a - b - c$$
$$\text{and } a - (b - c) = a - b + c$$

If we assume these results to be true for all values of a, b, and c and put $b = 0$ in them we have

$$a + (+c) = a + c \text{......I}$$
$$a + (-c) = a - c \text{......II}$$
$$a - (+c) = a - c \text{......III}$$
$$a - (-c) = a + c \text{......IV}$$

or these results may be established graphically thus:

We have seen (*vide* Art. 25) that + and — denote opposite directions, *i.e.*, $(+3)$ denotes 3 divisions to the right, and (-5), 5 divisions to the left. Thus $(+3) + (-5)$ will mean going 3 divisions to the right and then 5 divisions to the left or the net result is 2 divisions to the left, *i.e.*, -2.

$\therefore\ +3 + (-5) = -2 = +3 - 5$, and generally $a + (-c) = a - c$.

Also subtraction is the reverse of addition, *i e.*, if we subtract b fr m a and add b to the result we get a again; or symbolically $(a - b) + b = a$.

$\therefore\ \{3 - (-5)\} + (-5) = 3$

$\therefore\ \{3 - (-5)\} - 5 = 3$ (by the previous case.)

Adding $+5$ to both sides, $\{3 - (-5)\} = 3 + 5$.

And generally $a - (-c) = a + c$.

Note.—The wider use of the word *sum* in Algebra should be understood. In Arithmetic it is used to denote the sum of two positive numbers; but in Algebra a positive number may be added to a negative number and the result may be positive or negative.

In Arithmetic $5 - 4$ means the difference of 5 and 4. But in Algebra $5 - 4$ may mean the difference of 5 and 4 or the sum of 5 and -4.

In Arithmetic the difference between 5 and 4 means the greater — the smaller, *i.e.*, $5 - 4$. In Algebra $a - b$ means "b subtracted from a whether $a > b$ or $b > a$.

Exercise—Oral.

Subtract

1. $7a$ from $3a$.
2. $8ab$ from $10ab$.
3. $-7ab$ from $3ab$.
4. $-4y^2$ from $-9y^2$.
5. $-9xyz$ from $-24xyz$.

§ 102. Subtraction of expressions.

Ex. 1.—Subtract $3a - 4b - c$ from $5a - 6b - c$.

$$\begin{aligned}\text{The difference } &= 5a - 6b - c - (3a - 4b - c)\\ &= 5a - 6b - c - 3a + 4b + c\\ &= 5a - 3a - 6b + 4b - c + c\\ &= 2a - 2b.\end{aligned}$$

The work may be arranged as follows :—

$$\begin{array}{r} 5a - 6b - c \\ + 3a - 4b - c \\ \hline 2a - 2b \end{array} \qquad \text{or thus} \qquad \begin{array}{r} 5a - 6b - c \\ - 3a + 4b + c \\ \hline 2a - 2b. \end{array}$$

On the left, the like terms are written in the same vertical column and the subtraction is effected. Whereas on the right, the signs of the terms in the lower line are changed and like terms added together, the rationale of which the student can easily understand.

It is not necessary that the change of sign should be indicated in the working. It may be done mentally.

As in addition, unlike terms should be written in separate columns.

Ex. 2.—Subtract $6a - 3b - 9c - 4d$ from $9a + 3b + 4c$.

The work is arranged thus —

$$\begin{array}{r} 9a + 3b + 4c \\ 6a - 3b - 9c - 4d \\ \hline 3a + 6b + 13c + 4d. \end{array}$$

Exercise XII (e).

Subtract

1. $3a+4b-5c$ from $8a-9b-5c$.
2. $3x^2+4y^2-6z^2$ from $8x^2+6y^2-9z^2$
3. $8xy-6xz-9yz$ from $9xy+3xz-8yz-8ab$.
4. a^2-a+1 from a^3+a^2+2.
5. x^3+x^2+x+3 from x^4+4x^2+2.
6. $p^3+q^3+r^3-3pqr$ from $4p^3+6q^3-9r^3-18pqr$.
7. $5x^3y+6x^2y^2-9xy^3$ from $x^4+y^4-8x^2y^2-10xy^3$.
8. $-a^3-2a^2-4$ from a^4+3a^2+5.
9. $6x^3+8x^2y-9xy^2-10y^3$ from $18x^3-9x^2y+17xy^2-25y^3$.

10. What must be added to $a-b$ to get $a+3b$?

11. What must be subtracted from $6a-3b$ to get $5a+4b-3c$?

12. From $8a^2+9ab-3ac-2bc$ take away the sum of $9b^2+c^2+6bc$, $8a^2+b^2+7ab$ and $3a^2+11c^2+7ac$ and find the value of the result if $a=1$, $b=2$, $c=3$.

13. A boy works $a+b$ questions of which $c+d$ are right. How many are wrong?

14. A class contains $a+b$ boys out of whom p boys take leave and q boys are absent without leave. Find the number present.

15. A man walks $3a-4b$ miles due North from a fixed point O and then walks a distance $3a+4b$ miles due South. What is his final position with regard to O?

EQUATIONS.

§ 103. Truths in connection with equations.

The following truths are useful in working out equations:—

$$\text{If } x = p$$
$$x+4 = p+4$$
or in general $x+b=p+b$,

i.e., I. **If equals be added to both sides of an equation, the equation is not altered,** *i.e.*, it continues to be satisfied by the same value (or values) of the unknown.

Similarly

if $x=a$; $x-4=a-4$ or $x-b=a-b$,

i.e., if from equals we take away equals, the remainders are equal; in other words,

II. **If equals be subtracted from both sides of an equation, the equation remains unaltered.**

Again if $x = p$, $x \times 5 = p \times 5$ or $x \times a = p \times a$, *i.e.*, in general.

III. **If equals are multiplied by equals, the products are equal.**

Again if $x = p$, $x \div 4 = p \div 4$ or $x \div a = p \div a$, *i.e.*, in general,

IV. **If equals are divided by equals, the quotients are equal.**

Ex. 1.—Find the value of x which satisfies the equation

$$8x - 13 + 9x = 2x + 8 - 6x.$$

Collecting like terms on each side we have

$$17x - 13 = -4x + 8.$$

Adding $4x$ to both sides we get

$$17x - 13 + 4x = 8.$$

(The purpose of this is to transpose $4x$ to the left side. Note that when so transposed, it changes its sign, *viz.*, from $-$ to $+$).

Adding 13 to both sides we get

$$17x + 4x = 8 + 13.$$

(This is transposing 13 to the right side; and note that its sign also is changed from $-$ to $+$)

$$\therefore \quad 21x = 21 \qquad \therefore \quad x = 1.$$

It will be noted that in solving the equation we have taken all the terms containing the unknown quantity x to one side of the equation and the absolute terms to the other side without altering the equation.

The student should invariably verify the solution got, *i.e.*, see if the value got for x is right by substituting it for x on both sides and seeing if the two sides are equal.

Here, when we substitute 1 for x, the left-hand side

$$= 8x - 13 + 9x = 8 \cdot 1 - 13 + 9 \cdot 1 = 4.$$

Again the right-hand side $= 2x + 8 - 6x$

$$= 2 \cdot 1 + 8 - 6 \cdot 1 = 4.$$

Since the two sides are equal the solution is right.

Exercise XII (f).

Find the value of x satisfying the equations.

1. $8x - 4 = 12$
2. $4x + 5 = 10 + 3x.$
3. $8x - 7 = 7 + 6x.$
4. $9 - 3y = 6y - 72.$
5. $9z + 6 = 3z + 60.$
6. $-3y - 8 = 4y - 71.$
7. $3(2 + x) = 6x - 18.$
8. $8 - 9(x - 3) = 7x + 3.$
9. $4(x - 3) - 9(x - 5) = 9(8 - x) + 1.$
10. $8(x + 3) - 9(x + 5) = 6(x + 3) - 2(x + 32).$

Exercise XII (g).

1. One number is less than another number by 8, and their sum = 34. Find the numbers.

2. The sum of two angles of a triangle is 84° and their difference is 20°, find them. What is the third angle ?

3. I multiply a number by 5 and add 16 to the result and I get 56. What is the number ?

4. I subtract 8 from a certain number and multiply the result by 6. The final result is 84. What is the number ?

5. Find 4 consecutive numbers whose sum is 35.

6. Divide 600 rupees among A, B, and C so that A gets twice as much as C and B get three times as much as C.

7. Find three consecutive odd numbers so that their sum is 69.

8. A purse contians 20 coins consisting of rupees and quarter rupees amounting in value to Rs. 16-4. How many coins are there of each sort ?

9. A's age is twice B's; 4 years ago A was 3 times as old as B. Find their present ages.

10. Two men A and B have the same number of sheep. If A buys 25 and B sells 15, A will then have twice as many as B. How many has each ?

CHAPTER XIII.

AREA AND VOLUME.

§ 104. Take a rectangular piece of paper 3″ by 2″. With your ruler mark inches along the length and also along the breadth of the paper. Fold the paper at the points of division along the length, so that the paper is divided into 3 equal portions, lying one above another. Press with your finger-nails along the folds, so as to leave traces of the folds. Then unfold the paper and you will find it to be of the form shown in Fig. 91; now fold it at the point of division along the breadth so that the paper is divided into two equal portions one lying above the other. Press with your finger-nails along the fold so as to get a crease. If you unfold the paper you will find that the creases on the paper have divided it into 6 equal parts as in Fig. 92. What is the length of every one of these parts? and what is the breadth? There are 6 squares each of which is 1 inch long and 1 inch broad.

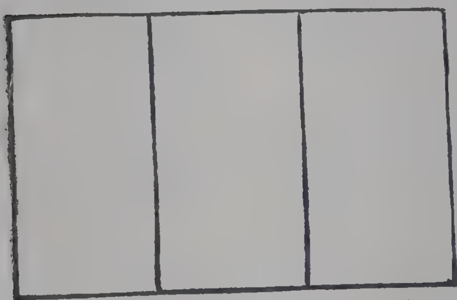


FIG. 91.

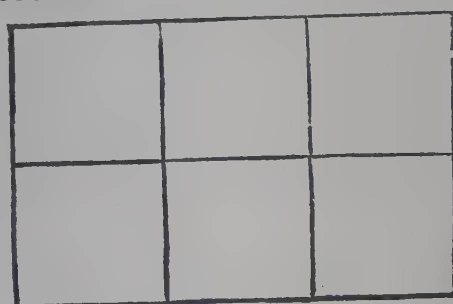


FIG. 92.

A square each of whose sides is one inch long is called a square inch, *i.e.*, **the amount of space enclosed within such a square is a square inch** (the shaded space given in the margin).

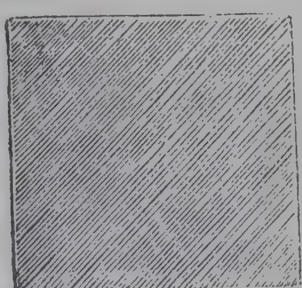


FIG 93.

And the paper we have taken contains 6 such squares or 6 square inches or the **area** (the space occupied) of the paper is said to be 6 square inches (written 6 sq. in.)

Note.—A square each of whose sides is 1 foot is called a square foot (1 sq. ft.).

Similarly we get a square yard (1 sq. yad.) a square mile, a square centimetre.

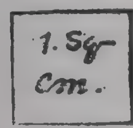


FIG. 94.

§ **105. Area.** Any space, on a plane, which is enclosed on all sides is called a *figure*. The amount of space in a plane figure is called its *area*. And the area is measured (as in the case of lengths), by finding how many times a certain standard area taken as the unit is contained in it. The British unit of area for measuring small areas is the square inch. For measuring large areas large units are employed such as a square foot, a square yard or a square mile.

In the French System, we have the units; a square centimetre, and also a square decametre, (called an *are*).

Exercise.—Practical.

1. Find by paper-folding the area of rectangles of the following dimensions:—

(*a*) Length 4 in. Breadth 2 in.
(*b*) ,, 7 cm. ,, 3 cm.
(*c*) ,, 5 in. ,, 3 in.
(*d*) ,, 8 cm. ,, 4 cm.

2. Construct on squared paper the following rectangles and find their area:—

(*a*) Length 5 in. by 4 in.
(*b*) ,, 8 cm. by 4 cm.
(*c*) ,, 4 cm. by 2·8 cm.

3. (*a*) Draw a rectangle 3 in, by 4 in. Divide the length and breadth so as to show inches. Through the points of division draw parallels to the sides of the rectangle. Count the squares so formed and give the area.

Similarly find the area of each of the rectangles.

(*b*) 6 cm. by 5 cm. (*c*) 8 in. by 6 in. (*d*) 10 cm. by 8 cm.

In all these cases you find you have a number of rows of squares each row containing a certain number. In the example of art. 104 you have 3 rows of 2 squares each or 2 rows of 3 squares each, or 6 squares in all. Without actually counting the squares, you may see that the area is obtained in each case by multiplication of the length by the breadth.

§ 106. The rectangle.

Example 1.—Find the area of a rectangular room 17 ft. by 11 ft.

Sol.—Divide the sides into 17 and 11 equal parts respectively, (see Fig. 95) so that each division is 1 ft. and draw parallels to the sides as in the figure. Then the rectangle will be found to consist of 17×11 small squares, a side of each being 1 ft. That is, the area $= 17 \times 11$ sq. ft. or 187 sq. ft.

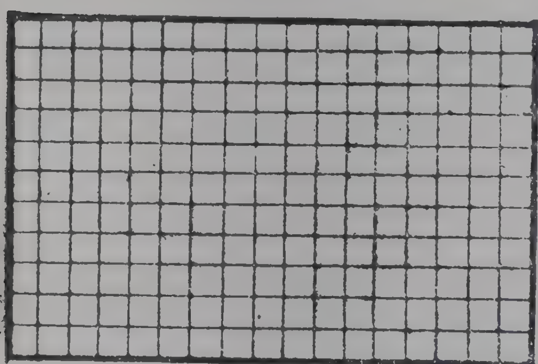


FIG. 95.

Similarly in general, if the adjacent sides of a rectangle measure L units of length and B units of length respectively, the rectangle may be divided into small squares, by dividing the sides into unit divisions and drawing, through those divisions of the sides, straight lines parallel to the sides of the rectangle. The area will thus be divided into $L \times B$ small squares, a side of each being one unit of length and therefore the area will be LB square units.

Thus the area is got by multiplying length by breadth.

Exercise—Oral.

What is the area of

1. a window 6 ft. by 3 ft.

2. a door 6 ft. by 4 ft.

3. a slate 10″ by 8″.
4. a page of manuscript 9″ by 6″.
5. a tennis field 40 ft. by 20 ft.
6. Find the area, by measuring the length and breadth, of—
   (*a*) the four walls of your room.
   (*b*) a page of your note-book.
   (*c*) the bottom of your instrument box.
   (*d*) your table.

§ 107. British and metric measures of area.—The student will have now no difficulty in understanding the tables of areas both British and Metric given in Chap. VIII. In the British table of areas the first relation '144 sq. inches make 1 sq. ft.' means that a rectangle whose length is 1 ft. or 12 in. and whose breadth is also 1 ft. or 12 in. contains 144 sq. inches. Similarly every other relation can be easily understood.

Example 1.—Find the area of a rectangular field of length 9 ch. 4 lks., and breadth 3 ch. 30 lks., expressing the answer in acres and cents.

Length = 9 ch. 4 lks. = 9·04 chains (for 100 links = 1 chain)
Breadth = 3 ch. 30 lks. = 3·3 chains
∴ Area = 9·04 × 3·3 sq. ch. = 29·832 sq. chains
= 2·9832 acres. (10 sq. ch. = 1 acre)
= 2 acres 98 cents.

$$\begin{array}{r} 9{\cdot}04 \\ 3{\cdot}3 \\ \hline 27{\cdot}12 \\ 2{\cdot}712 \\ \hline 29{\cdot}832 \end{array}$$

Example 2.—The area of a field whose length is a mile is 8 acres. Find the breadth of the field.

Area = Length × Breadth.
∴ Breadth = Area ÷ Length.
The area of the field = 8 × 4840 sq. yds.
And the length = 1 × 1760 yds.
∴ the breadth = 38720 ÷ 1760 yds. = 3872 ÷ 176 yds. = 22 yds.

Example 3.—A room is 16 ft. 8 in. long, and 12 ft. 8 in. broad. How many bricks each 8 in. square will be required to floor it?

The area of the room = 200 × 152 sq. in. = 30,400 sq in.

The area covered by 1 brick = 64 sq. in.

∴ number of bricks required = 30,400 ÷ 64 = 475.

Exercise—Practical.

Draw figures to show that (1) 1 sq. yd. = 9 sq. ft.

(2) 1 sq. ch. = 16 sq. poles (1 ch. = 4 poles)

(3) 1 sq. ch. = 484 sq. yds. (1 ch. = 22 yds.)

Exercise XIII (a).

1. Fill up the blanks in the following table regarding rectangles:

Length.	Breadth.	Area.
18 ft.	13 ft.	......
16 ft.	......	192 sq. ft.
......	17 ft.	340 sq. ft.
21 ft.	1 yd.	......
6 yds. 2 ft.	......	100 sq. ft.
18 ft. 9 in.	......	273 sq. ft. 63 sq. in.

2. Reduce 145689 sq. in. to sq. yds

3. Reduce 1486958 sq. ft. to acres (1 acre = 4840 sq. yds.)

4. A wall is 32 ft. long and 6 ft. 8 inches high. Find its area.

5. A sheet of drawing paper is 27 in. by 9 in. How many square inches are there on both sides of it?

6. A tennis lawn is 25 yds. by 14 yds. It is situated in a grassy field 75 yds. by 32 yards. What is the area occupied by the grass?

7. A wall is 30 ft. by 8 ft. There are two doorways each 6 ft. by 4 ft. and two windows each 5 ft. by 3 ft. What is the area of the wall excluding these openings?

8. A hall is 39 ft. 3 in. long and 28 ft. 6 in. broad. Two carpets each of which is 20 ft. long and 15 ft. broad are spread on the floor. What is the portion of the floor not covered by these carpets? Draw a neat figure taking 1 cm. to represent a foot.

9. A desk 4 ft. long and 1 ft. broad contains 4 holes each 3 sq. in. in extent to hold ink bottles. Find the area of the remaining portion.

10. The section of a stone pillar is in the form of a rectangle 12·5″ by 9·5″ with a hollow square inside each side being 4·5″. Find by calculation the area of the section.

11. The floor of a room is paved with 100 Cuddapah slabs each being a square whose side is 2 ft. Express the area in sq. yds. and sq. ft. If the breadth of the room is 16 ft., what is the length ?

12. The width of a sheet of paper is 16 cm. and its length is 28 cm. Find (i) its perimeter in centimetres and (ii) its area in square centimetres.

13. A rectangular field is 250 m. long and 110 m. wide. Find its area (i) in square metres and (ii) in hectares.

14. A postage stamp is 15 mm. by 12 mm. Find its area in (i) square millimetres and (ii) square centimetres.

15. A roll of carpet contains 48 sq. m. If it is 32 m. long, what is its width in decimetres ?

16. If a square metre of sheet brass is cut into strips 20 cm. wide, what will be the total length of the strips ?

17. Find how many tickets each 12 cm. by 5 cm. can be cut from a sheet of paper 2 m. broad and 2·5 metres long ?

18. The pressure of the wind was about 18 lbs. per square inch on a certain day. How much would this be in lbs. on a floor 30 ft. long and 25 ft. wide ?

§ 108. Representation by a plan.

Example 1.—Draw on squared paper a plan of a room 16 ft. by 12 ft. 1 division representing 1 foot and find the area of the room.

Draw a rectangle having 16 divisions for its length and 12 divisions for its breadth. Since 1 division represents 1 foot, 1 small square represents 1 square foot. Area of the rectangle = 16 × 12 or 192 sq. ft.

Example 2.—Draw a plan of a rectangular field 210 ft. by 140 ft. Scale 70 ft. to an inch ; and find its area.

A length of 70 ft. is to be represented by 1 inch.

∴ ....210 ft. ....3 inches.

Similarly a breadth of 140 ft. ....2 inches.

∴ a rectangle whose length is 3 inches and whose breadth is 2 inches will represent the room.

The area of this rectangle is 3×2 or 6 sq. in.
but each square inch represents 70×70 or 4900 sq. ft.

$\therefore$ 6 sq. inches represent 6×4900 or 29400 sq. ft.

$\therefore$ the area of the field = 29400 sq. ft.

Exercise XIII (b)—Practical.

(Apparatus, squared paper, dividers, scale, rulers).

1. Draw the plan of a room 30 ft. by 22 ft. representing each foot by a millimetre. Divide it up into strips each 2 ft. wide. In how many ways can this be done?

2. In the previous question if the room is to be carpeted, find the length of the carpet required in both the cases. Check your result by working it out in another manner.

3. Suppose the width of the carpet is 5 ft., arrange the carpet so that its width is along the direction of (1) the length of the room, (2) the breadth of the room. State the length of the carpet required in the first case and give the wastage in the second case in square feet.

4. Draw the plan of a room 15 ft. by 12 ft. (scale 1 centimetre to the foot). Find the length of carpet required to cover the floor with carpet 2 ft. 6 inches wide so that there may be no wastage.

5. A hall 40 ft. long by 32 ft. wide is paved with Cuddapah slabs each 4 ft. by 2 ft. Draw a plan and find the number of slabs required (4 ft. to be represented by 1 cm.)

6. Draw a representation of a roof 36 ft. by 30 ft. (6 ft. to be represented by 1 inch.) The roof is to be covered with Mangalore tiles each 1 ft. 4 in. long and 9 in. wide. How many tiles are required if there is no overlapping of one tile over another? Show it by drawing.

7. If in the previous example each tile lapped over the one below it to a distance of 2 inches, how many tiles will be required if the length of each tile is parallel to the breadth of the roof?

8. A hall is 50 ft. long and 30 ft. broad. Leaving a pathway 5 ft. wide all round, the remainder is to be covered with a carpet 3 ft. wide. Arrange the carpet so that the wastage may be the least and find the length of carpet required (1 ft. to be represented by 1 mm.)

9. A hall is 40 ft. long 27 ft. broad. Leaving a pathway 5 ft. wide in the middle of the hall running parallel to the length of the hall, on either side are arranged benches 1 ft. wide parallel to the breadth of the room. If the benches are 2 ft. apart from one another, find the length of each bench and the number of benches?

10. A room is 15 ft. long and 12 ft. broad and 8 ft. high. Cut out a piece of paper which when folded round would represent the four walls of the room and find the area of the four walls (scale 2 ft. to be represented by 1 cm.)

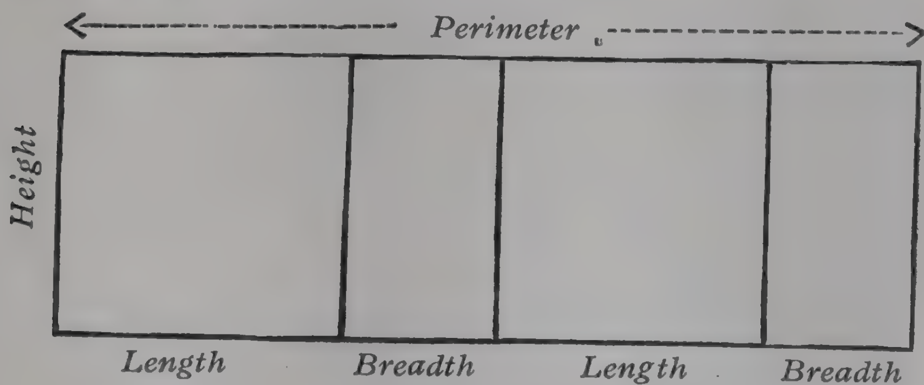


FIG. 96.

Fig. 96 though not drawn to scale gives the form of such a paper when unfolded.

11. A room is 20 ft. long 15 ft. broad and 10 ft. high. Find the area of the four walls as in example 10 and find the cost of white-washing at 3 as. per square (an area of 100 sq. ft. is technically called *a square*.

12. Cut out a piece of paper which when folded round would represent the four sides and the base of a box 2 ft. long, 1·5 ft. wide and 1 ft. high (scale 1 ft. to be represented by 1 inch). What is the area of the inner surface?

13. Represent as in the preceding example a box 3 ft. long, 2 ft. broad and 1 ft. 6 in. high. Find the total area of the inner surface (4 sides and base) and the cost of painting it at 1 anna a sq. ft.

14. Draw a representation of the four walls of a room 18 ft. long, 12 ft. broad and 9 ft. high, mark on it also a window in the

centre of one of the breadthwise walls and a doorway in the middle of one of the lengthwise walls, the dimensions being

window :—5 ft. high and 3 ft. broad.

doorway :—7 ft. high and 4 ft. broad.

(Represent 1 ft. by 1 mm. and calculate the area of the 4 walls excluding these windows and doorways).

* § **109.** If L, B and H denote the length, breadth and height of a room, find the area of—

(1) the floor.
(2) the ceiling.
(3) a lengthwise wall.
(4) a breadthwise wall.
(5) the 2 lengthwise walls.
(6) the 2 breadthwise walls.
(7) all the 4 walls.

Hence learn that

(1) the area of the floor or the ceiling or $A, = L \times B$ and (2) the area of the 4 walls or $A_w. = 2\,H\,(L + B)$.

Example 1.—The length of a room is 18 ft. and its height is 8 ft. If the cost of painting the walls at 6 pies per sq. ft. be Rs. 15, find the cost of matting the floor at 1 anna per sq. ft.

Sol.—As the cost of painting the walls is Rs. 15 at 6 píes a sq. ft., the area of the four walls = (Rs. 15 ÷ 6 pies) sq. ft. = 480 sq. ft.

But the area of the four walls = 2 H × (L + B)
= 16 ft. × (Length + Breadth.)

∴ 16 ft. × (Length + Breadth) = 480 sq. ft.

∴ Length + Breadth = 480 ÷ 16 or 30 ft.

But the Length = 18 ft. Hence the Breadth = 12 ft.

∴ The area of the floor = 18 × 12 sq. ft. = 216 sq. ft.

And as the cost of matting the floor is 1 anna per sq. ft., the cost required is 216 as. = Rs. 13-8. *Ans.*

Example 2.—The cost of carpeting a room is Rs. 45. If the length be 5 feet less, the cost would be only Rs. 30. Find the length of the room.

Sol.—If the length be 5 ft. less the cost would be Rs. (45 — 30) or Rs. 15 less;

that is :—Rs. 15 is the cost of carpeting a rectangle 5 ft. in length and of breadth equal to the breadth of the room, *i.e.*, an area of $5 \times B$ sq. ft. (where B is the breadth in feet).

$\therefore$ Rs. 45 (or 3 times Rs. 15) is the cost for $5B \times 3$ or $15B$ sq. ft.

$\therefore$ the area of the room $= 15 \times B$ sq. ft.

$\therefore$ the length is 15 ft. *Ans.*

Example 3.—A room 20 ft. by 17 ft. has a verandah 5 ft. wide about it. Find the area of the verandah.

Sol.—Let ABDC be the room (Fig. 97), and the shaded portion the verandah. The length of the room ABDC is AB (= 20 ft.) and its breadth is AC (= 17 ft.)

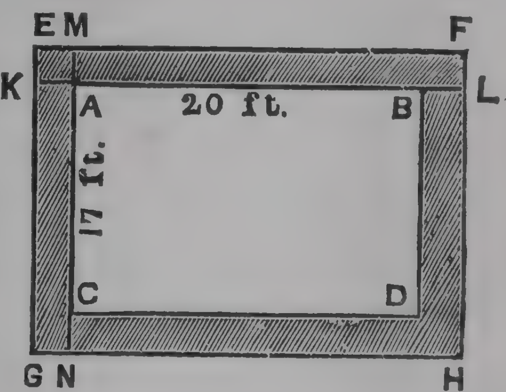


FIG. 97.

Now, since the width of the verandah is 5 ft.

AK = BL = AM = 5 ft.

Hence KL = 20 ft. + 5 ft. + 5 ft. = 30 ft.; and MN = 17 ft. + 5 ft. + 5 ft. = 27 ft.

Hence EF (= KL) is 30 ft., and EG. (= MN) is 27 ft.

Now, EF × EG = the area of EFHG, which is equal to the sum of the area of the room and the area of the verandah.

Hence the area of the verandah + that of the room

$= 30 \text{ ft.} \times 27 \text{ ft.} = 810 \text{ sq. ft.}$

But the area of the room $= 20 \text{ ft.} \times 17 \text{ ft.} = 340$ sq. ft

$\therefore$ The area of the verandah $= (810 - 340)$ sq. ft.

$= 470$ sq. ft. *Ans.*

Or thus: The verandah can easily be seen to consist of 2 strips like MFLA (Fig. 97) whose length is AL or 25 ft. and breadth AM or 5 ft. and 2 strips like AKGN whose length is KG or 22 ft. and breadth AK or 5 ft. $\therefore$ the area of the verandah $= 2(25 \times 5 + 22 \times 5)$ sq. ft. $= 47 \times 5 \times 2$ or 470 sq. ft.

Example 4.—A rectangular garden 40 ft. by 30 ft. is to be enclosed with a path 4 ft. wide, which is to run all round it on the inside. Find the cost of paving this path at 3 as. per sq. ft.

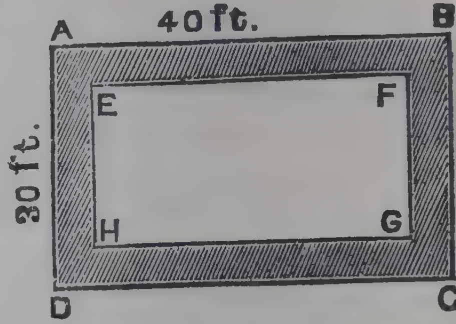


FIG. 98.

Sol.—Let ABCD be the garden (Fig. 98). Since AB is 40 ft. in length, EF = 40 ft. − 4 ft. − 4 ft. = 32 ft.

Similarly EH = 22 ft. [See Example 3.]

Hence the area of the path
= (40 ft. × 30 ft.) − (32 ft. × 22 ft.)
= 1,200 sq. ft. − 704 sq. ft.
= 496 sq. ft.

∴ The cost of paving the path = 496 × 3 as. = Rs. 93. *Ans.*

*Exercise XIII (c).

1. If $L = 15{\cdot}4$ ft., $B = 12{\cdot}9$ ft., find A.

2. If $A = 16{\cdot}96$ sq. ft. and if $L = 4{\cdot}2$ ft., what is B?

3. If $A = 25{\cdot}625$ and if $B = 5{\cdot}25$, what is L?

4. Find the area of (1) a square room 12 ft. 6 in. long, (2) a square plot of side 660 yds.

5. On a map drawn to a scale of half a mile to an inch the sides of a rectangular field are ·52 and ·34 in. respectively. Find the area of the field in acres.

6. On a map drawn to a scale of 1 to 6600, an estate measures 8·4 cm. by 3·2 cm. Find the area of the field in ares.

7. Required the length of a room 24 ft. 6 in. wide, which requires for carpeting the floor 150 yards of carpet 6 ft. wide.

8. $A_W = 2H(L + B)$, find A_W if
(1) $H = 12$ ft.; $L = 20$ ft.; $B = 15$ ft.
(2) $H = 13{\cdot}8$ ft.; $L = 26{\cdot}4$ ft.; $B = 15{\cdot}3$ ft.
(3) $H = 10$ ft.; $L = 34$ yds.; $B = 12$ yds.

9. Find H if
(1) $A_W = 150$ sq. ft.
$L = 10$ ft.
$B = 8$ ft.

(2) $A_W = 200$ sq. yds.
$L = 80$ ft.
$B = 60$ ft.

10. Find L if
(1) $A_W = 800$ sq. ft.
$H = 10$ ft.
$B = 20$ ft.

(2) $A_W = 685$ sq. ft.
= 8 ft.
= 18·4 ft.

11. Find B if

(1) $A_W = 698$ sq. yds.
$L = 80$ ft.
$H = 12$ ft.

(2) $A_W = 893{\cdot}4$ sq. ft.
$L = 30{\cdot}7$ ft.
$H = 12$ ft.

12. Find the cost of painting the walls of a room 16 yds. by 12 yds. and height 6 yds. at 1*s*. 9*d*. a sq. ft.

13. Find the cost of papering the walls of a room 19 ft. 6 in. by 13 ft. 4 in. and 9 ft. 6 in. high with paper at 3 as. a sq. yd.

14. A room 20 ft. long, 15 ft. broad and 10 ft. high has two windows each 5 ft. by 3 ft. and one doorway 6 ft. by 4 ft. 6 in. Find the cost of papering the walls at 4 as. a sq. yd.

15. A room 40 ft. 9 in. long, 21 ft. 3 in. broad and 15 ft. high has two doorways each 7 ft. by 4 ft. and 8 windows each 5 ft. 3 in. by 4 ft. 6 in. Find the cost of papering the walls at 2 as. 6 ps. a sq. yd.

16. Find the length of paper required for the walls of a room

(1) 15 ft. by 12 ft. and 10 ft. high; paper 1 ft. 6 in. wide.

(2) 48 ft. 6 in. by 20 ft. 8 in., and 12 ft. 9 in. high; paper 2 ft. 3 in.

17. Find the cost of papering the walls of the rooms in Question 16 if paper costs Re. 1-8 per piece of 12 yds.

18. A room 22 ft. long, 12 ft. 6 in. wide and 10 ft high is to be papered with rolls of paper each 3 ft. 2 in. square. How many such rolls are required, the area occupied by doors and windows being one-fifth of the whole area?

19. The area of the 4 walls of a room whose length is 3 times its breadth is 960 sq. ft. If the height is 12 ft., find the length and breadth.

20. The height of a room is 12 ft and the area of the 4 walls 120 sq. yds. If the length is twice the breadth, required the breadth and the area of the floor.

21. The length of a room is 17 ft. and its height is 9 ft. If the cost of painting the walls at 4*s*. a sq. ft. amounts to £108, find the cost of matting the floor at 5 as. a sq. yd.

22. A room is 22 ft. 5 in. long and 17 ft. 7 in. broad. If the cost of painting the walls at 5 as. a sq. ft. is Rs. 300, find the height of the room.

23. The cost of carpeting a room 24 ft. long at 3*s*. 9*d*. per sq. yard is £8 and the cost of painting its walls at 1 a. 6 ps. per sq. foot is Rs. 90. Find the breadth and the height of the room.

24. The cost of carpeting a room is Rs. 80; had the room been 2 yds. less in length, the cost would have been only Rs. 75. Find the length of the room.

25. The expense of carpeting a room 15 yards long is £24-6*s*.; if the breadth be 5 yds. less it would cost only £15 to carpet the room. Find the breadth of the room.

26. A piece of a tennis net is 5′ 6″ by 4′ 6″ and each mesh is a square of 1·8 sq. in. How many meshes are there?

27. In a football ground the portion in which the game is played, is 140 yds. by 90 yds., and a space of 6 yds. in width is roped off all round. What is the area of the enclosure and the length of the rope?

28. A picture frame is 24″ by 18″. If the frame is 1·5 in. wide, find the area of the front of the frame.

29. A room 20 ft. by 15 ft. has a verandah (4 ft. wide) about it. Find the area of the verandah.

30. The area of a verandah 3 ft. broad round a room whose length is 15 ft. is 198 sq. ft. Find the breadth of the room.

31. Find the inner surface of a box 5 ft. long, 2·5 ft. broad and 2·5 ft. deep.

32. In a school of 600 boys, each boy is to be supplied with 2 sheets of drawing paper each 9″ × 3″. Each big sheet of drawing paper is 27″ × 48″. How many big sheets have to be ordered for to provide all the boys with the paper and what will remain?

33. A copy of the Hindu newspaper is composed of 4½ sheets, each sheet measuring 20″ × 13″. Find the area of paper used for this copy as a compound quantity If copies are issued every day excepting Sunday and if a subscriber uses all the paper he receives in April 1911, the first of April falling on a Saturday, for papering the walls of his house, what is the area covered?

34. Find the area in acres of a field represented by a map 15 in. by 12 in. (scale: 6 inches to the mile).

35. A piece of 40 yds. of cloth whose breadth is 36 in. costs Rs. 8-8 as.; while another piece of cloth containing only 20 yards whose breadth is 40 inches costs Rs. 9-6 as. Which is cheaper and by how much per sq. yard?

§ **110. The use of squared paper.** The area of any small figure may be practically measured by describing the figure on squared paper and counting the number of squares in the figure. In counting fractions of squares, fractions less than half are ignored, while those greater than half are counted as whole squares. The area thus computed can be only approximate; but enough accuracy for practical purposes can be secured by representing the unit of length by a large number of divisions on squared paper.

Ex. 1. Estimate the area of each of the following figures by counting squares:—

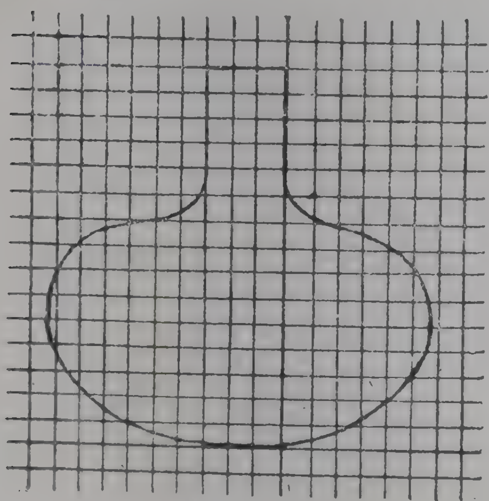


FIG. 99.

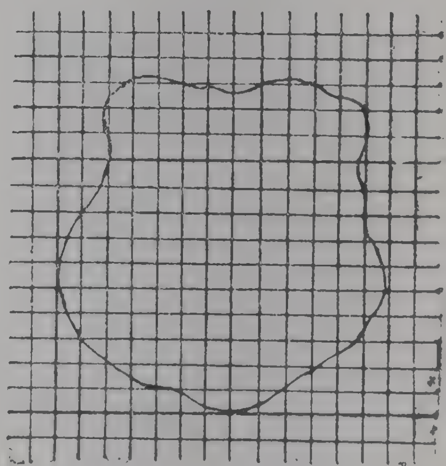


FIG. 100.

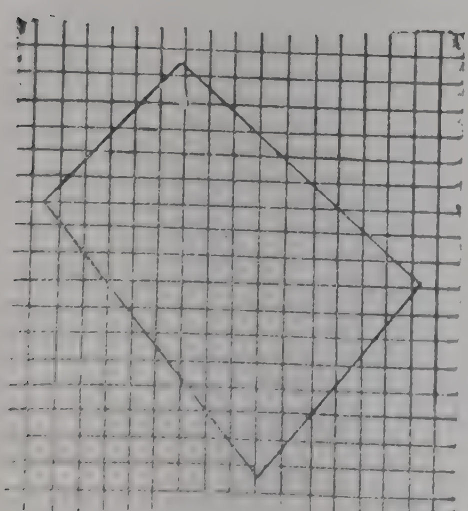


FIG. 101.

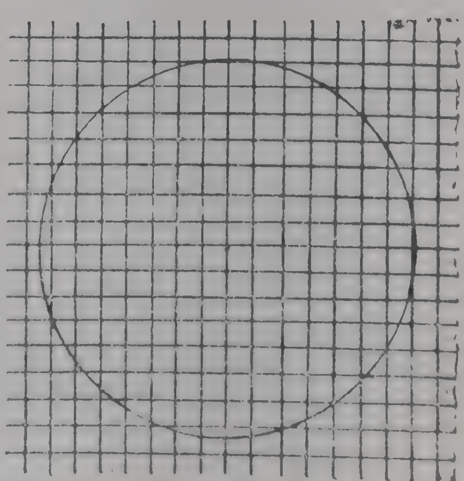
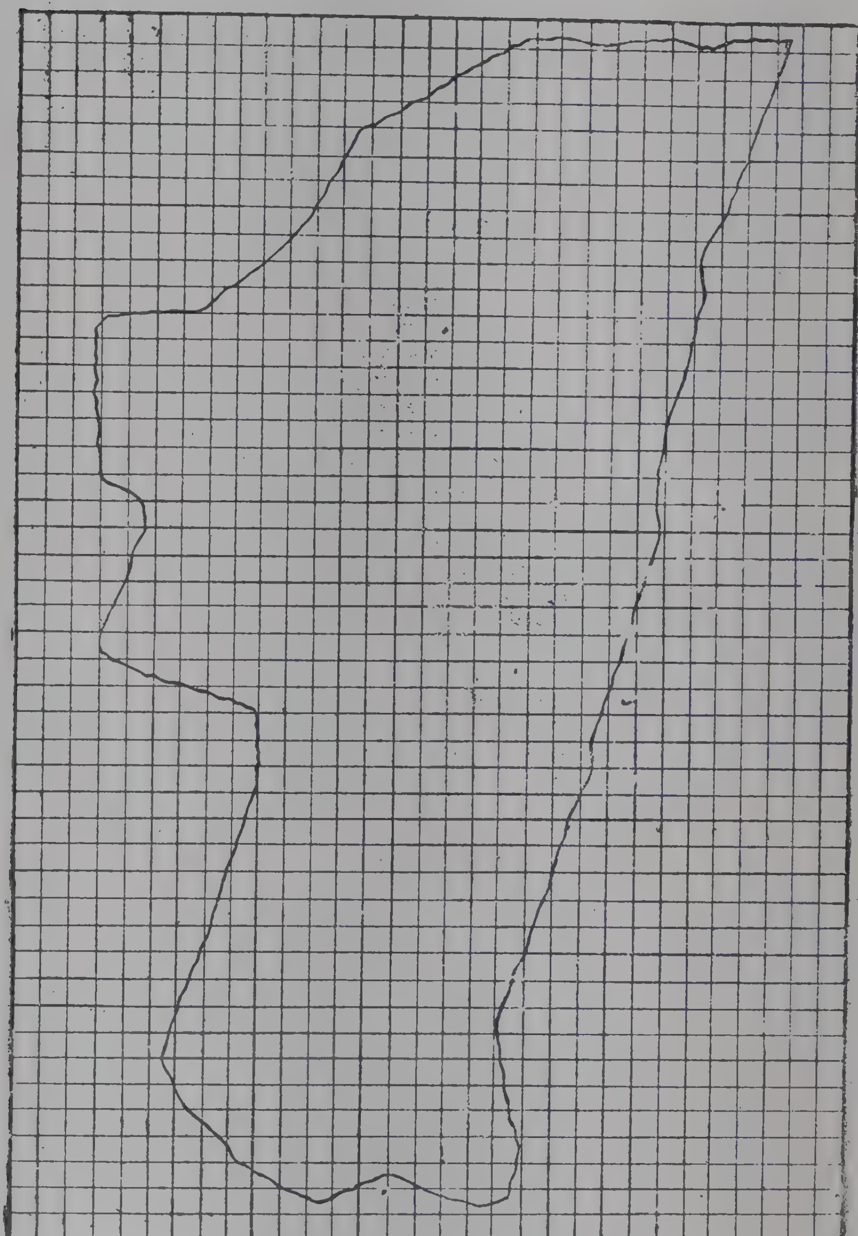


FIG. 102.

Ex. 2. Find, by counting squares, the area of the city of Madras from the accompanying map :—



SCALE 5 DIVISIONS = 1 MILE.

FIG. 103.

VOLUME.

§ 111. The cube.—Copy the annexed drawing

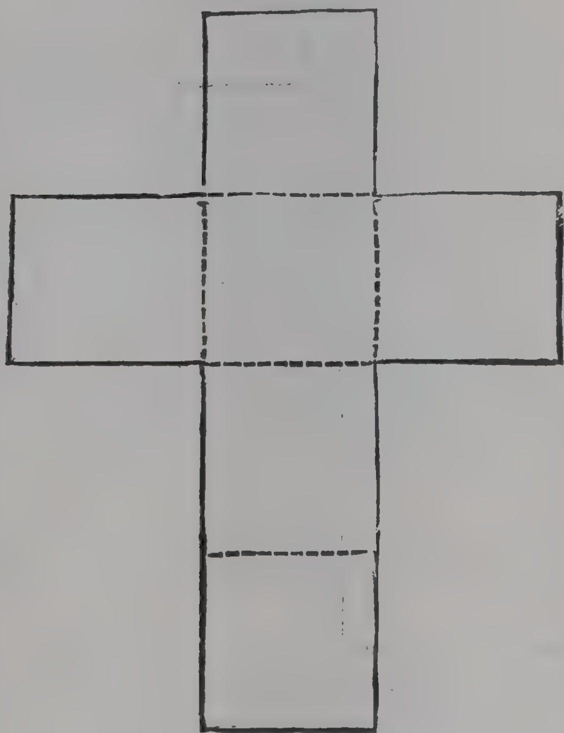


FIG. 104.

(Fig. 104) on thin card-board, run a knife half through the card-board along the dotted lines and bend the flaps through a right angle. The resulting form is called a *cube* (see Fig. 105).

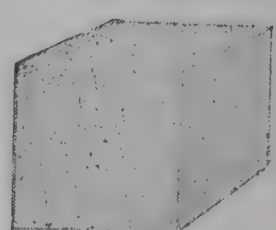


FIG. 105.

Each of the six squares in Fig. 104 becomes a side of the cube. Thus the cube has six equal sides or *faces*. It can rest on any of its faces as base, and the length, breadth and height of a cube are the same, being equal to a side of the base.

§ 112. The cuboid.—Draw the accompanying drawing (Fig. 106) on card-board, and, as for the cube, cut along the dotted lines and fold the flaps. The resulting solid form is called a *cuboid* (Fig. 107.)

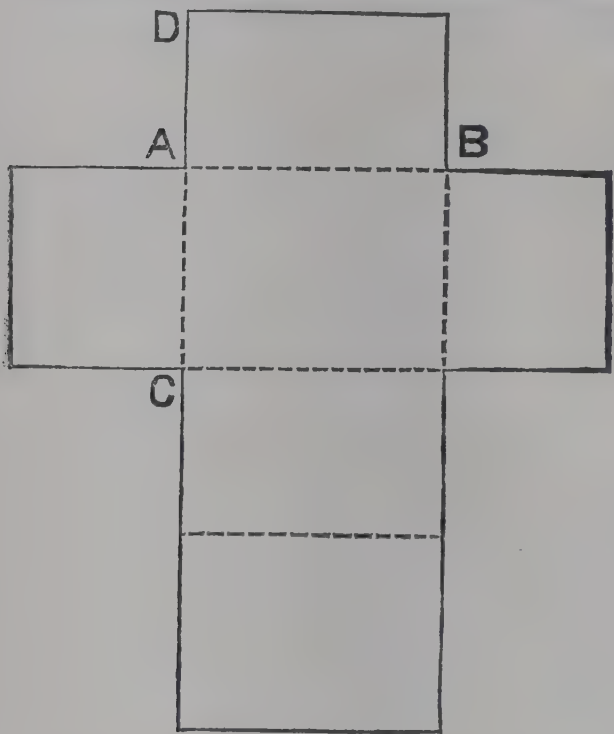


FIG. 106.

As in a cube, so also in a cuboid, the faces are rectangular and are at right-angles to one another; but they need not be squares. AB is the length and AC the breadth of the base, while AD is the height of the cuboid. Thus, the length, breadth and height of a cuboid are not all equal. The cuboid is also called a rectangular solid.

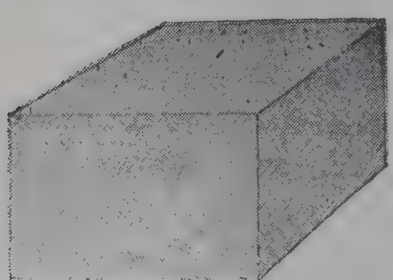


FIG. 107.

A cuboidal wooden block of the form in Fig. 108 may be cut into several small cubes. Suppose that the length, breadth and height measure 5 in., 4in., and 3 in., respectively, and that the edges are divided into inches. By sawing through these inch-divisions perpendicularly to the sides and edges, *i.e.*, along the lines indicated in the figure, the cuboid will be divided into $5 \times 4 \times 3$ or 60 small blocks of the same form as the small figure shown by the side. Each of these blocks

will be a cube, each edge measuring one inch. Thus the

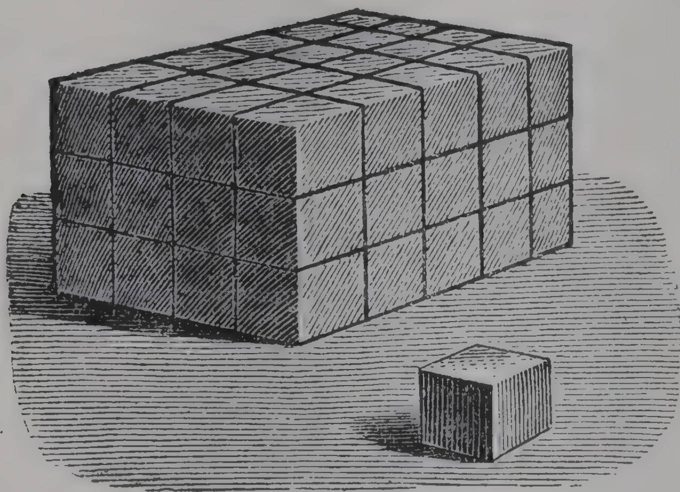


FIG. 108.

cuboid takes up the same space as these 60 one-inch cubes.

§ **113. Volume.**—The amount of solid space occupied by any substance is called its *volume.* The British unit of volume is a cubic inch, which is a cube one of whose edges is an inch. The unit may also be a cubic foot or a cubic yard, *i.e.*, a cube, an edge of which measures a foot or a yard.

In the French system, the unit may be a cubic centimetre or a cubic decimetre. Volumes of solids are generally expressed in terms of the unit employed, as p cubic inches or q cubic feet or r cubic centimetres. The volume of the solid in the previous article = 60 cubic inches.

§ **114. The volumes of a cube and a cuboid** and the **areas of their surfaces.**

(*a*) **The cube.**—The volume of a cube whose edge measures a units $= a^3$ cubic units.

The total surface, *i.e.*, the area of the six faces $= 6a^2$ square units.

(*b*) **The cuboid.**—The volume of a cuboid whose length, breadth and height measure l, b, and h units $= lbh$ cubic units. It may also be expressed as **base × height.**

The area of the six faces $= 2(lb + lh + bh)$ square units. The truths of these will be readily seen from Figs. 134 and 136.

Exercise—Oral.

1. Place 8-inch cubes one over another so as to form a pile. What are the dimensions of the pile, *i.e.*, the length, breadth and height ? How many cubic inches are there in the pile ?

2. Place 2 inch cubes side by side in a row. How many such rows should you place side by side in order to form a square base ? How many cubic inches are there in the whole solid so formed ?

3. Place an inch cube upon each one of these cubes. How many cubes or cubic inches are there altogether ? What are the dimensions of the solid figure so formed ?

4. Place one more cube upon each of the cubes. How many layers of cubes have you now ? What is the height and what is the area of any one of the layers ?

Exercise XIII (d).

1. On each inch square into which a rectangle 3" × 2" can be divided an inch cube is placed. What is the volume in cubic inches of the solid so formed ? What are its dimensions ?

2. Repeat Ex. (1) with the following rectangles :—

(*a*) 4" × 2". (*b*) 3" × 4". (*c*) 5" × 4".

3. If an inch cube is placed on each of the inch cubes in Exercise (1) and (2), what are the dimensions of the volume of the new solids so formed ?

4. Write down the volumes of the following rectangular solids :—

(a)	Length	6 in.,	Breadth	3 in.,	Thickness	1 in.
(b)	,,	10·5 in.	,,	9 in.	,,	5 in.
(c)	,,	12·3 in.	,,	10·6 in.	,,	8 in.
(d)	,,	20 cm.	,,	18 cm.	,,	10 cm.
(e)	,,	x cm.	,,	y cm.	,,	z cm.
(f)	,,	p in.	,,	q in.	,,	r in.

§ 115. The table of cubic measure.

Place 12 inch cubes in a row, and arrange 12 such rows. What are the dimensions of the solid so formed ? Length 12″, breadth 12″, height 1″. You have a solid containing 12 × 12 or 144 cubic inches. On each inch cube put 1 in. cube so as to get two such layers, each layer consisting of 144 inch cubes, the dimensions of the new solid so formed being length 12″, breadth 12″, height 2″. On each inch cube put 10 inch cubes one lying above another, so as to get 10 more layers or 12 layers on the whole. The dimensions of the solid of 12 layers are : length 12″, breadth 12″ and height 12″ and since in each layer there are 144 inch cubes, there must be 144 × 12 or 1728 inch cubes in the solid whose length is 12 inches or 1 foot, whose breadth is 12 inches or 1 foot and whose height is also 12 inches or 1 foot.

∴ 1 cubic foot = 1728 cubic inches.

Similarly the student can construct for himself the tables of cubic measure both British and Metric.

Exercise—Practical.

1. In a cube (side 1 ft.) what is the area of each of the faces ?

2. On each square inch in the bottom, of a cube (side 1 ft.) consisting of inch cubes, how many inch cubes stand in a pile ?

3. What is the total number of inch cubes in the cube whose side is 1 ft. ?

4. Take centimetre cubes and by arranging them in rows of ten and then forming layers of 10 one above another, show that a cubic decimetre contains 1000 cubic centimetres.

5. Draw diagrams to shew that—

(*a*) 27 cubic feet make one cubic yard.

(*b*) 1000 cubic millimetres make one cubic centimetre.

Exercise XIII (e).

1. Find the cubical contents of a wall 20 ft. long, 8 ft. high, and 2 ft. 8 in. thick.

2. A room is 30 ft. long, 19 ft. broad, and 12 ft. high. Find the quantity of air the room contains.

3. What are the cubical contents of a box 4′ 3″ long, 3′ 9″ broad, and 2′ 1″ deep ?

4. A reservoir is 20 metres long, 6 metres wide and 70 cm. deep. What quantity (in kilogrammes) of water does it hold when full ?

5. $V = L^3$, find the value of V, when

(*a*) L = 3′ 2″.

(*b*) L = 4·9 cm.

6. V = LBH, find—

(*a*) V, when L = 20″, B = 18·5″ H = ·12·3″

,, L = 60′ B = 25′, H = 15′

(*b*) L when V = 540 cu. ft., B = 9 ft., H = 3 ft.

,, V = 680 cu. yds., B = 27 ft., H. = 20 ft.

(*c*) B when V = 960 cu. cm., L = 30 cm., H = 80 mm.

,, V = 2420 cu. ft., L = 33 ft., H = 20 ft.

(*d*) H when V = 840 cu. m., L = 21 m. B = 4m.

,, V = 1800 cu. ft., L = 30 ft., B = 24 ft.

7. What is the thickness of a block of wood the length of which is 18 ft., the breadth 4 ft. 6 in. and the cubical contents 28 cu. ft. 1164 cu. in.

8. A room has a capacity of 4285 cu. ft. Its length is 20 ft. 3 inches, its height is 11 ft. 4 inches. What is its breadth ?

9. The dimensions of a class-room are 20 ft.; 12 ft. and 10 ft. and there are 38 boys reading in the class. How many cubic feet of air is allowed to each boy ?

10. Find the cost of digging a trench 6′ 8″ long, 5′ 3″ wide and 3 yds. deep at 6 as. 8 pies a cubic foot.

11. What is the cost of a log of wood 28 ft. long 18 in. wide and 12 in. thick at Re. 1-5-4 per cubic foot ?

12. A rectangular rod of iron is 30 ft. long, 2 in. wide and 0·25 in. thick. Find its weight if a cubic foot of iron weighs 500 lbs.

13. A log of wood 1 ft. 8 in. square measures 22 ft. 4 in. in length. What is the value at Rs. 2-4-6 per cu. yard ?

14. Arrange the following in the order of their capacity :— (1) A cubical bin whose edge is 10 ft. ; (2) a bin whose edges are 12, 10 and 8 ft. ; (3) a bin containing as much as three cubical boxes whose edges are 5, 3 and 2 ft. respectively.

15. A block of stone is 2 yds. 1 ft. 5 in. long, 18 in. broad and 15 in. thick. Find its solid contents and its value at 2*s*. 3*d*. per cubic foot.

16. What length of wire of cross section ·589 sq. mm. can be drawn out of a bar of copper weighing 9 kg., a c. cm. of copper weighing 8·65 g.

17. The pedestal of a statue having a square base the side of which is 28·5 in. contains 10356·1875 cubic inches. Find the height of the pedestal.

18. A wall 22 ft. long, 18 in. thick, and 10 ft. high, has two windows each 16 in. by 1 ft. 3 in. and a doorway 6 ft. by 4 ft. 6 inches. Find the cubic contents of the wall.

19. How many bricks each 10 in. long, 9 in. broad, 2·4 in. thick, are required to build a wall 70 ft. long, 1·5 ft. thick and 15 ft. high ?

20. A reservoir is 57 ft. 8 in. long and 46 ft. 3 in. wide ; how many cubic feet of water must be drawn off to make the water sink (1) 1 foot ; (2) 4·5 feet.

21. In a certain vessel there are 100 cub, ft. of water and the depth of the water is 2 ft. 6 inches. A flat piece of stone 2 ft. 8 in. by 1 ft. 6 in. is dropped into the water when it is found that the surface of the water has risen half an inch exactly. Find the thickness of the stone.

22. If gold be beaten out so thin that an oz. will form a leaf of 20 sq. yds., how many leaves would make an inch thick, the weight of a cubic foot of gold being 10 cwts. 95 lbs.

23. If half a mile of wire can be made out of one cubic foot of copper, find the area of the cross section in decimals of a square inch to two significant figures.

24. 40,000 gallons of rain water fall over 4 acres of ground. If one cubic foot of water weighs 6·25 gallons, find in inches the amount of rainfall.

25. The external dimensions of a rectangular open vessel are 14, 10 and 9 in. and the thickness of the material is $\frac{1}{2}$ inch. When the vessel is empty, it weighs 950 oz., and when it is filled with water 1679 oz. Find the weight of a cubic foot of water and a cubic foot of the material.

26. The external length, breadth and height of a closed box are 4 ft., 12 ft. and 16 in. respectively and the wood of which it is made is 1 in. thick. Find (1) the number of cubic inches of wood in the box, and (2) the cost of lining it with metal at 4*d.* per sq. foot.

27. The external dimensions of a closed cistern are 16, 13, and 10 in. and the thickness of the material $\frac{1}{2}$ in. If when the vessel is empty it weighs 200 oz. What does it weigh when full of water? (a cubic foot of water weighing 1000 oz.) ?

28. An open box of wood measures externally 30 cm. × 25 cm. × 15 cm. and the thickness of the wood is 2 cm. Find the volume of wood used and the number of litres of water it can hold.

29. A closed vessel formed of material 1 inch thick, weighs 6 cwt. 2 qrs. and its external dimensions are 9′ 5,″ 6′ 7″ and 3′ 4″. What would be its weight if it were all solid ?

30. A cistern 10 ft. long, 3 ft. wide and 9 in. deep contains pulp for making paper. If half the volume of the pulp is lost in drying, how many sheets of paper 27″ × 8″ will be obtained if 400 sheets in thickness go to an inch.

31. The cubic contents of a room 12·5 ft. long and 6 ft. 3 in. broad is 937·5 cub. ft. Find the cost of painting its walls at 14 as. a sq. yard.

32. Given that mercury weighs 13·5 times as much as water (bulk for bulk), find the volume of 18 grammes of mercury. To what depth will 400 kg. of mercury fill a rectangular vessel 4 metres by 15 mm. ?

33. Find the number of cubic feet in the flooring of a room 80 ft. by 40 ft., if the boards are 1 in. thick.

34. A path 40 yds. long, and 2·5 yds. wide, is covered to a depth of 4 in. with gravel. Find the weight of gravel used if 30 cu. ft .of the gravel weigh one ton.

35. If 4832 ft. of copper wire can be made out of 2 cu. ft. of copper, find the area of its cross section in square inches.

36. If the area of the cross section of a coil of copper wire is 6 sq. mm., how many metres of it can be made from 150 c. c. of copper ?

37. The cross section of a pipe is 8 sq. inches., find what quantity of water flows through the pipe in 5 hours if water flows at the rate of 3 miles an hour ?

38. A square foot of sheet iron weighs 6 ℔. Find its thickness given that a cubic ft. of wrought iron weighs 480 lbs.

39. A room is 12 ft. long, 10 ft. wide and 10 ft. high. The Madras Education Department requires 50 cu. ft. for each pupil. Can the room accommodate a class of 40 ? If not what is the least number of pupils that can be admitted ?

40. Find the total number of cubic feet of timber required for constructing a gable roof which requires a main rafter $20' \times 6'' \times 4''$ and 12 pieces to be placed on each side of the rafter each being $6' \times 4'' \times 2''$; and find the cost of the timber if the rafter sells at 1 rupee per every running foot and the pieces at 4 a. 3 p. per every running foot.

Exercise XIII (f).

1. If 12,000 copies of a newspaper be issued daily, each copy consisting of 5 sheets and each sheet 4 ft. by 2 ft. How many acres will the editions of a week cover (no edition on Sunday) ?

2. The length and breadth of a room measured by a certain yard-measure were found to be 50 and 24 ft. respectively. If the measure were 2 inches too short, by what area was the room less than the supposed area ?

3. The external dimensions of a cistern without a lid (half inch thick) are 4 ft. 3 in. long, 2 ft. 5 in. wide, and 2 ft. 1 in. high. Find the cost of painting its internal surface at 4 as. a square foot.

4. A pond whose area is an acre is frozen over with ice 3 inches thick; find in tons to the nearest unit the weight of the ice if a cubic foot of it weighs 56 lbs.

5. A cubic foot of a certain wood weighs ·825 as much as a cubic foot of water whose weight is 1000 ounces. What is the weight of a beam of the wood 13 ft. 4 in. long, 1 ft. 3 in. deep and 1 ft. 3 in. thick?

6. A seam of coal 18 ft. thick weighs 21,674 tons. If a cubic foot of water weighs 1,000 oz. and if a cubic foot of coal weighs ·0008 times a cubic foot of water, over what area was the coal spread?

7. Find the expense of digging out the gravel of a pit 40 yds long, 18 ft. wide and 3 ft. deep at 10 as. per cubic yard. If the gravel is sold at Re. 0-15-6 a cubic yard, after being carried to a distance of 50 miles, what are the net proceeds derived from the sale of the gravel if the cartage comes to 1 p. per cubic yard per mile?

8. A stream 2 inches square in section pours into a cistern at the rate of 20 ft. per minute; how many gallons enter the cistern in 2 hours (1 gallon = 277 cubic inches nearly)?

9. A cistern is 10 ft. long, 8 ft. wide and 3 ft. deep and contains 160 cu. ft. of water; how many bricks each 10 in. long, 6 in. broad and 2 in. thick must be thrown into the cistern to make the water rise to the top if each brick absorbs water to the extent of ·25 of its volume.

10. A reaping machine cuts down a patch of corn a metre wide each time it goes round a rectangular field. If the field is *originally* 60 m. by 50 m., what is the length, breadth, and area of the patch uncut after the machine has gone round: (1) 6 times; (2) 20 times; (3) 15 times?

11. Find the volumes of the solids in the following figures, (*a*) FIG. 109, (*b*) FIG. 110 where (1) AB = AC = 3 ft.; (2) BE = DF = GH = 9 in.; ED = FG = 1 ft.

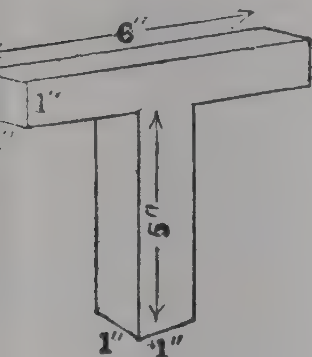


FIG. 109.

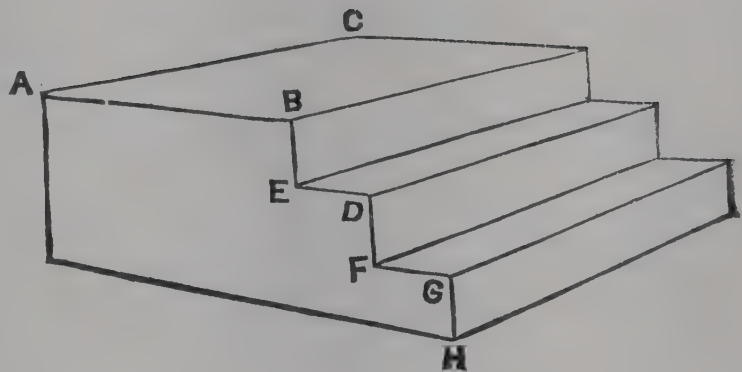


FIG. 110.

12. If the specific gravity of a sample of salt water is 1·063, what is the weight of (1) a litre, (2) a cubic foot of it.

13. The basin of a river is 7620 sq. miles in area and the yearly rainfall 30 inches; a third of the rain that falls is evaporated and the rest flows to the sea. Find to the nearest million gallons the discharge of water to the sea per hour.

14. A wooden box with its covering is internally an exact cube of 2 ft. edge what volume of lead 0·75″ thick is required to line it ?

15. Find the areas of the following figures and of the shaded portions in Figs. 118 and 119 by measurement ; given that each inch represents 10 ft,

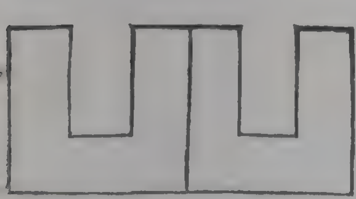


FIG. 111.

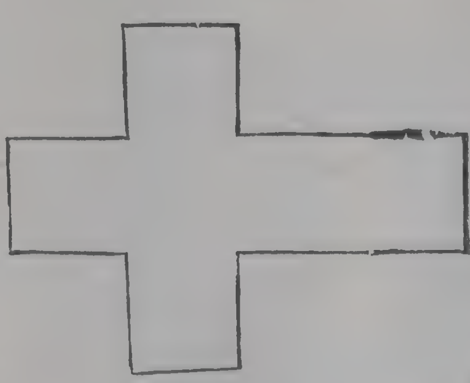


FIG. 112.

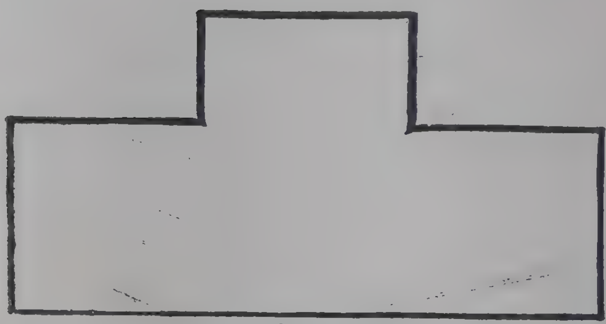


FIG. 113.

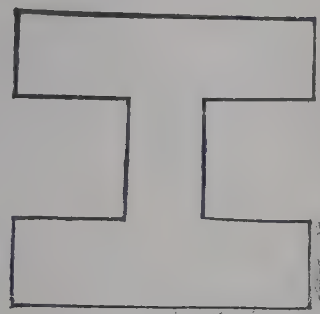


FIG. 114.

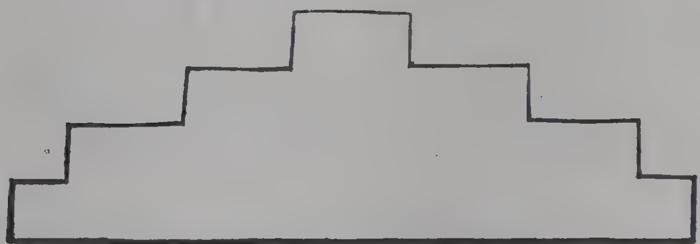


FIG. 115.

FIG. 116.

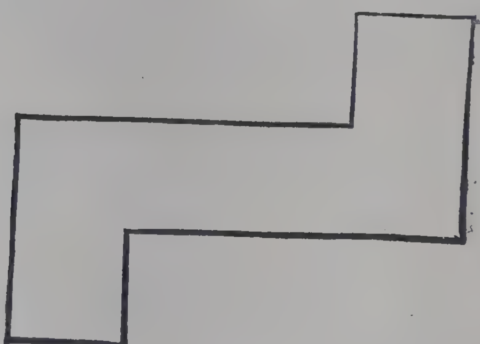


FIG. 117.

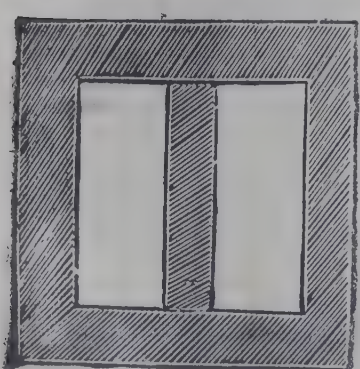


FIG. 118.

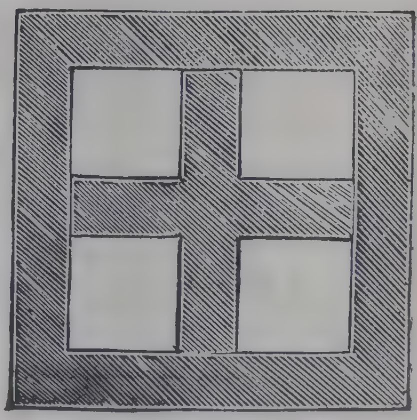


FIG. 119.

§ 116. Volume of an irregular body.

(Apparatus, Burette, Glass, Tumbler, Pebbles, Chemical Balance, Weights).

1. Find by a burette the number of cubic centimetres in the volume of a pebble.

Fill the burette with water so that the water reaches any convenient mark. Put the pebble into the burette. Note the mark the water now reaches. The difference between the two readings is the volume of the pebble.

2. Measure the volume of a pin, a shot, a button, any irregular mass of lead.

3. Measure the dimensions of a given tin box and calculate its cubical contents in cubic inches and find by a burette the number of cubic centimetres the box holds. Hence allowing for the thickness of the material of which the box is made, calculate the number of cubic centimetres equivalent to one cubic inch.

4. Find the volume of an irregular piece of any substance. Weigh it in grammes and find the weight of an inch cube of the substance in grammes. The volume is got by dividing the weight of the mass by that of the inch cube of the substance.

CHAPTER XIV.

BISECTION OF LINES AND ANGLES.

§ **117.** Take a string. How will you find the middle point of the string? Hold the two ends of the string together between the fingers of the left-hand and keep it tight with the right-hand. You now see that the string is divided into two equal portions and the middle point is exactly where it is held by the right-hand.

Draw a straight line AB on a piece of paper. How will you similarly find the middle point of AB?

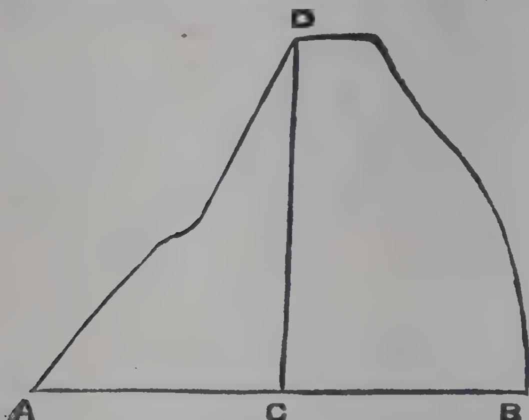


FIG. 120.

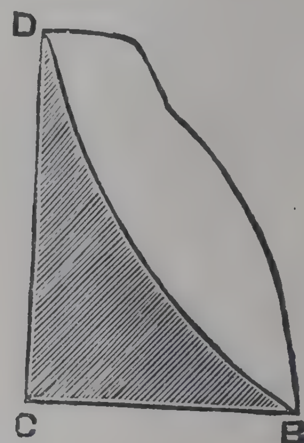


FIG. 121.

Fold the paper so that the extremity A of the line may fall on the extremity B (tracing paper is recommended for the purpose) and press the fold with your finger-nails so as to leave a crease. Let C be the point where the crease cuts the line AB. Measure the portions AC and CB with your dividers you will find that AC and CB are equal. Hence C is the middle point. But you know that AC and CB are equal without the help of the dividers. How Because AC and CB coincide with one another.

You have also seen (Art. 32) that when a line is folded back on itself the angles made by the crease with the line are right angles. You therefore say that the crease bisects (divides into two equal parts) the line at right-angles, *i.e.*, the crease is the perpendicular bisector of the line AB.

Take any point P on the crease. Join PA and PB (Fig. 122). On folding the paper again about the crease, what do you find with regard to the lines PA and PB? They coincide. What do you conclude? They are equal. Take any other point Q on the crease and do the same. You find that QA = QB. You infer from these experiments **that any point on the crease or the perpendicular bisector of a straight line such as AB is equidistant from the extremities of the line.**

Also if you take points R, S, etc., equidistant from A and B (say 2 in., 2·5 in., etc.), by the construction of (Art. 93), it will be found that these points lie on the crease, *i.e.*, the perpendicular bisector of the line AB. **In other words, the locus of a point equidistant from two given points (as for instance A and B) is the perpendicular bisector of the line joining the points (*viz.*, AB).**

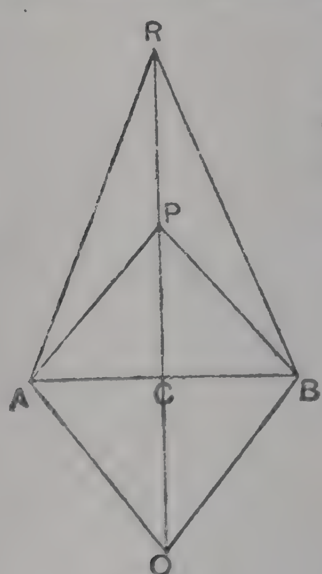


FIG. 122.

Also if any point R be taken outside the crease as in Fig. 123, it can be easily proved that RA is not equal to RB. For in this case either RA or RB must cut the crease PQ at some point

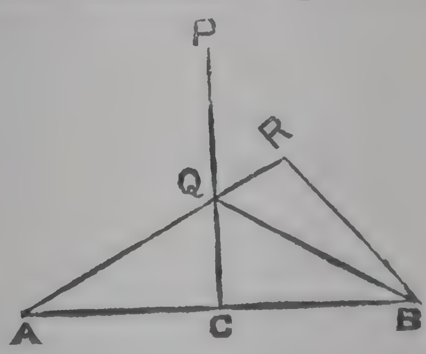


FIG. 123.

(say, Q as in Fig. 123). Then QA = QB as shown above and BQ + QR > BR, *i.e.*, AR > BR.

The triangles PAB and QAB (Fig. 122) are isosceles triangles because PA = PB and QA = QB in those triangles; and, since the triangles PCA and PCB are found to coincide on folding, it may be concluded that

1. $\angle$APC = $\angle$CPB, i.e., that PC bisects the angle APB, i.e., the line joining the vertex of an isosceles triangle to the middle point of the base bisects the vertical angle.

2. $\angle$PCA = $\angle$PCB and each of them is a right angle, i.e. the line joining the vertex of an isosceles triangle to the middle point of the base is perpendicular to the base or the bisector of the vertical angle is also perpendicular to the base and bisects it.

3. $\angle$PAB = $\angle$PBA, i.e., the base angles of an isosceles triangle are equal, i.e., the angles opposite to the equal sides are equal.

4. If there are two isosceles triangles on the same base, the line joining the vertices (or the line produced) bisects the base at right angles.

§ 118. Bisection of straight lines.

Suppose you are required to bisect AB without folding. You know that the line joining the vertices of two isosceles △s bisects the base at right angles. Hence you must construct any two isosceles triangles, one above AB and the other below AB, and from (Art. 93) you know that to construct two such triangles we have to describe circles of equal radii with A and B as centres. So with centres A and B describe circles of any convenient radius intersecting at P and Q Join PQ meeting AB at C Then C is the middle point of AB.

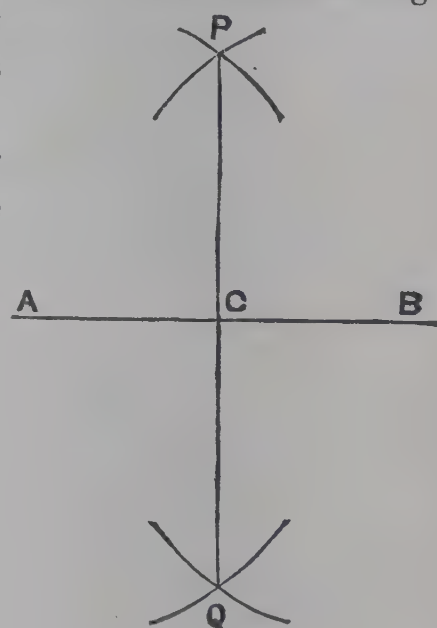


FIG. 124.

N.B.—It is not necessary to show the circles in full ; enough if the portions of the circumference intersecting at P and Q are shown.

Exercise—Oral.

1. Suppose AB in Fig. 124 is 2 inches. If you describe circles of radii one inch long, what will happen ?

2. What must be the least length of the radius of the circles if AB = 2 in. ; 4 in. ; 6 in. ; 8·6 cm. ?

3. Is it necessary to construct the isosceles triangles on opposite sides of AB ? How will you get the middle point if both triangles are described on the same side ?

4. Mark the points A, B and C (Fig. 122) on tracing paper and turning it round, place the trace of A on B and the trace of B on A. Where does the trace of C fall ? How does this prove that C is the middle point of AB ?

5. If the circle drawn with centre A had a greater radius than that drawn with B as centre would C still be the middle point of AB ?

Exercise—(Practical).

1. Draw any line AB and bisect it.

2. Repeat Ex. 1 with 3 more straight lines. First guess the middle point by making a pencil mark.

3. Draw a line PQ = 5·7 cm. and bisect it at R. Compare PR, RQ with your dividers and thus verify your construction.

4. Draw a line 4·3 in. long. Find the middle point by measurement. Bisect it by the construction of (Art. 118) and see if the bisector obtained by construction passes through the middle point found by measurement.

5. Draw a line AB. Divide it into 4 equal parts.

6. Divide any line PQ into 8 equal parts.

7. Divide a given straight line into 2 parts such that one part is seven times the other.

8. Take two points A and B. Find 4 points equidistant from A and B. What is the locus of those points ?

§ 119. Bisection of angles.

Draw an angle $ABC = 60°$. Fold the paper care-

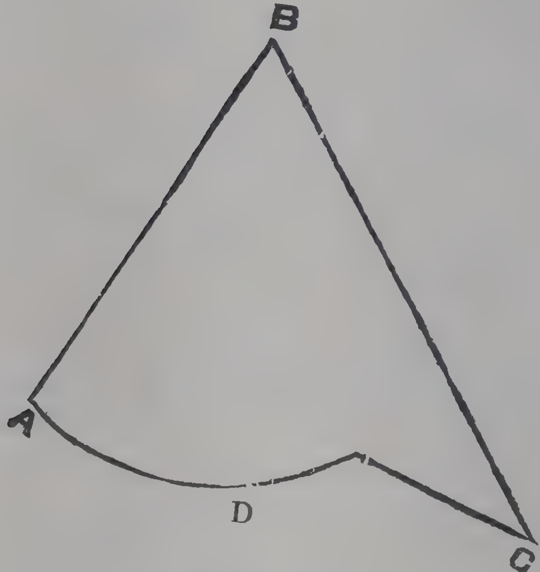


FIG. 125.

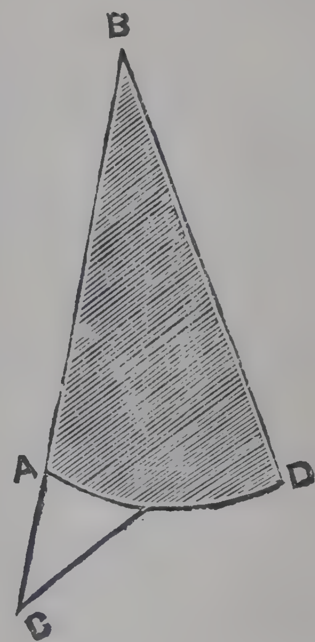


FIG. 126.

fully so that BC falls along BA. Mark the crease BD and unfold the paper. Now measure the angles ABD and CBD. These angles must be equal, each measuring $30°$, because they coincided on folding. The angle ABC is said to be bisected by the line BD.

Suppose an angle is to be bisected without folding; you have seen that the vertical angle of an isosceles triangle is bisected by the line joining it to the middle point of the base. Let AOB be the angle to be bisected (Fig. 127). (To get an isosceles triangle whose vertex is O) describe a circle of any radius (conveniently long) with centre O cutting OA and OB in P and Q respectively and join PQ. To get the perpendicular bisector of the base PQ, we must get another isosceles triangle on the other side of PQ, hence, with P and Q as centres describe two circles of equal radii inter-

secting at X join OX. Then the two angles AOX and BOX will be found to be equal. Also the two arcs into which PQ is divided by the line OX will be found to be equal.

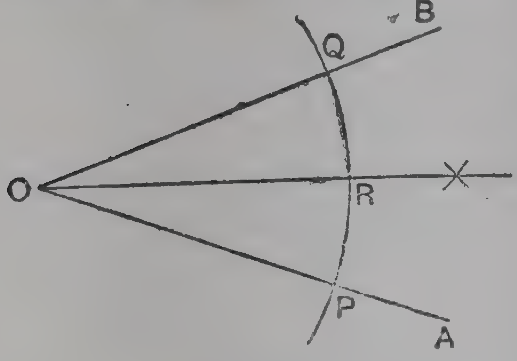


FIG. 127.

N. B.—In order that your constructions may be accurate your lines must be fine; your circles must have their centres exactly at the right points; in joining two points you should take the utmost care to make the pencil pass exactly through the points, before you actually draw the line.

Exercise—Oral.

1. What must be the least length of the radius of circles described with P and Q as centres?

2. When will these circles not intersect?

3. Ascertain by folding if the line OX bisects the angle AOB.

4. Suppose that the circles described with centres P and Q had different radii, would OX be still the bisector of the angle AOB?

Exercise—(Practical.)

1. Draw an angle of 70° and bisect it. Test your work with the protractor.

2. Draw half a dozen angles, make a dot with your pencil in each case to mark a point through which you think the bisector will pass. Then bisect each angle by construction and see if your guess is right.

3. Draw an angle of 160° with your protractor, and divide it into 4 equal parts with ruler and compasses.

4. With ruler and compasses construct an angle of 60°. Obtain from it an angle of 15°.

5. Divide a given angle into 4 equal parts.

6. Divide a given angle into 8 equal parts.

7. Divide an angle into two parts such that one may be 3 times the other.

8. On the bisector of an angle take a point. Draw perpendiculars on the arms of the angle (with set squares), show with your dividers that the perpendiculars are equal and by folding also.

9. Repeat Ex. 8 with a number of points on the bisector.

10. What do you infer from (8) and (9) ?

Hence learn that **any point on the bisector of an angle is equidistant from the arms of the angle.** [The distance of a point from a line is the length of the perpendicular drawn from the point to the line].

§ **120. Drawing perpendiculars.**—You have seen how to draw perpendiculars to given straight lines from a given point (whether on the lines or without them) by means of set squares. This can also be done by means of paper-folding. Draw a line AB on a piece of paper and take a point C on it or outside it. Fold the paper so that a part of the line AB containing the point B may fall on the part containing the point A and so that the crease may pass through C. The crease will then be the perpendicular drawn from C on AB.

If however we have to draw a perpendicular to AB from C a point on AB by means of accurate construction, we have to make use of the property that the line drawn from

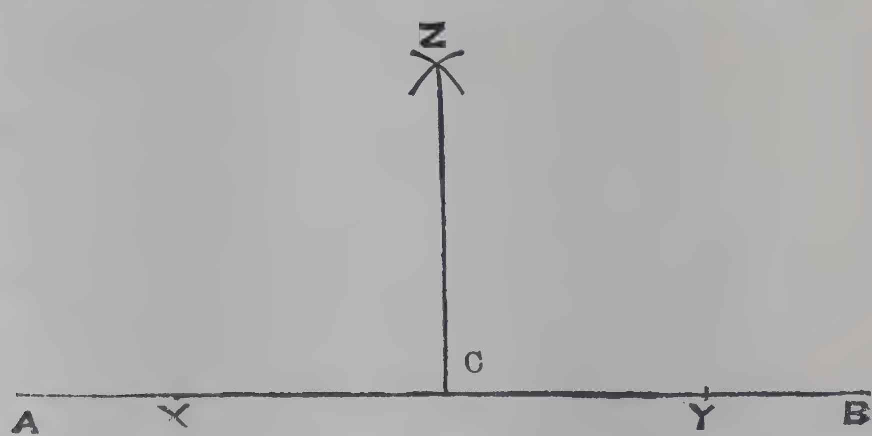


FIG. 128.

the vertex of an isosceles triangle to the middle point of the base is perpendicular to the base. (To make C the middle point of the base) mark off two points X and Y on the line AB equidistant from C. (To get the vertex of the isosceles triangle) describe with centres X and Y circles of the same radius so as to intersect at Z. Then ZC is perpendicular to AB. Test the accuracy of your construction with set-square and ruler.

N.B.—The line AB may be produced both ways in marking points equidistant from C. Thus to draw a straight line perpendicular to AB from the point A you may produce BA and get points X and Y on either side of A, equidistant from it.

To draw a straight line perpendicular to a straight line from a point without it, we make use of the property of two isosceles triangles standing on the same base, *viz.*, that the line joining their vertices bisects the base at right angles. Let AB be the given straight line and C a point without it.

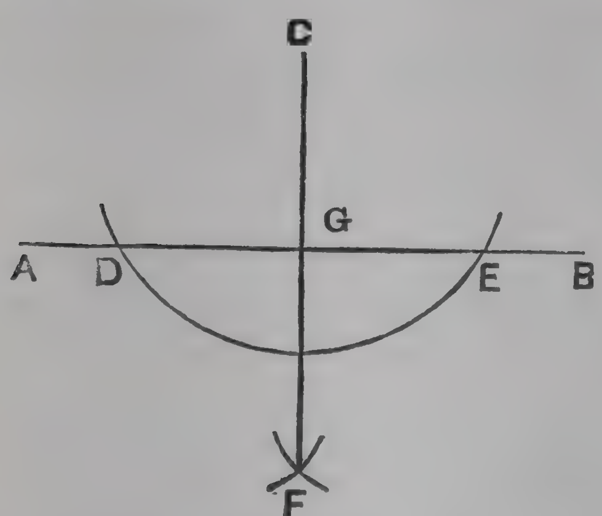


FIG. 129.

(To get one isosceles triangle having C as vertex) with centre C describe a circle of such a radius that it cuts AB in D and E (then CDE is one such triangle). (To get

another isosceles triangle having DE as the base) describe equal circles (*i.e.*, circles of equal radii) with D and E as centres such that they intersect at a point F. Join CF cutting AB in G. Then CG is perpendicular to AB. (Test the perpendicularity with your set-square and ruler).

Exercise—Oral.

1. When the point C is in the line AB; what must be the least length of the radius so that the intersection of the circles may be possible.

2. When will these circles not intersect ?

3. When the point C is without the line AB, what must be the least length of the radius of the circle described with C as centre and what must be the least length of the radius of the circles described with D and E as centres ?

4. When will the circle described with C as centre not cut the straight line ?

5. When will the circles described with D and E as centres not intersect ?

6. Show by means of paper folding that CZ and CF are $\perp$ to AB in the two figures 128 and 129.

7. Why should the circles with X and Y or D and E as centres have equal radii in the two figures ?

Exercise—Practical.

1. Draw a straight line 3″ long. At points 1·6 inches from each end erect perpendiculars. Why are these lines parallel ?

2. Draw a line AB 5 cm. long. At each end erect perpendiculars AP, BQ each 8 cm. long. Join PQ. By what test can you say that PQ is paralled to AB ?

3. Without set-squares and protractors, draw lines making with a given line AB, angles of 90°, 45°, 22½°.

4. Without using set-squares and protractors construct angles of 135°, 112½°, 67½°.

5. Divide a right angle into two parts so that one part may be twice the other.

6. Divide a right angle into two parts so that one part may be 5 times the other.

7. Construct an isosceles right-angled triangle. Calculate the number of degrees in each acute angle. Check your result by measurement.

8. AB is a given straight line and C a point without it. Draw through C a line parallel to AB by using the fact that the perpendicular distance between parallel straight lines is the same at every part.

§ **121. Circles and chords**.—Suppose you are given a circle, *i.e.*, you see a circle on a piece of paper. The position of the centre is not known. How will you find the centre of the circle?

Cut out a circle out of paper and fold it about the middle so that the two parts of the circumference coincide. What is the crease so formed? A diameter. Again fold the circle similarly so as to get another crease. This also represents a diameter. The point where these two creases intersect must be the centre of the circle.

* We have seen that if a circle is folded about its diameter the two parts coincide with one another or in other words the circle is said to be symmetrical about its diameter.

Let us now find a *construction* by means of which we can find the centre of a circle without folding.

Take any two points A and B on its circumference, join AB.

(The line joining any two points on the circumference of

a circle is called a chord) If O be the centre of the

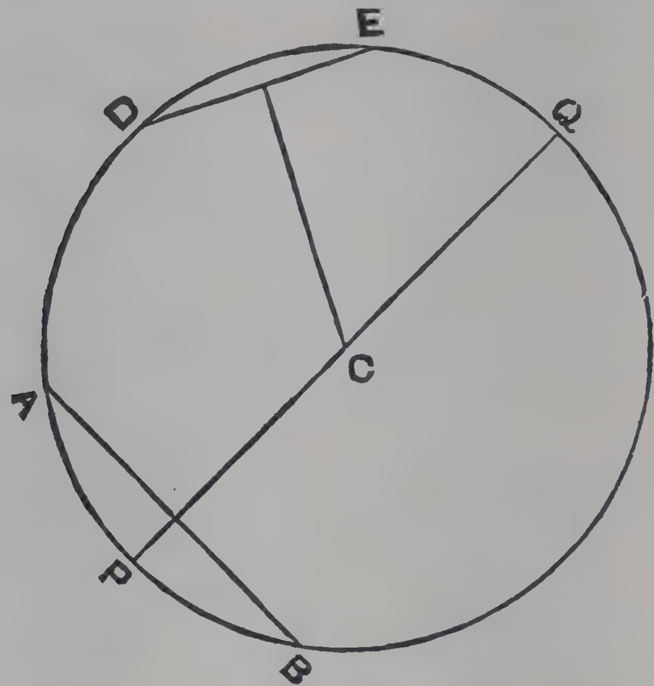


FIG. 130.

circle OA = OB being radii, *i.e*, O is equidistant from A and B. ∴ O must be on the perpendicular bisector of AB. Hence we conclude that **the centre is on the perpendicular bisector of a chord of a circle** or **the line bisecting a chord at right angles passes through the centre**.

Bisect AB at right angles by PQ then the centre is in PQ, *i.e.*, PQ is a diameter. Bisect PQ at C. Then C is the required centre.

Or draw another chord DE. Bisect DE at right angles by RS. RS contains the centre. ∴ the centre is the intersection of PQ and RS.

Draw any triangle ABC. Suppose that a circle passes through the angular points of the triangle ABC; how will you find its centre? (If a circle passes through A, B and C, then AB, AC and BC are the chords of the circle).

By drawing the perpendicular bisectors of AB and AC,

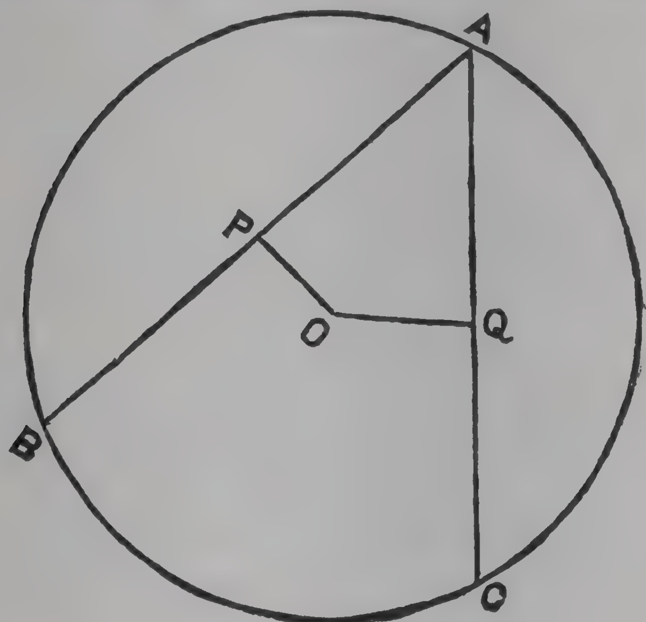


FIG. 131.

viz., PO and QO and finding their point of intersection the centre can be determined. Since PO is the perpendicular bisector of AB, O is equidistant from A and B. ∴ OA = OB. Similarly O is equidistant from A and C, *i.e.*, OA = OC ∴ OA = OB = OC. With O as centre and a radius equal to OA describe a circle; it will pass through B and C.

The circle described so as to pass through the three angular points of a triangle is called the **circum-circle** of the triangle, and the centre of this circle is called the **circum-centre** and the radius of the circle is called the **circum-radius.**

Similarly if A, B, C be any 3 points not in the same straight line you can describe a circle passing through them.

Exercise—Oral.

1. About what line should an isosceles triangle be folded so that the two parts might coincide?

2. About what line should an equilateral triangle be folded so that the two parts might coincide? How many lines are there about which the triangle is symmetrical?

3. Take a rectangle and mention any lines about which it is symmetrical.

4. Is a rectangle symmetrical about its diagonals?

5. Is a square symmetrical about its diagonals?

6. How many circum-circles can be described about a $\triangle$?

Exercise—Practical.

1. Construct a triangle whose sides are 5 cm. 4 cm. and 3 cm. Describe the circum-circle. Where is the circum-centre and what is the circum-radius? Measure also the least angle.

2. Construct a triangle whose sides are 15 mm., 8 mm. and 17 mm. Describe the circum-circle, note the position of the centre and the length of the radius. Measure also the angle opposite to the side 17 mm.

3. Construct a $\triangle$ having $a = 8{\cdot}3$ cm., $b = 9{\cdot}6$ cm. and $C = 120°$. Describe the circum-circle. Note the position of the centre.

4. Construct a $\triangle$ having $a = 6{\cdot}9$ cm., $B = 32°$, $C = 23°$. Describe the circum-circle and note the position of the centre.

5. Construct a $\triangle$ having $a = 2{\cdot}4''$, $b = 3{\cdot}6''$ and $C = 80°$. Describe the circum-circle and note the position of the centre.

6. Fill up the following tabular form noting the details connected with each of the triangles constructed in the Exs. (1 to 5.)

Triangle.	Right-angled or obtuse-angled or acute-angled.	Length of the radius	Position of the circum-centre inside or outside the $\triangle$.	Inferences.
In exercise 1				
,, 2				
,, 3				
,, 4				
,, 5				

Hence it may be concluded

I. That in a right-angled triangle

(*a*) **the circum-centre is the middle point of the hypotenuse.**

(*b*) **the hypotenuse is the diameter of the circum-circle.**

(*c*) **the line drawn from the right-angle to the middle point of the hypotenuse (being a radius of the circum-circle) is half the hypotenuse (which is a diameter).**

II. That in an obtuse-angled triangle the circum-centre falls without the triangle.

III. That in an acute-angled triangle the circum-centre falls within the triangle.

§ 122. Chords and diameters.—If AB is a chord of a circle and O the centre, what kind of a triangle is OAB? An isosceles triangle. Bisect AB at C. Join OC. Measure the angles OCB and OCA with your protractor. You will find that they are right angles. Can you prove that they are right angles by using any of the properties of isosceles triangles you have already learnt? Yes; the line drawn from the vertex to the middle point of the base is perpendicular to the base. Repeat the experiment with 3 other chords. What do you infer?

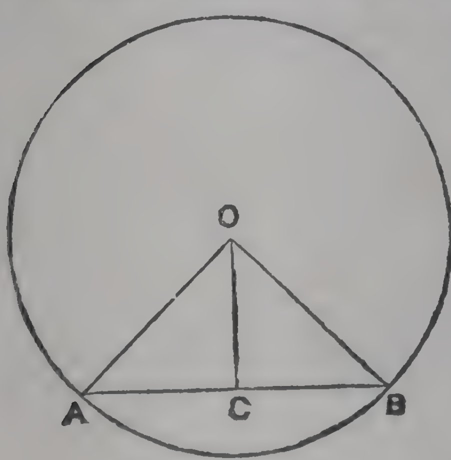


FIG. 132.

We infer:—**If a straight line passing through the centre of a circle (or a diameter) bisect a chord which does not pass through the centre it cuts it at right angles.**

Draw another chord PQ. From O draw OR ⊥ to PQ. Measure with your dividers PR, and RQ. What can you

say about R? R is the middle point of PQ. Can you show otherwise that PR = RQ? Yes. What property of isosceles triangles do you use and which is the isosceles triangle you have here? PQO is an isosceles triangle. The line drawn from O, the vertex, ⊥ to the base, namely OR, bisects the base PQ. Repeat the experiment with 3 other chords. What do you infer?

We infer:—**If a straight line passing through the centre of a circle (or a diameter) cuts a chord which does not pass through the centre at right angles, it bisects it.**

Exercise—Practical.

1. Describe a circle of 2·5 in. radius. Mark off along a diameter of this circle distances of half an inch and at each point draw a chord at right angles to the diameter. Measure parts of the chords on one side of the diameter only and from these measurements give the lengths of the whole chords.

2. Take distances of ·5 in. along a diameter of a circle of 3 inches radius and at each point draw chords making an angle of 60°. Can you give the length of the chords by measuring parts on one side of the diameter as in the preceding question? If not, why not?

3. You are given a triangle ABC and also its circum-centre O. Show how to bisect the sides making use of your set-square only.

4. Describe a circle of 8 cm. radius and draw a number of parallel chords at distances of 1 cm. Draw a diameter at right-angles to these chords, cutting them at M_1, M_2, M_3......; what are these points with regard to the corresponding chords? Hence what is the locus of the middle points of a system of parallel chords of a circle?

5. *A is a point within a circle. Show how to draw a chord through A so that it may be bisected at A.*

6. Two circles whose centres are P and Q intersect at A and B, M is the middle point of AB. Show that PM and QM are in the same straight line. Verify with your ruler.

7. Given a line AB (1 inch long) one half of a chord, (A being on the circumference) and also the length of the radius to be 3 inches, describe the circle without producing AB.

CHAPTER XV.

GREATEST COMMON MEASURE AND LEAST COMMON MULTIPLE.

§ **123. Measure and multiple.** We have already seen what is meant by the factors of a product or a number. 2 is a factor of 6, because 2 when multiplied by 3 gives us 6. Also, 6 when divided by 2 leaves no remainder. Thus a product is always exactly divisible by each of its factors. Each of the factors is also said to be a **measure** of the product and the product itself is said to be a **multiple** of each of the factors. Thus 2 is a *measure* of 6, because 2 is contained in 6 an exact number of times, *i.e.*, *measures* 6 exactly. **A measure of a number is a number which divides that number exactly**. 6 is said to be a *multiple* of 2 for 6 is got by *multiplying* 2 by a whole number. **A multiple of a number is a number which is exactly divisible by that number.** Thus *measure* and *multiple* are correlative terms. If a is a measure of b, then b is a multiple of a.

Exercise—Oral.

1. Give the measures of 30, 56, 63, 64, 72, 81, 84, $9x^2y$, $7x^2z$, $8x^2yz$, $9xyz$; (x, y, z being whole numbers).

2. Give all the multiples of 2 less than 74; of 3 less than 60; of 4 less than 79; and of 5 less than 96.

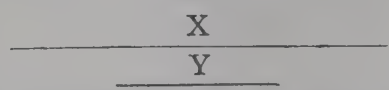
3. What must be added to 19, 23, 29, 31 so that the sums may be exact multiples of 3, 5, 4 and 9 respectively?

4. What must be subtracted from 23, 29, 31 so that the sums may have respectively 4, 5 and 7 as measures?

Exercise—Practical.

1. Draw two straight lines AB and CD 6·3 inches long and ·9 inches long. Show that CD is an exact measure of AB and that AB is a multiple of CD.

2. Of the two lines X and Y given here, examine if Y is a measure of X.



If not, produce X to the nearest point so that the whole may be an exact multiple of Y.

3. AB is a straight line 10 cm. long, CD is 1·5 cm. long. Find a point X in AB nearest to B, so that CD may be a measure of AX. Will BX be a measure of CD ?

4. Draw a rectangle 15 cm. by 12 cm. Show that the rectangle 2·5 cm. by 4 cm. is a measure of CD ?

5. Show graphically that 16 is a multiple of 4.

§ 124. Common measure and multiple.

2 is a measure of 6, *i.e.*, divides 6 exactly ; it is also a measure of 8. Thus 2 is a common factor or measure of 6 and 8.

A common measure of two or more numbers is a number which will divide each of the numbers without any remainder.

Again 6 is a multiple of 2 because 2 divides 6 exactly. It is also a multiple of 3. ∴ 6 is a common multiple of 2 and 3.

A common multiple of two or more given numbers is a number which is exactly divisible by each of the given numbers.

Exercise—Oral.

1. Give the common measures of 4 and 8 ; 12 and 18 ; 24 and 36 ; 80 and 96 ; 20, 35 and 40 ; 16, 18 and 24 ; 108, 120 and 144 ; $9x$ and $7x$; $8y$, $16y$ and $12y$.

2. Give a common multiple of 2 and 3 less than 25; of 6 and 8 less than 100; of 7 and 9 less than 80; of 9 and 5 less than 200.

3. What smallest numbers must be added to 15 and 21 so that 4 may be a common measure of the sum in each case?

4. What smallest number must be subtracted from 26, 40, 58, so that the remainders may be multiples of 6?

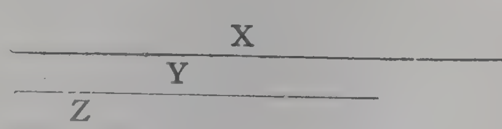
5. What smallest numbers must be added to and subtracted from 93 and 104 respectively so that the sum and difference may be exact multiples of 12?

Exercise—Practical and Graphical.

1. Take a stick a yard long and 3 sticks 4 in., 6 in. and 8 in. long respectively. Measure the yard with each of these. Find the greatest length which measures the yard.

2. Draw two straight lines AB and CD, 8 cm. and 6 cm. long. What is the length of a line which is a common measure of AB and CD? Show graphically that the lengths you propose are common measures of the given lines.

3. Examine if Z is a common measure of X and Y. If not, produce X and Y to the nearest points so that Z may be a common measure of the whole lines thus produced.



4. In the previous example, find points in each of the lines X and Y nearest to one extremity so that Z may be a common measure of the distances of these points from the other extremity.

5. Draw two straight lines AB and CD, 3 cm. and 2 cm. long respectively. Show graphically that AB—CD is a common measure of AB and CD.

6. Draw on squared paper two lines AB and CD, 3″ and 2″ long respectively so that A and C are on the same vertical line. Produce AB and mark off lengths equal to AB along the prolongation of AB. Do the same with CD. See where the marks fall again on the same vertical line.

7. Repeat experiment 6 again by producing the lines still further so as to get another pair of points on the same vertical line.

8. Continue the experiment until you get half a dozen pairs of such points. Measure the distances between A, C and the first pair of points; between A, C and the second pair; between A, C and the third pair and so on; and show that these distances are common multiples of AB and CD.

9. Show graphically that 5 is a common measure of 10 and 15; and 12 is a common multiple of 4 and 6.

10. Show that the rectangle $1{\cdot}5'' \times 1{\cdot}6''$ is a common measure of the rectangles $4{\cdot}5'' \times 4{\cdot}8''$ and $6'' \times 8''$.

11. Show that the rectangle 10·8 cm. by 8·4 cm. is a common multiple of the rectangles 2·7 cm. by 2·8 cm., and 1·8 cm. by 2·1 cm.

§ 125. Greatest Common Measure and Least Common Multiple.

2 is a common measure of 18 and 24
3 also............................18 and 24
6 ....18 and 24

Of all the common measures of 18 and 24, *viz.*, 2, 3 and 6; 6 is the greatest. Hence 6 is the greatest common measure of 18 and 24, *i.e.*, 6 is called the G.C.M. of 18 and 24 (using the first letters of these words). **The greatest common measure** of two or more numbers is the greatest number which divides each of the numbers exactly. The greatest common measure or G.C.M. is also known as the highest common factor or H.C.F. (*factor* being, as we have already seen, another word for *measure*).

12 is a common multiple of 2, 3, 4 and 6.
24................................2, 3, 4 and 6.
36................................2, 3, 4 and 6.

Of all these common multiples of 2, 3, 4 and 6, *viz.*, 12, 24 and 36, 12 is the least. Hence 12 is said to be the least common multiple of 2, 3, 4 and 6 or the L.C.M. of 2, 3, 4 and 6 (using the first letters). **The least common multiple** of two or more numbers is the least number which is exactly divisible by each of the given numbers.

Exercise—Oral.

1. Give the G.C.M. or H.C.F. of :—

(*a*) 6 and 8. (*b*) 8 and 12.
(*c*) 18 and 24. (*d*) 48 and 64.
(*e*) 15, 20 and 25. (*f*) 20, 40 and 60.

2. Find by inspection the H.C.F. of :—

(*a*) 3×3, $2 \times 3 \times 4$ and $2 \times 4 \times 7$.
(*b*) $2 \times 3 \times 5$, $2 \times 3 \times 7$ and $2 \times 3 \times 9$.
(*c*) $2^2 \times 3 \times 7$, $2 \times 3^2 \times 11$ and $2 \times 3 \times 15$.
(*d*) $4 \times 3 \times 5$, $3^2 \times 5 \times 2$ and $3 \times 5^2 \times 11$.

3. Give the L.C.M. of the following pairs, by using the multiplication table and noting what products are common.

(*a*) 3 and 5. (*b*) 4 and 6. (*c*) 8 and 6. (*d*) 9 and 3.
(*e*) 9 and 12. (*f*) 14 and 12. (*g*) 16 and 12. (*h*) 13 and 14.

4. Find the L.C.M. of :—

(*a*) 2×3 and 3. (*b*) 3×2^2 and 2^2.
(*c*) $3 \times 4 \times 5$ and 3×4. (*d*) $4 \times 5 \times 3 \times 2$ and $5 \times 3 \times 2$.

Exercise—Graphical.

1. Draw on squared paper two straight lines AB and CD, 6·3 cm. and 8·4 cm. long respectively, so that A and C are on the same vertical line. *Produce the lines constructing scales of 6·3 and 8·4 cm.* along them respectively and see where any two marks again fall on the same vertical line. Read off the distances of such marks from A and C respectively. Hence find the L.C.M. of AB and CD.

§ 126. To find the G.C.M. or H.C.F. and L.C.M. of two or more numbers by factors. Prime numbers are those which are incapable of being split up into factors. Such numbers are divisible only by unity and themselves, *e.g.*, 7, 11, 13, 17, 19, 23, 29, are prime numbers.

Example 1.—Find the G.C.M. of 240, 186 and 36.

Resolve the numbers into the prime factors, *i.e.*, factors that are divisible only by unity and themselves;

when thus resolved : $240 = 3 \times 2^4 \times 5$
$186 = 3 \times 2 \times 31$
$36 = 3^2 \times 2^2$.

The factors common to the 3 numbers are 3×2.

N.B.—(of 3, 3 and 3^2, evidently 3 is a common factor and not 3^2. Similarly of 2^4, 2 and 2^2; 2 is a common factor and not 2^2 or 2^4).

$\therefore$ The H.C.F. $= 3 \times 2$ or 6.

The H.C.F. of two or more numbers contains all the factors common to those numbers raised to the lowest powers in which they occur.

Example 2.—Find the L.C.M. of 1530, 16800, 684.

$$\begin{aligned} \text{Now} \quad 1530 &= 17 \times 3^2 \times 5 \times 2 \\ 16800 &= 7 \times 3 \times 2^5 \times 5^2 \\ 684 &= 19 \times 3^2 \times 2^2. \end{aligned}$$

The L.C.M. is the least number divisible by 1530, 16800, and also 684. $\therefore$ the L.C.M. should contain as factors 17, 3^2, 5 and 2 to be divisible by 1530; should also contain 7, 3, 2^5 and 5^2 to be divisible by 16800; should also contain 19, 3^2 and 2^2 to be divisible by 684.

$\therefore$ the L.C.M. should contain all the prime factors in the given numbers, *viz*., 17, 3, 5, 2, 7 and 19. But 3 and 2 and 5 occur in different powers, the power of 3 to be included in the L.C.M must be divisible by 3^2 and 3. $\therefore$ it must be 3^2 (3 is contained in 3^2).

Similarly the power of 5 to be included in the L.C.M. must be divisible by 5, and 5^2. $\therefore$ it must be 5^2; the power of 2 to be included in the L.C.M. must be divisible by 2, 2^5 and 2^2, $\therefore$ it must be 2^5.

$\therefore$ the L.C.M. is $17 \times 3^2 \times 5^2 \times 2^5 \times 7 \times 19$.

The L.C.M. contains all the prime factors of the given numbers each raised to the highest power in which it occurs in the given numbers.

Exercise XV (a).

Find (by factors the G.C.M. or the H.C.F. of :—

1. 120 and 180. **2.** 260 and 156.

3. 460, 116 and 920. **4.** 2904 and 429.

5. 864, 1296 and 1728. **6.** 824, 927 and 1648.

7. 1356, 5424 and 6780. **8.** 5555, 9999 and 12221.

Find (by factors) the L. C. M. of :—

9. 16, 32 and 9. **10.** 112, 108 and 116.

11. 160, 156 and 172. **12.** 88, 99 and 132.

13. 460, 840 and 720. **14.** 385, 425 and 625.

15. 1080, 1264 and 1680. **16.** 330, 792, 864 and 480.

17. Draw up a table of all the prime numbers between 1 and 100.

18. A person gets into a hackney carriage whose registered number lies between 2000 and 3000. He remembers that there is a zero in the number and that it is divisible by 72. What is the number ?

19. Construct a table of numbers each divisible by 3, 4, 5 and 6, lying between 200 and 300.

20. What number less than 10,000 is divisible by 11, 17 and 519?

21. Find the H.C.F. of :—

(a) $15a^2bc$, $20a^3b^2c^2$, $35a^3b^3c^3$.

(b) $17x^2y^2$, $21x^3y^2z^2$. $35xyza^4$.

(c) $100xy$, $120xyz$, $150x^2y^2z$.

22. Find the L.C.M. of :—

(a) $8a^3b^2$, $9a^2b^3c$, $15a^2b^2d$, $18abcd^2$.

(b) $7a^2c$, $14a^2bc$, $42a^2bcd$, $38abcd^3$.

(c) $8a^3c$, $10a^3b$, $25x^2yza$ and $30xy^2z^2b$.

§ 127. Graphical introduction to the general method of finding the H.C.F.

Example.—The length and breadth of a hall are 68 ft. and 28 ft. respectively. It is proposed to arrange largest pictures mounted on frames of the same size along the walls of the hall. What is the size of the frame employed ?

Construct a rectangle ABCD 6·8 cm. by 2·8 cm. (let each foot be represented by a millimetre). Mark off along the length AD a distance AE equal to AB (the breadth). From ED again cut off EF = AB. Through

F draw FK parallel to AB. The number of pictures along

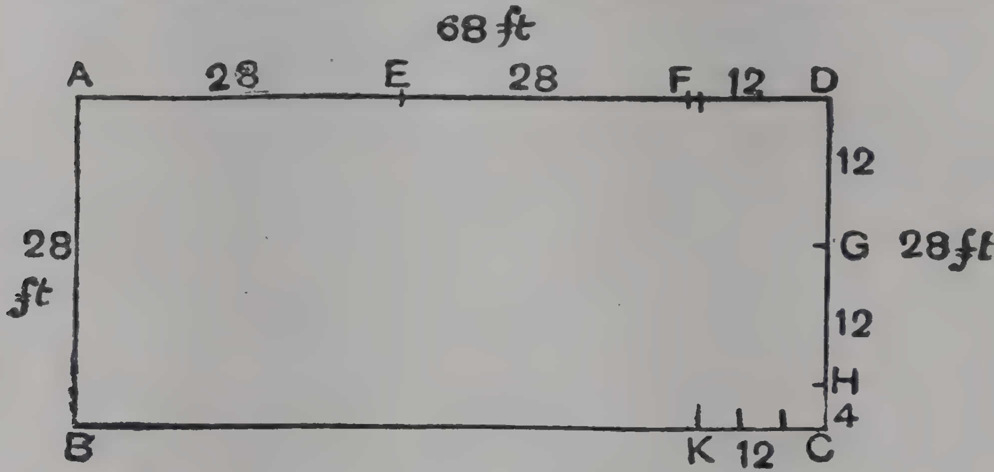


FIG. 133.

AE, EF is the same as the number of pictures along AB, *i.e.*, the pictures will exactly fit BK, for BK = AF. The pictures must also fit FD exactly. From DC set off distances DG, GH, equal to FD reaching as far as H. Then the pictures fit exactly DG and GH. And they must also fit HC. Mark off lengths equal to HC along CK. This can be done once, twice, thrice. ∴ the breadth of each picture must be 4 ft., for KC represents 12 ft.

Exercise—Graphical and Practical.

1. A room is 16·8 ft. long and 12·6 ft. broad. The floor of the room is to be covered with large square tiles none of which are to be cut. Show on your drawing (to any convenient scale) the plan of the floor with the tiles. Find also the largest tile which could be used for the room.

2. Find graphically the G.C.M. of :—

(*a*) 84 and 96.

(*b*) 63 and 72.

(*c*) 125 and 175.

3. A rectangle is 16·8 cm. long and 12·4 cm.; what is the largest size of the squares into which it might be cut ?

4. Draw on squared paper a line 4 in. long and another 3 in. long and show how you would use them to find the G.C.M. of 36 and 27.

§ 128. G.C.M.—General Method.

The graphical method described in the Art. 127 suggests the following method for finding the H.C.F. of 28 and 68.

Divide 68 by 28 ; the quotient is 2 and the remainder 12. The required G.C.M. is the G.C.M. of 12 and 28. Divide 28 into 12 ; the quotient is 2 and the remainder 4. The G.C.M. is also the G.C.M. of 4 and 12. ∴ the G.C.M. of the given numbers is **4.**

Since the remainders alone are wanted, the Italian method of division is employed.

28) 68 (2
12) 28 (2
4) 12 (3
...

Example 1.—Find the G.C.M. of 2076 and 3287.

The H.C.F. is 173.

2076) 3287 (1
1211) 2076 (1
865) 1211 (1
346) 865 (2
170) 346 (2

the whole work may be shortened into the following form :

1	2076	3287	1
2	865	1211	1
	173	346	2

Example 2.—Find the H.C.F. of 12, 144 and 1728.

The H.C.F. of 12 and 144 is 12.

Again the H.C.F. of 12 and 1728 is 12.

∴ 12 is the H.C.F. of 12, 144 and 1728.

Some hints for shortening the Method.

1. If at any stage in this method of division the prime factors of a remainder can be seen, it is generally good to revert to the system of finding the G.C.M. by factors.

2. Any factor that is not common to the remainder and the previous divisor may be rejected at once.

3. To find the H.C.F. of three or more numbers, first find by division the H.C.F. of the two smallest. That H.C.F. may be resolved into factors. Then the method of factors can be employed in finding the H.C.F. of that H.C.F. and the remaining numbers.

To Find the G.C.M. of 2 concrete quantities.

Reduce them to the same denomination and then find the G.C.M. The G.C.M. of 8 a. 4 p. and Re. 1-1-4, is the G.C.M. of 100 pies and 208 pies. ∴ the G.C.M. is **4 pies.**

Exercise XV (b).

Find the H.C.F. of :—

1. 572 and 676.

2. 2047 and 1780.

3. 857142 and 428571.

4. 17171 and 20493.

5. 38556 and 36720.

6. 98, 112, 224.

7. 176, 256, 1600.

8. 81576, 67980, 108768.

9. A blacksmith has two long bars of iron one 88 inches long, the other 32 inches long; he wants to cut them up into an exact number of pieces each of the same length to be used for windows. What is the length of the longest piece that he can obtain under these conditions? How many windows can be got made if each window requires 4 such pieces?

10. Two Societies contributed to a charity fund Rs. 2751-4-0 and Rs. 3,001-4-0 respectively. Each member of the first Society contributed as much as each member of the second. What is the highest possible amount that any member could have subscribed?

11. A room is 14 ft. 3 in. long, 12 ft. 9 in. broad, and 10 ft. 6 in. high. It is to be filled up with cubical packets which are to be arranged in rows, one layer being placed above another. What is the largest size of each cube and how many packets will there be in the room?

12. What is the greatest weight that measures exactly 3 can. 4 mds. 20 plms. and 5 can. 0 mds. 4 viss 30 plms.?

13. Find the greatest number which will divide 7181 and 90514 respectively without any remainder and what remainders will be left if that greatest number divides 7184 and 90520.

14. Hence find the greatest number which will divide 7184 and 90520 leaving as remainders 3 and 6 respectively.

15. Find the greatest number which will divide 3000 and 3876 leaving remainders 20 and 2 respectively.

16. The perimeter of the floor of a room is 972 inches and that of another is 1296 inches. What is the length of the longest piece of string which may be used to compare their perimeters?

17. Two regiments of soldiers numbering 2592 and 3672 are to be arranged in the form of 2 rectangular solids so that the front in each contains the same number of men. What is the greatest number of men that can be placed in the front and how many deep will each solid be?

18. Show that any number that divides each of 2 numbers divides also (1) their sum, (2) their difference, (3) any multiples of either, (4) the sum of any multiples of the numbers, (5) the difference of any multiples of the numbers.

19. If p is the G.C.M. of two numbers A and B, show that A and B may be expressed in the form pl and pm where l and m do not contain any factor common to A and B.

20. If p is a common factor of A and B, show that p is also a common factor of $A+B$, $A-B$, mA, nB, $mA+nB$, $mA-nB$. Hence deduce the principle on which the general rule for finding the H. C. F. is based?

§ 129. Graphical method of finding the L.C.M. of two or more numbers.

Example.—In a hostel attached to a college, the rooms of students are built in three parallel rows each of the same length. In the first row each room accommodates 4 and is 24 ft. long. In the second row each room accommodates 2 and is 18 ft. long. In the third row each room is 8 ft. long and is used only by one. What is the least number of students that can reside in the hostel?

On a piece of squared paper draw AD = 12 divisions (let each division represent 2 ft.). On the line below AB draw CD so that C may be exactly below A and CD = 9 divisions. On the line below CD, draw EF = 4 divisions so

that A, C and E are on the same vertical line (Fig. 134).

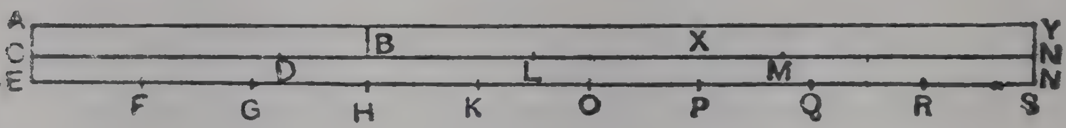


FIG. 134,

Along the first horizontal line step off AB, with a pair of dividers, a number of times marking the points as you proceed. Along the second and third lines step off lengths equal to CD and EF respectively marking the points as you proceed. See if any three marks on these horizontal lines as you proceed are together on the same vertical line. In the figure you will find that this happens at a distance of 36 divisions from the vertical line ACE. Hence the least length of a row is 36×2 or 72 ft.

In the first row there are 3 rooms each used by 4. $\therefore$ 12 students reside in it. In the second row there are 4 rooms each used by 2. $\therefore$ 8 students reside in it; in the third row there are 9 rooms and 9 students live in it $\therefore$ the least number of students is **29**.

Exercise—Graphical.

1. On one side of a road there are trees 72 ft. apart and on the other lamp posts 32 ft. apart. If the first lamp post be opposite to the first tree, how far off is the next tree which is exactly opposite to a lamp post?

2. In a street there are two equal parallel rows of houses, the houses in the same row being of the same lengths; but the lengths of the houses one in each row are 42 ft. and 27 ft. If at one end of the street, the extremeties of the two first houses are opposite to each other, find the least number of houses in the street.

3. In a yard measure one side is graduated to read tenths of an inch and is divided into 360 equal divisions. The other side is graduated to read eighths of an inch and is divided into 288 equal parts. What other divisions beside the zeroes are opposite to one another?

4. Two scales are placed with their edges alongside one another. In one of the scales the divisions are 0·5 in. apart, in the other ·05 in. apart. What is the smallest distance between divisions opposite to one another?

5. In a hall, along its entire length, are to be placed either a number of cots each 6·5 ft. long or a number of sofas each 4·5 ft. long or a number of tables each 3·5 ft. long. What must be the least length of the hall?

6. Find graphically the L.C.M. of 125 and 150.

§ **130.** If A and B are two numbers and H their H.C.F., then $A = H \times$ some factor, say p, so that $A = pH$.

Similarly $B = H \times$ some factor (this need not be the same as the other factor, hence call it q); so that $B = qH$ (p and q have no common factors, for H contains all the factors common to A and B.)

By the method of prime factors. L.C.M. of A and B
$$= p \times q \times H.$$

$$\therefore \text{L.C.M.} \times \text{G.C.M.}\ (i.e., H) = p \times q \times H \times H$$
$$= p \times H \times q \times H \text{ (commutative law)}$$
$$= A \times B \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$
$$\therefore \text{L.C.M.} = A \times B \div \text{G.C.M.} \quad \ldots\ldots\ldots\ldots\ldots\ldots(2)$$
$$= (A \div \text{G.C.M.}) \times B \text{ or}$$
$$A \times (B \div \text{G.C.M.}) \quad \text{Commutative law.}$$

Hence we learn that

(1) the product of H.C.F. and L.C.M. of two numbers = the product of the numbers.

(2) the L. C. M. of two numbers = the product of the two numbers ÷ their H.C.F.

(3) the L.C.M. of two numbers = product of one of the numbers and the quotient got by dividing the other number by the H.C.F.

Example 1.—Find the L.C.M. of 16872 and 34029. These are very large numbers whose factors we cannot easily see.

Find the H.C.F. of 16872 and 34029.

$$16872 \mid 34029$$
$$\mid \quad 285$$

The factors of 285 are 5, 19 and 3. Now 5 is not a factor of 16872 but 19 and 3 are factors. ∴ 57 is the H.C.F.

∴ The L.C.M. $= 34029 \times (16872 \div 57)$
$= 34029 \times 296$
$= 10072584.$

Example 2.—Find the L.C.M. of 4326, 4944 and 6489.

Find the H.C.F. of 4326 and 4944

$$4326 \mid 4944$$
$$\mid \quad 618$$

618 is the H.C.F.

$4326 = 618 \times 7 = 2 \times 3 \times 103 \times 7$

$4944 = 618 \times 8 = 2 \times 3 \times 103 \times 8$

$6489 = 7 \times 3^2 \times 103$

The L.C.M. $= 2^4 \times 3^2 \times 103 \times 7$
$= 16 \times 9 \times 103 \times 7 = 103824.$

Exercise XV (c).

Find the L C.M. of :—

1. 1872 and 3120. **2.** 3549 and 3887.

3. 1960 and 1920. **4.** 26910 and 26013.

5. 145, 261 and 348. **6.** 187, 209 and 1900.

7. 639, 852 and 923. **8.** 1430, 3696 and 2457.

To find the L.C.M. of concrete quantities : reduce them to the same denomination and then find the L.C.M.

Find the L.C.M. of 8 a. 4 p., 4 a. 2 p. and 6 a. 8 p. ; *i.e.*, 90 pies, 50 pies and 80 pies,

$$90 = 3^2 \times 10 \, ; \, 50 = 5 \times 10 \, ; \, 80 = 2^3 \times 10$$

∴ The L.C.M. $= 3^2 \times 5 \times 2^3 \times 10$ or 3600 pies.
i.e., Rs. 18-12-0.

Exercise XV (d).

(H.C.F. and L.C.M.)

1. Find the least number that is exactly divisible by 18, 16 and 13.

2. Find the least number that is exactly divisible by each of the even numbers from 3 to 21.

3. Find the least number of sheets of paper that a teacher must have to distribute equally among 16, 24, 28, or 30 boys.

4. Find the least number which when divided by 12, 15, 16, 18, leaves in every case a remainder 9.

5. Find the smallest number which when divided by 24, 36 and 84, leaves in each case a remainder which is 6 less than the corresponding divisor.

6. What is the smallest number which when divided by 85, leaves a remainder 70 and when divided by 65 leaves a remainder 50?

7. What is the smallest sum of money which when divided among 32 men leaves a remainder of Rs. 11, when divided among 28 men leaves a remainder of Rs. 7, and when divided among 48 men leaves a remainder of Rs. 27?

8. A number of coins can be exactly made into heaps of 23 coins each; but if they be made into heaps of 24 coins or 28 coins each there are 2 left, find the least number of coins in the heap.

9. Find the least sum of money which contains 14 as., Rs. 2-0-8, and Re. 1-5-0 an exact number of times. Find also the G.C.M. of these three sums.

10. Find the least sum of money that can be paid in half annas, quarter annas, two-anna pieces and four-anna pieces.

11. Find the least sum of money that can be paid in florins, half-crowns, crowns, and shillings.

12. Taking a kilometre as 39037·9 inches, find the shortest distance that can be expressed as an exact number of miles and also as an exact number of kilometres.

13. Find the weight of the least capacity that contains an exact number of pounds or viss. (1 viss = $3\frac{1}{8}$ lb.)

14. The forewheel of a carriage is 10 ft. in circumference and the hind wheel 13 ft. 4 in. In what distance will they have made a complete number of turns for the first time?

15. Four bells begin to toll at intervals of 4, 5, 6 and 8 minutes respectively; how long after will they simultaneously ring a second time?

16. Four planets are observed to be together in the same sign of the zodiac and they complete a revolution in the zodiac in $365\frac{1}{4}$ days, $27\frac{1}{2}$ days, $12\frac{1}{4}$ years, and $30\frac{1}{2}$ years respectively. Find when they will be together again in the same sign.

17. Three boys who run at the rates of 3, 4 and 5 miles per hour respectively start from the same point and go round a circle $1\frac{1}{2}$ miles in circumference. In how many hours will they be together again at the starting point?

18. The G.C.M. of two numbers each consisting of 3 digits is 108 and their L.C.M. is 6048. Find the numbers.

19. Show why the H.C.F. and L.C.M. must be wrong in the following case and give reasons for your answer:—35 is the H.C.F. of 70 and 210 and 415 is their L.C.M.

20. Show that the G.C.M. of any two numbers is a factor of the L.C.M.

CHAPTER XVI.

VULGAR FRACTIONS, RATIO, &c.

§ 131. You have already seen that any part of a quantity is called a *fraction* of that quantity. In the chapter on decimals we considered quantities which are divided into **10** equal parts, into **100** equal parts, into **1000** equal parts and so on. We may however divide a quantity into any number of equal parts, say 12 equal parts or 16 equal parts, *e.g.*, 1 anna is divided into 12 equal parts, each part being a pie; again a rupee is divided into 16 equal parts, each part being an anna.

You know that one-fourth of a rupee means that a rupee is divided into 4 *equal* parts of which 1 part is taken; one-fourth is symbolically written $\frac{1}{4}$; similarly three-fourths of a rupee means that a rupee is divided into 4 equal parts out of which 3 are taken; three-fourths is symbolically written $\frac{3}{4}$.

In the accompanying figure let AB (a length of 4 cm.)

represent one rupee. Divide AB into 4 equal parts AP, PQ, QR, RB (each part being 1 cm.) Then AP represents a fourth of a rupee; AQ two-fourths of a rupee, AR three-fourths and A.B. four-fourths, *i.e.*, the whole rupee. The fractions are written, $\frac{1}{4}$, $\frac{2}{4}$, $\frac{3}{4}$, $\frac{4}{4}$.

Generally a fraction is expressed by two numbers one placed above the other with a line between them. The number below the line or the lower figure shows into how

many equal parts the unit is divided, *i e.*, it gives the *name* of each part and is hence called its **denominator**; and the number above the line shows how many such parts are taken to form the fraction, *i e.*, gives the *number* of such parts and is called the **numerator.**

$\frac{7}{16}$ of a ℔. means that a ℔. is divided into 16 equal parts of which 7 parts are taken.

Draw a rectangle 2 in. by 2 in. and divide each of the

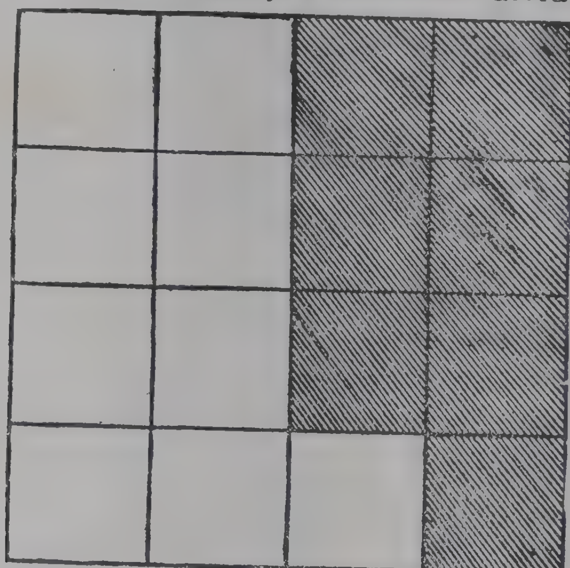


FIG. 135.

adjacent sides into 4 equal parts and divide the figure into 16 equal parts by drawing parallels to the opposite sides through the points of division. Take seven of these parts and shade them. The shaded portion $= \frac{7}{16}$ of the whole rectangle.

Generally the fraction $\frac{a}{b}$ means that a certain quantity regarded as a unit is divided into b equal parts and out of these a parts are taken to form the fraction. Fractions like $\frac{1}{4}, \frac{3}{4}, \frac{9}{16}, \frac{a}{b}$...... are generally called **vulgar fractions**.

Exercise—Oral.

1. Express as vulgar fractions—

(*a*) Four-sevenths. (*b*) Three-elevenths.

(*c*) Seven-twelfths. (*d*) Five-sixths.

2. Read off the following fractions and give their meaning :—

(*a*) $\frac{1}{3}$, $\frac{1}{8}$, $\frac{3}{12}$, $\frac{7}{16}$, $\frac{9}{25}$. (*b*) $\frac{11}{18}$, $\frac{17}{24}$, $\frac{19}{25}$, $\frac{16}{121}$.

3. State the value of the following :—

$\frac{5}{8}$ of a rupee ; $\frac{3}{12}$ of a sh. ; $\frac{11}{20}$ of a pound ; $\frac{7}{16}$ of a lb. ; $\frac{5}{16}$ of 1 viss. ; $\frac{9}{16}$ of 1 cwt.

Exercise—Practical.

1. Draw a line AB to represent a unit and draw lines to represent the following fractions :—$\frac{5}{6}$, $\frac{7}{8}$, $\frac{11}{12}$, $\frac{17}{36}$.

2. Take a rectangular piece of paper and fold it a number of times and cut off portions that will represent $\frac{3}{8}$, $\frac{5}{22}$, $\frac{13}{16}$ of it.

3. Draw rectangles measuring (1) 6 in. by 2 in. (2) 8 cm. by 3 cm. (3) 9 cm. by 4 cm. Taking each rectangle as the unit, represent the following fractions by shaded portions :—

(1) $\frac{2}{3}$, $\frac{5}{6}$, $\frac{11}{12}$. (2) $\frac{3}{4}$, $\frac{4}{6}$, $\frac{11}{24}$. (3) $\frac{5}{18}$, $\frac{11}{12}$, $\frac{19}{36}$.

§ 132. Fractions of compound quantities.

Example 1.—Find the value of $\frac{3}{16}$ of Rs. 10-8-0.

Divide Rs. 10-8-0 into 16 equal parts ; each part is Re. 0-10-6. Take 3 such parts. The value = Re. 0-10-6 × 3 = Re. 1-15-6.

Example 2.—Find the value of $\frac{18}{29}$ of 2 ton 11 cwt. 3 qr. 4 lb.

We have to divide 2 ton 10 cwt. 3 qr. 4 lb. by 29 and then muitiply the result by 18.

2 ton 11 cwt. 3 qr. 4 lb. ÷ 29 = 1 cwt. 3 qr. 4 lb.

1 cwt. 3 qr. 4 lb. × 18 = 1 ton 12 cwt. 16 lb.

Thus $\frac{18}{29}$ of 2 ton 11 cwt. 3 qr. 4 lb. = 1 ton 12 cwt. 16 lb.

We may also first multiply by 18 and then divide by 29. The result is even then the same. The nature of any particular example will suggest which of these two methods will have to be followed in solving it.

Example 3.—Evaluate $\frac{16}{17}$ of £3-3-9.

16 × £3-3-9 = £51.

£$\frac{51}{17}$ = £3.

Exercise XVI (a).

Find the value of the following :—

1. $\frac{4}{9}$ of Rs. 15-12-0.
2. $\frac{13}{25}$ of £200 8*s*. 4*d*.
3. $\frac{15}{28}$ of 3 cwt.
4. $\frac{17}{81}$ of 27 yd. 2 ft. 3 in.
5. $\frac{17}{75}$ of 1 m. 8 dm. 7 cm. 5 mm.
6. $\frac{8}{9}$ of Rs. 3-6-0 + $\frac{10}{11}$ of Rs. 122-5-1 — $\frac{2}{23}$ of Rs. 47-8-11.
7. $\frac{1}{3}$ of 1 ton 4 cwt. — $\frac{1}{6}$ of 2 ton 1 qr. 10 lb. + $\frac{17}{28}$ of 4 ton 3 qr.

§ **133**. We have pointed out above that in evaluating $\frac{18}{29}$

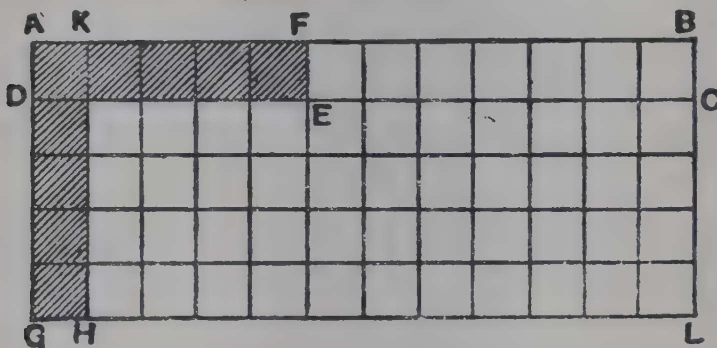


FIG. 136.

of a concrete quantity we may divide the quantity by 29 and multiply by 18 or multiply the quantity by 18 and then divide the result by 29, *i.e.*, 18 times the quantity may be divided by 29. There are thus two interpretations of a fraction ; *e g*., $\frac{5}{12}$ of a foot, may mean that a foot is to be divided into 12 equal parts and then 5 of these are to be taken or that 5 ft. is to be divided into twelve equal parts and one of those parts is to be taken. In other words five-twelfths of a unit = a twelfth of 5 such units. This may be graphically illustrated thus—

Take a rectangle ABCD divided into 12 equal parts. Then 5 of these parts as in ADEF will represent $\frac{5}{12}$ of a unit (*viz.*, the whole rectangle). Now AGLB contains 5 such rectangles, *i.e.*, represents 5 units. AGHK is $\frac{1}{12}$ of AGLB. $\therefore$ AGHK is $\frac{1}{12}$ of 5 units. The rectangle AGHK = the rectangle ADEF for each of them contains 5 squares. $\therefore$ 5 times $\frac{1}{12}$ of a unit = $\frac{1}{12}$ of 5 such units.

Exercise—Graphical.

1. Show graphically that

(*a*) $\frac{2}{5} = \frac{1}{5}$ of 2 or $2 \div 5$.

(*b*) $\frac{5}{6} = \frac{1}{6}$ of 5 or $5 \div 6$.

(*c*) $\frac{7}{8} = \frac{1}{8}$ of 7 or $7 \div 8$.

§ **134. Ratio.** Take two straight lines AB and CD 2 cm. and 10 cm. long respectively. On comparing the lengths of AB and CD we find that CD is 5 times AB. Hence if AB is taken as the unit, CD is 5 times

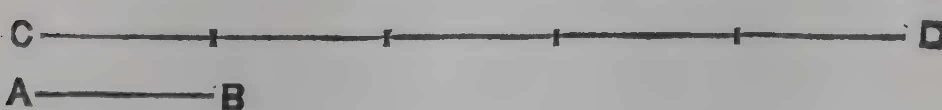


FIG. 137.

the unit; also AB is a fifth part of CD, *i.e.*, if CD is taken as the unit AB is $\frac{1}{5}$. When we compare two lengths or two rectangles or, more generally, two quantities of the same kind, we express that one quantity is so many times the other or is so many parts of the other. The relation that thus exists between any two quantities of the same kind is called their **Ratio**.

The ratio of one quantity A to another B of the same kind may be expressed as a fraction whose numerator is A and denominator is B, *i.e.*, by $\frac{A}{B}$. The ratio of A to B is

usually expressed with two dots placed between A and B: thus:—A : B(read A to B). The first term A is called the antecedent and the second term B the consequent of the ratio. In Fig. (137) the ratio of AB : CD $= \dfrac{2 \text{ cm.}}{10 \text{ cm.}} = \frac{1}{5}$; the ratio of CD : AB $= \dfrac{10 \text{ cm.}}{2 \text{ cm.}} = 5$. The ratio in each case is an abstract number. Symbolically $A : B$, $A \div B$, $\dfrac{A}{B}$ all express the ratio of A to B.

Exercise—Graphical.

1. Draw on $\frac{1}{10}''$ squared paper two rectangles 1·5″ by 1·4″ and 2·5″ by 2·4″. Find the ratio of the former to the latter (by comparing the number of small squares in each.)

2. Draw on squared paper two straight lines AB and CD 3 inches and 4 inches long respectively. E and F are points in AB and CD so that AE = ·6″ and CF = ·8″. Find the ratio of (1) AE to AB (2) CF to CD (3) AE to EB (4) CF to FD (5) AE : CF (6) AE : FD (7) EB : CF (8) EB to CD.

When two quantities A and B are compared, A may be so many times B, when A is greater than B, or A may be a part of B, when A is less than B. Let A be 53 oz. and B, 1 lb. or 16 oz. Then the fraction $\dfrac{A}{B}$ or the ratio $A : B = \frac{53}{16}$.

If A = 3 oz. and B = 1 lb. or 16 oz. then the fraction $\dfrac{A}{B}$ or the ratio $A : B = \frac{3}{16}$. When the numerator of a fraction is greater than its denominator, the fraction is said to be **improper**, *e.g.*, $\frac{53}{16}$ or when the antecedent of a ratio is greater than the consequent, it is said to be **a ratio of greater inequality**, *e.g.*, 53 : 16.

When the numerator is less than the denominator the fraction is said to be **proper**, *e.g.*, $\frac{3}{16}$; or when the antece-

dent of a ratio is less than the consequent, it is said to be a **ratio of less inequality**. *e.g.*, 3 : 16.

Now $\frac{53}{16}$ means that 53 parts are taken each part being one-sixteenth of the unit. 48 such parts make 3 units and the remaining 5 parts make $\frac{5}{16}$ of the unit. Thus $\frac{53}{16}$ equals 3 and $\frac{5}{16}$ (generally written $3\frac{5}{16}$). This $3\frac{5}{16}$ is called **a mixed number** because it consists of a whole number called its integral part and a proper fraction called its fractional part. Every improper fraction can be expressed as a mixed number and *vice versa*.

Example 1.—Express as a mixed number $\frac{57}{16}$.

$\frac{57}{16}=3\frac{9}{16}$ as shown above. Divide by 16, the quotient 3 gives the integral part and the remainder 9 the numerator of the fractiona part while the denominator is unaltered.

Example 2.—Convert $2\frac{3}{10}$ into an improper fraction.

Here the unit is divided into 10 equal parts. ∴ 2 units are equal to 20 such parts. In $\frac{3}{10}$ we have 3 such parts. ∴ we have in $2\frac{3}{10}$ 20+3 or 23 such parts, each part being $\frac{1}{10}$ ∴ $2\frac{3}{10} = \frac{23}{10}$. The rule therefore is to multiply the whole number by the denominator and add the numerator to the product ; the result will be the numerator of the improper fraction and the denominator of the required fraction is the same as the denominator of the fraction in the mixed number.

Exercise XVI (b).

1. Express as mixed numbers the following fractions :—

(*a*) $\frac{28}{3}$. (*b*) $\frac{485}{12}$. (*c*) $\frac{564}{13}$. (*d*) $\frac{684}{13}$. (*e*) $\frac{897}{31}$. (*f*) $\frac{1084}{625}$.

2. A house costs $4\frac{3}{5}$ times as much as its furniture which costs Rs. 1260. What is the cost of the house?

3. How much is $8\frac{7}{12}$ of Rs. 240 less than $120\frac{5}{16}$ of Rs. 160-8-0

4. What must be added to $6\frac{7}{8}$ of 5 can. 9 md. 3 viss to make 40 candies?

5. When 5 pieces each of $4\frac{7}{9}$ yards have been cut off from a mull piece of 40 yds., find what length remains.

6. Evaluate $7\frac{7}{12}$ of Rs. 16-5-4+$8\frac{9}{16}$ of Rs. 176-7- 0— $13\frac{16}{23}$ of Rs. 139-1-3.

§ 135. Constant Ratio. I. Let ABC be an equilateral triangle having each of its sides equal to 1 inch. From A draw AD perpendicular to BC. Measure AD and you will find it to be more than ·86 or about ·87 of an inch.

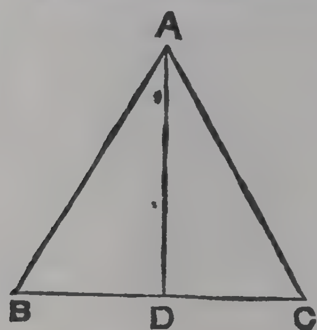


FIG. 138.

∴ the ratio of AD : AB = ·87 : 1 or ·87.

Draw another equilateral triangle XYZ having each of its sides = 2 inches. From X draw XU ⊥ to YZ and find the ratio $\frac{XU}{XY}$. XU = 1·74 nearly and XY = 2 inches.

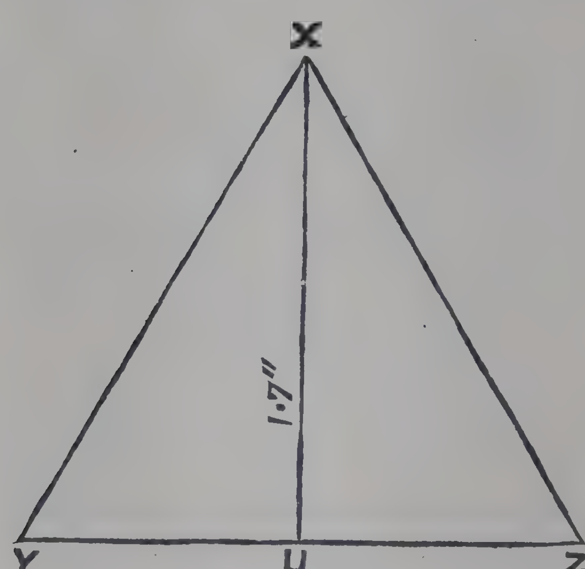


FIG. 139.

$$\therefore \quad \frac{XU}{XY} = \frac{1{\cdot}74}{2} = {\cdot}87.$$

Similarly in every equilateral triangle the ratio of the

altitude to a side will be found to be about ·87 and is therefore constant.

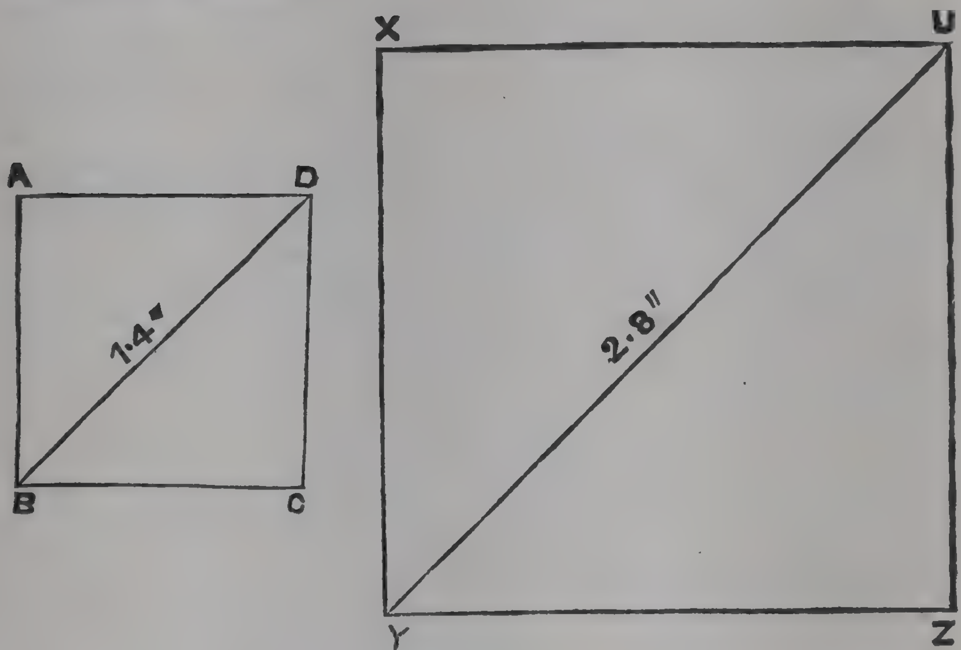


FIG. 140.

II. Construct a square ABCD on a base of 1 inch. Join BD. Measure BD and you will find it to be about 1·4 in.

∴ the ratio of BD : AB = 1·4 : 1 or 1·4.

Construct another square XYZU on a base of 2 inches. Join YU and find the ratio YU : YZ.

YU = 2·8 and YZ = 2 inches.

$$\therefore \quad \frac{YU}{YZ} = \frac{2 \cdot 8}{2} = 1 \cdot 4.$$

Similarly in every square the ratio of the diagonal to a side of the square is about 1·4.

Exercise XVI (c)

1. Find the height of equilateral triangles having bases of—
(*a*) 3·6 in. (*b*) 4·8 cm. (*c*) 7·3 in. (*d*) 9·8 cm.

2. In a right-angled triangle one of the acute angles is 60°. Find the length of the side opposite to this angle if the hypotenuse measures 6·8 cm.

3. The hypotenuse of an isosceles right-angled triangle is 8·4 cm. Find the length of each of the equal sides.

4. Find the length of the diagonal of a square whose side is
(*a*) 20 cm. (*b*) 12·8 in. (*c*) 30 yds. (*d*) 48 yds.

5. A boy draws a square and finds its diagonal to be 20·8 cm. Find the length of a side.

6. Find the shadow of a tree 15 ft. high when the altitude of the sun is (1) 60°, (2) 30°, (3) 45°.

7. Construct a number of right-angled triangles having one acute angle in each equal to 75°. Find by measurement in each triangle the ratio of the side opposite to 75° to the hypotenuse. What do you infer ?

8. A ladder 35 ft. high makes an angle of 60° with the ground and rests against a wall. (1) Find the height of the point it reaches on the wall. (2) Find the same when it slips down and make an angle of 45° with the ground.

§ 136. Division of a straight line in a given ratio.

Draw a straight line AB 6 cm. long. Take C, D, E, F and G along AB each equal to 1 cm. Take H any point outside AB. Join AH and BH. Through the points C, D, E, F, G draw CC′, DD′, EE′, FF′ and GG′ parallel to BH meeting AH in C′, D′, E′, F′ and G′ respectively. Measure with your dividers the parts of the line AH. You will notice that they are all equal. From A draw two other lines to meet BH in K and L. With your dividers measure the parts of AK and AL. You will find that

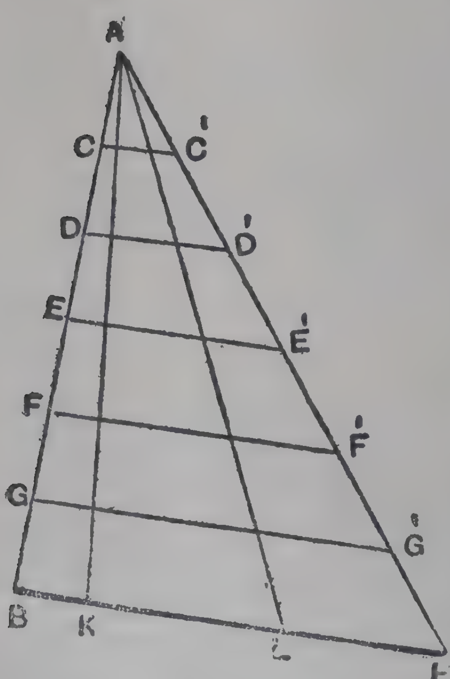


FIG. 141.

AK and AL are each divided into 6 equal parts just like AH. Repeat the experiment by changing the length of AB and by dividing it into 5 or 4 equal parts and drawing parallels.

From these experiments you infer that **if there are three or more parallel straight lines and the intercepts (portions cut off), made by them on any transversal (straight line that cuts them) are equal, then the corresponding intercepts on any other transversal are also equal.**

The above property may be utilized in dividing a straight line, into a number of equal parts or in any given ratio; say, we have to divide a line AB 5 cm. long into 6 equal parts. At A make any angle BAC. Along AC step off with your dividers 6 equal lengths, AD, DE, EF, FG, GH, HK. Join BK. Through H, G, F, E and D draw HH′, GG′, FF′, EE′, DD′, parallel to BK. Now AB is divided into 6 equal parts.

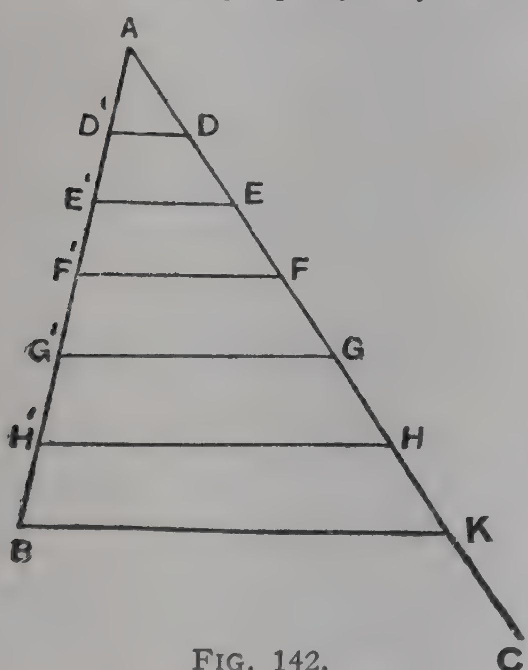


FIG. 142.

Similarly we may divide AB into any number of equal parts.

Note that $AD' = \frac{1}{6}$ AB, $AE' = \frac{2}{6}$ AB, $AF' = \frac{3}{6}$ AB, $AG' = \frac{4}{6}$ AB, $AH' = \frac{5}{6}$ AB.

In the figure the same line AB is divided at D′ in the ratio of 1 : 5; for AD′ contains one part and BD′ contains

5 parts, *i.e.*, AD′ : BD′ equals 1 : 5. Similarly E′ divides the same line in the ratio of 2 : 4, F′ in the ratio of 3 : 3, etc. To divide a line in a given ratio, it is not necessary to draw all the parallels. For example to find the point (*viz.*, D′) in AB which divides it in the ratio of 1 : 5, through D the 1st point in AK draw DD′ parallel to BK. Then AB is divided at D′ in the ratio of 1 : 5.

Exercise—Practical.

1. Divide a straight line 3·4 inches long into three equal parts. Test your work by measuring each part with your dividers.

2. Divide a straight line 6·8 in. long into 5 equal parts.

3. Draw a straight line equal to one inch and divide it into 10 equal parts. See if the divisions correspond to tenths on your scale.

4. Draw a straight line a decimetre long, divide it into 10 equal parts and see if the divisions correspond to the centimetre divisions on your scale.

5. Draw AB a decimetre long. Divide it into 7 equal parts. From your figure find by measurement the value of $\frac{1}{7}$, $\frac{2}{7}$, $\frac{3}{7}$, $\frac{4}{7}$, $\frac{5}{7}$, $\frac{6}{7}$ to two places of decimals.

6. Similarly divide AB a decimetre long into (1) 6 equal parts, (2) 11 equal parts and find the decimal equivalents of the following fractions to two places of decimals :—$\frac{5}{6}$. $\frac{5}{11}$, and $\frac{9}{11}$.

§ 137. One concrete quantity expressed as a vulgar fraction of another. Equivalent fractions.

To find the part 6 as. is of Rs. 10, *i.e.*, to find the ratio of 6 as. to Rs. 10.

(1) We may reduce both to two-anna pieces and then find the ratio.

$$6 \text{ as.} = 3 \text{ (two-anna pieces)}$$
$$\text{and Rs } 10 = 80 \text{ (two-anna pieces)}$$

∴ 6 as. is obtained by dividing Rs. 10 into 80 equal parts and taking 3 of these parts,

i.e., 6 as. = $\frac{3}{80}$ of Rs. 10.

(2) We reduce both quantities to annas.

6 as = 6 as.

Rs. 10 = 160 as.

∴ 6 as. is obtained by dividing Rs. 10 into 160 equal parts and taking 6 of these parts,

i.e., 6 as. is $\frac{6}{160}$ of Rs 10.

(3) We may reduce both to quarter annas and then determine the ratio

6 as. = 24 (quarter annas)

Rs. 10 = 640 (do.)

∴ 6 as. is obtained by dividing Rs. 10 into 640 equal parts and taking 24 of these parts,

i.e, 6 as. = $\frac{24}{640}$ of Rs. 10. The fractions $\frac{3}{80}$, $\frac{6}{160}$, $\frac{24}{640}$ all express what part 6 as. is of Rs. 10, *i.e.*, the ratio of 6 as. to Rs. 10; and therefore the fractions $\frac{3}{60}$, $\frac{6}{160}$, $\frac{24}{640}$ are **equivalent.**

Generally a fraction $\frac{a}{b}$ is equivalent to the fraction $\frac{n \times a}{n \times b}$

In other words the ratio $a : b$ is equivalent to the ratio $na : nb$.

A fraction is unaltered if both the numerator and the denominator are multiplied by the same quantity. In other words a ratio is unaltered if both the antecedent and the consequent are multiplied by the same number.

Again $\frac{24}{640}$, $\frac{6}{160}$ and $\frac{3}{80}$ are all equal, *i.e.*, $\frac{24}{640}$, $\frac{24 \div 4}{640 \div 4}$ and $\frac{24 \div 4 \div 2}{640 \div 4 \div 2}$ are all equal. Generally a fraction $\frac{a}{b}$ is

equivalent to the fraction $\dfrac{a \div n}{b \div n}$; in other words the ratio $a : b$ is equivalent to the ratio $\dfrac{a}{n} : \dfrac{b}{n}$,

i.e., a fraction is unaltered, if both the numerator and the denominator are divided by the same quantity. In other words a ratio is unaltered if both the antecedent and the consequent are divided by the same quantity.

§ 138. Graphical illustrations of Equivalent fractions.

We have seen that $\frac{4}{5}$ is equivalent to $\dfrac{4 \times 3}{5 \times 3}$ or $\dfrac{12}{15}$. This can be illustrated by a figure thus :—

Let ABCD be divided into 5 equal parts, each part being equal to EFCD. Then $\frac{4}{5}$ of ABCD will be represented by ABFE which contains 4 of the 5 equal parts.

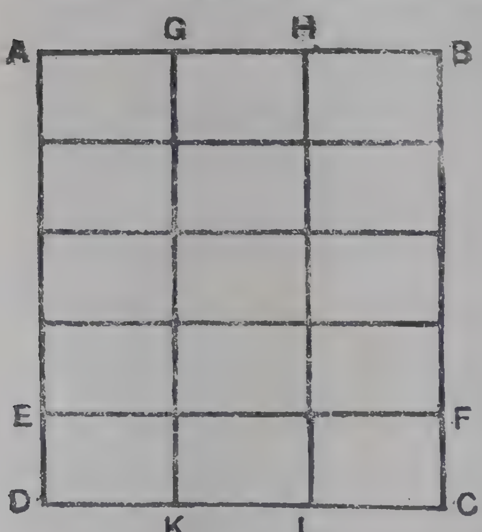


FIG. 143.

Now divide AB into 3 equal parts, *viz.*, AG, GH and HB; and draw lines GK and HL as in Fig. 143. The figure ABCD is now divided into 15 equal parts. Figure AEFB (which $= \frac{4}{5}$ of ABCD) contains 12 of the 15 equal parts into which the whole is divided and $\therefore$ represents $\frac{12}{15}$. $\therefore$ $\frac{4}{5} = \frac{12}{15}$.

The very important proposition that has been just enunciated enables us *to change the form of a vulgar fraction without altering its value.*

To show graphically that $\frac{1}{2} = \frac{3}{6} = \frac{12}{24}$.

Draw a line AB 2·4 cm. long to represent a unit. Divide it into 2 equal parts at C. Then AC $= \frac{1}{2}$ a unit.

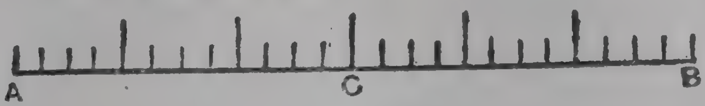


FIG. 144.

Divide each of the 2 equal parts into 3 parts. The whole line AB is then divided into 6 equal parts and AC contains 3 of them and is therefore $\frac{3}{6}$ of the unit. Divide each of these six parts into 4 equal parts. Then AB is divided into 24 equal parts and AC contains 12 of them. ∴ AC $= \frac{12}{24}$ of the unit.

Since each fraction expresses the part AC is of AB the unit, $\frac{1}{2} = \frac{3}{6} = \frac{12}{24}$.

When the numerator and denominator of a fraction do not contain any common factor the fraction is said to be in its **lowest terms**.

§ 139. Equality of fractions and ratios. In the example we have taken in Art. 137 the fractions $\frac{3}{80}$, $\frac{6}{160}$, $\frac{24}{640}$ are all equal; in other words the ratios 3 : 80, 6:160, 24 : 640 are all equal. The equality of the fractions in the above case is symbolically represented thus: $\frac{3}{80}=\frac{6}{160}=\frac{24}{640}$ and generally, if the fractions $\frac{a}{b}$, $\frac{c}{d}$, $\frac{e}{f}$ are all equal the fact is expressed symbolically thus: $\frac{a}{b} = \frac{c}{d} = \frac{e}{f}$ (read 'a by b equals c by d equals e by f.') Similarly the equality of the ratios in the above case is symbolically represented thus:—$3 : 80 = 6 : 160 = 24 : 640$, and if the ratios $a : b$, $c : d$, $e : f$ are all equal, the fact is expressed symbolically thus:—$a : b = c : d = e : f$ (read "a to b equals c to d equals e to f.")

Exercise—Oral.

1. What fraction is 7 cwts. of 1 ton? Find by reducing both—(1) to cwts., (2) to qrs., (3) to lbs. Are these fractions equal? If so what is your reason?

2. Mention the fraction 5 viss is of a candy obtained by reducing both to (1) viss; (2) seers; (3) palams.

3. What is the ratio of 6 miles 3 furlongs to 7 miles 2 furlongs?

4. What fraction is a annas of b rupees?

5. What is the ratio of p square feet to q square yards?

6. Reduce the following fractions to their lowest terms:—

$$\frac{10}{15}, \frac{12}{35}, \frac{24}{40}, \frac{25}{30}.$$

7. Express the following ratios in their simplest form:—

(*a*) 18 : 12; 16 : 20; 24 : 30; 40 : 84.

Symbolical.

Reduce to lowest terms:

1. $\frac{ac}{bc}, \frac{aba}{bcd}, \frac{a^2c}{ac^2}, \frac{b^2c}{bc^2}, \frac{6a^2bc}{7abc^2}.$

2. Express the following ratios in the simplest form:—

$$ma : mb;\ mac : mbc;\ ma^2 : m^2a;\ 6mabc : 7mabd.$$

Exercise—Graphical or Practical.

1. Draw a line 4″ in length and regard it as a unit; divide it into 12 equal parts and mark off a portion to represent the fraction $\frac{4}{12}$; show from your figure that the fractions $\frac{4}{12}, \frac{2}{6}, \frac{1}{3}$ are equivalent.

2. Take a line AB = 12 cm. as the unit and divide it into 16 equal parts, mark off a portion to represent the fraction $\frac{12}{16}$; show from your figure that the fractions $\frac{6}{8}, \frac{3}{4}$ are equivalent.

3. Draw a square of 8 cm. side and consider it as a unit area, mark off portions representing $\frac{3}{4}, \frac{6}{8}$. Show that they are equivalent fractions.

4. Draw a rectangle 10 cm. by 8 cm. and consider it as a unit area. Mark off portions of it to represent $\frac{70}{80}, \frac{35}{40}, \frac{14}{16}, \frac{7}{8}$. Show that these are equivalent.

5. Show graphically by taking suitable squares or rectangles that the following sets of fractions in each case are equivalent :—

(*a*) $\frac{2}{5}$, $\frac{4}{10}$, $\frac{6}{15}$. (*b*) $\frac{5}{12}$, $\frac{10}{24}$, $\frac{15}{36}$.

6. Divide a line AB 6 cm. in length in the ratio of 2 : 3 ; divide it also in the ratio of 4 : 6 and show that these ratios are equal.

Exercise XVI (d).

1. Express in their lowest terms—

(*a*) $\frac{120}{132}$. (*b*) $\frac{144}{156}$. (*c*) $\frac{192}{200}$. (*d*) $\frac{168}{338}$.

2. Express in their simplest form—

(*a*) 168 : 144. (*b*) 135 : 175. (*c*) 184 : 138. (*d*) 182 : 217.

3. Arrange together the equivalent fractions among the following :—$\frac{8}{24}$, $\frac{7}{15}$, $\frac{6}{23}$, $\frac{24}{72}$, $\frac{35}{75}$, $\frac{32}{96}$, $\frac{72}{276}$, $\frac{120}{360}$, $\frac{78}{299}$, $\frac{105}{225}$.

4. Write down fractions equivalent to the following : (1) with smaller numerators (2) with larger denominators :—$\frac{875}{625}$, $\frac{138}{207}$, $\frac{480}{576}$.

5. Write down 3 ratios equivalent to the ratios given in exercise 2—(1) with smaller antecedents, (2) with larger consequents.

6. Assign values to x in the fractions—

(*a*) $\frac{x}{162}$. (*b*) $\frac{60}{x}$. (*c*) $\frac{x}{252}$. (*d*) $\frac{100}{x}$. (*e*) $\frac{140}{x}$. (*f*) $\frac{x}{288}$

so that they may be equivalent to (1) $\frac{1}{2}$ (2) $\frac{2}{3}$ (3) $\frac{5}{6}$ (4) $\frac{4}{9}$.

7. What weight is the same fraction of 15 can. 30 mds. as (*a*) 72 lbs. is of 216 lbs. ; (*b*) Rs. 5-5 is of Rs. 6-6 ; (*c*) a mile is of a furlong.

8. A boy gets in a certain examination 40 marks out of 100 marks. If the maximum is raised to 150 how should his marks be altered so that he may get the same fraction of the maximum as before ?

9. A man sets apart always a certain fraction of his annual income for charity every year. In a certain year his monthly income was Rs. 60 and the annual sum reserved for charity was Rs. 125. What sum was reserved for charity in a year in which his monthly income was Rs. 75 ?

§ 140. Lowest Terms. We have seen how a fraction may be reduced to its lowest terms by dividing its numerator and denominator successively by their common factors. But we may also divide the numerator and denominator by one number which contains all the factors common to both the numerator and the denominator, *i.e.*, by the H. C. F. or G. C. M. of the numerator and the denominator.

Ex. Reduce $\frac{142857}{999999}$ to its lowest terms.

The G. C. M. of 142857 and 999999 is 142857.

$$\therefore \quad \frac{142857}{999999} = \frac{142857 \div 142857}{999999 \div 142857} = \frac{1}{7}.$$

Reduce to lowest terms by finding the H. C. F.

(1) $\frac{2905}{3885}$. (2) $\frac{2553}{8967}$. (3) $\frac{7493}{19749}$. (4) $\frac{428571}{999999}$. (5) $\frac{9375}{15625}$.

Express the following as mixed numbers reducing the fractional parts to lowest terms :—(6) $\frac{12875}{625}$. (7) $\frac{65748}{864}$. (8) $\frac{124784}{99}$. (9) $\frac{1578346}{28792}$.

§ 141. Comparison of fractions. *Method* I.

Ex. Compare the fractions $\frac{2}{6}$, $\frac{3}{6}$, $\frac{5}{6}$.

Here $\frac{2}{6}$ means that the unit is divided into 6 equal parts and out of them 2 parts are taken. In $\frac{3}{6}$ we take 3 such parts. $\therefore$ $\frac{3}{6}$ is greater than $\frac{2}{6}$. Similarly $\frac{5}{6}$ can be seen to be greater than $\frac{3}{6}$ or $\frac{2}{6}$.

Now compare the fractions $\frac{1}{2}$ and $\frac{2}{3}$.

Here the numerators are different from one another and also the denominators. If we reduce them to equivalent fractions having equal denominators we can compare them as in the above. We have seen that in finding equivalent fractions we multiply the numerator and denominator by the same number, *i.e.*, one denominator is a multiple of the other. Here we have to find fractions equivalent to $\frac{1}{2}$ and $\frac{2}{3}$ and having the same denominator. $\therefore$ that denominator must be a multiple of 2 and 3 ; 6 is such a multiple and $\frac{1}{2} = \frac{3}{6}$ and $\frac{2}{3} = \frac{4}{6}$; now $\frac{4}{6} > \frac{3}{6}$ $\therefore$ $\frac{2}{3} > \frac{1}{2}$.

Similarly compare $\frac{2}{3}$, $\frac{3}{5}$, and $\frac{5}{6}$. Here we must as before find a common multiple of 3, 5 and 6 and reduce the given fractions to equivalent ones with that multiple as denominator ; 30 is such a multiple and the corresponding equivalent fractions are $\frac{20}{30}$, $\frac{18}{30}$, $\frac{25}{30}$.

Evidently $\frac{25}{30}$ or $\frac{5}{6}$ is the greatest and $\frac{18}{30}$ or $\frac{3}{5}$ is the least, *i.e.*, $\frac{5}{6} > \frac{2}{3} > \frac{3}{5}$.

Method 2. We can also compare fractions by reducing them to a common numerator. For that we find a common multiple of the numerators. In the example worked above, 2, 3, 5 are the numerators and 30 is a multiple and we have

$$\frac{2}{3} = \frac{2 \times 15}{3 \times 15} = \frac{30}{45}$$

$$\frac{3}{5} = \frac{3 \times 10}{5 \times 10} = \frac{30}{50}$$

$$\frac{5}{6} = \frac{5 \times 6}{6 \times 6} = \frac{30}{36}.$$

Of these the greatest is $\frac{30}{36}$ or $\frac{5}{6}$.

Method 3. By reducing to decimals. For converting vulgar fractions into decimals, *vide* Art. 143.

$$\frac{2}{3} = \cdot 66.., \quad \frac{3}{5} = \cdot 6, \quad \frac{5}{6} = \cdot 83...$$

Evidently ·83...or $\frac{5}{6}$ is the greatest and ·6 or $\frac{3}{5}$ is the least.

Exercise XVI (e).

Reduce the following sets of fractions to fractions having the least common denominator and compare them :—

1. $\frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}$.
2. $\frac{2}{7}, \frac{3}{5}, \frac{4}{9}, \frac{5}{11}$.
3. $\frac{7}{15}, \frac{6}{20}, \frac{8}{30}, \frac{9}{35}$.
4. $\frac{8}{16}, \frac{13}{14}, \frac{18}{24}, \frac{19}{36}$.

Arrange the following fractions in the ascending order of magnitude by working out by each of the three methods shown above :—

5. $\frac{15}{16}, \frac{17}{18}, \frac{20}{24}$.
6. $\frac{120}{143}, \frac{160}{169}, \frac{224}{260}$.
7. $\frac{5}{6}, \frac{8}{11}, \frac{11}{12}, \frac{13}{15}, \frac{15}{16}$.
8. $\frac{13}{14}, \frac{18}{19}, \frac{21}{24}, \frac{28}{30}, \frac{48}{64}$.
9. Arrange the above fractions in the descending order of magnitude.

§ 142. Addition and subtraction of fractions.

Example 1.—Add $\frac{3}{4}$ and $\frac{2}{3}$.

Addition is possible only in the case of things of the same kind. Here three-*fourths* and two-*thirds* are not of the same kind. So we first reduce them to fractions of the same kind, *i.e.*, having the same *denominator* and then add. Thus

$$\frac{3}{4} + \frac{2}{3} = \frac{9}{12} + \frac{8}{12} = \frac{9+8}{12} = \frac{17}{12} = 1\frac{5}{12}.$$

The same may be worked *graphically*. Let ABCD (Fig. 145) represent a unit. It is divided into 4 equal parts by the vertical lines in the figure. AEFD contains 3 such parts and therefore represents $\frac{3}{4}$.

Again ABCD is divided into 3 equal parts by the horizontal lines. AGHB contains 2 of them; and therefore represents $\frac{2}{3}$.

Now, by the vertical and horizontal lines, ABCD has been divided into 12 equal parts. AEFD contains 9 of them, and AGHB, 8 of them. Therefore the sum of AEFD and AGHB (*i.e.*, the sum $\frac{3}{4}$ and $\frac{2}{3}$) contains 17 of the equal parts, and is therefore equal to ABCD + 5 of its 12 parts, *i.e.*, $= 1\frac{5}{12}$, and is represented by the figure RBCPQ.

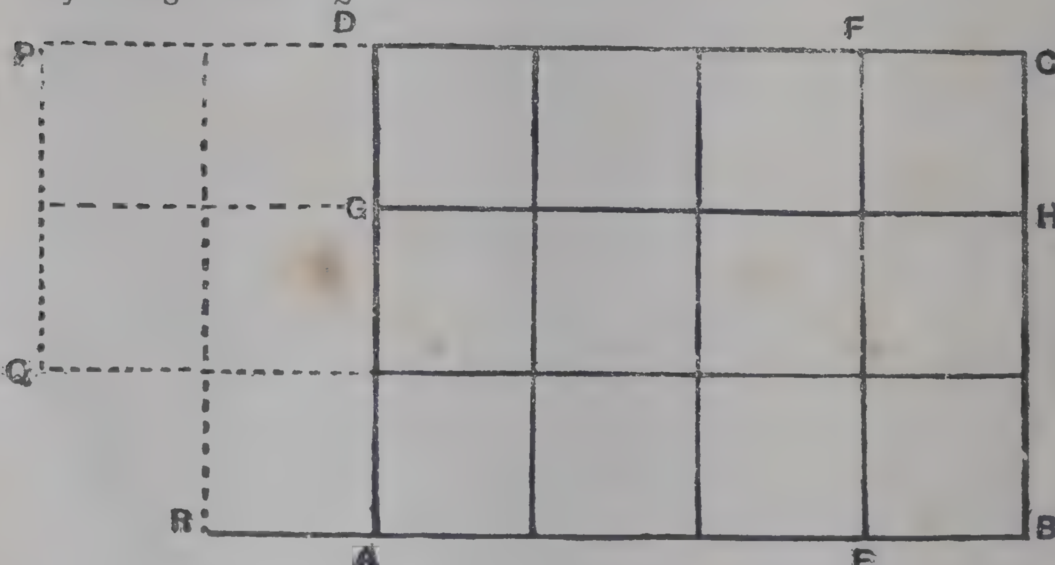


FIG. 145.

Symbolically $\frac{a}{b} + \frac{c}{d} = \frac{ad}{bd} + \frac{bc}{bd} = \frac{ad+bc}{bd}$.

Example 2.—Subtract $\frac{2}{3}$ from $\frac{3}{4}$.

$$\frac{3}{4}-\frac{2}{3}=\frac{9}{12}-\frac{8}{12}=\frac{9-8}{12}=\frac{1}{12}.$$

Graphically. In the figure of previous example, AEFD — AGHB = (9 squares — 8 squares) = one square = $\frac{1}{12}$ of ABCD.

Symbolically $\frac{a}{b}-\frac{c}{d}=\frac{ad}{bd}-\frac{bc}{bd}=\frac{ad-bc}{bd}$.

Exercise.—Oral.

Find the value of

(1) $\frac{1}{2}+\frac{1}{4}$. (2) $\frac{1}{4}+\frac{3}{4}$. (3) $\frac{3}{4}+\frac{1}{3}$. (4) $\frac{2}{3}+\frac{1}{4}$. (5) $\frac{2}{9}+\frac{1}{6}$.
(6) $\frac{1}{4}-\frac{1}{4}$. (7) $\frac{3}{4}-\frac{2}{3}$. (8) $\frac{7}{8}-\frac{1}{2}$, (9) $1-\frac{1}{3}$. (10) $2-\frac{2}{3}$.
(11) $3-\frac{1}{4}$. (12) $9-\frac{13}{10}$. (13) $1\frac{1}{4}+3\frac{3}{4}$. (14) $2\frac{3}{4}+1\frac{1}{2}$.
(15) $3\frac{1}{2}-1\frac{3}{4}$. (16) $7\frac{5}{6}-2\frac{2}{3}$.

Exercise—Graphical.

Represent graphically on squared paper, taking (1) straight lines, (2) rectangles, the following additions and subtractions :—

(1) $\frac{1}{2}+\frac{1}{4}$. (2) $\frac{5}{6}+\frac{7}{8}$. (3) $\frac{9}{10}+\frac{11}{12}$. (4) $\frac{31}{16}+\frac{5}{12}$. (5) $\frac{11}{12}-\frac{3}{5}$.

Find graphically the value of

(6) $2\frac{7}{12}+3\frac{3}{4}-1\frac{1}{8}$. (7) $3\frac{1}{8}+4\frac{1}{6}-3\frac{7}{8}$.

Exercise XVI (f).

Evaluate and test the correctness of your answer by dealing with them as compound quantities :—

1. Re. $\frac{1}{3}$ + Re. $\frac{1}{5}$ + Re. $\frac{1}{6}$.
2. Re. $\frac{1}{4}$ + Re $\frac{1}{6}$ + Re. $\frac{1}{7}$.
3. £$\frac{3}{5}$ + £$\frac{1}{8}$ + £$\frac{2}{3}$.
4. Re. $\frac{8}{9}$ — Re. $\frac{6}{13}$.

Find the value of

5. $\frac{11}{24}-\frac{3}{20}$.
6. $\frac{3}{7}+\frac{5}{8}+\frac{6}{13}+\frac{7}{18}$.
7. $\frac{11}{12}+\frac{14}{15}+\frac{9}{20}+\frac{11}{32}$.
8. $\frac{27}{70}+\frac{34}{40}+\frac{89}{36}+\frac{21}{32}$.
9. $\frac{101}{156}-\frac{87}{143}$.
10. $\frac{81}{144}-\frac{79}{150}$.
11. $\frac{8}{3}+\frac{6}{5}-\frac{7}{9}+\frac{1}{3}$.
12. $\frac{2}{3}+\frac{5}{6}+\frac{7}{8}-\frac{9}{10}-\frac{11}{14}$.

In the addition and subtraction of mixed numbers, in the following examples, take the whole numbers by themselves and the fractions by themselves :—

13. $2\frac{1}{4}+1\frac{3}{4}+3\frac{7}{8}$.
14. $8\frac{9}{16}+16\frac{2}{15}+32\frac{4}{7}+68\frac{[illegible]}{9}$.
15. $19\frac{1}{2}-7\frac{1}{4}$.
16. $107\frac{3}{4}-91\frac{7}{8}$.

(Hint : In 16, $\frac{7}{8}$ is greater than $\frac{3}{4}$. $\therefore$ subtract it from $1\frac{3}{4}$ taking 1 from 107).

17. $303\frac{5}{16} - 178\frac{15}{17}$. **18.** $248\frac{151}{192} - 203\frac{199}{200}$.

Find the value of

19. $\frac{18}{19} \div \frac{21}{32} - \frac{23}{25} + 1\frac{5}{6}$. **20.** $8\frac{1}{2} + 9\frac{1}{3} - 6\frac{10}{17} - 3\frac{15}{19}$.

21. $18\frac{3}{4} + 6\frac{25}{42} + 9\frac{46}{68} - 12\frac{31}{42}$.

22. What must be added to (*a*) Rs. $23\frac{15}{16}$ to make Rs. $30\frac{5}{6}$. (*b*) £$15\frac{17}{40}$ to make £$120\frac{5}{21}$?

23. Find the difference between the greatest and the least of the fractions in :—

(*a*) $\frac{2}{3}, \frac{15}{19}, \frac{16}{17}$ and $\frac{41}{56}$. (*b*) $\frac{5}{9}, \frac{16}{21}$; $\frac{18}{24}$ and $\frac{41}{48}$.

24. What fraction added to the sum of $\frac{11}{12}, \frac{5}{14}, \frac{6}{19}$ and $\frac{31}{32}$ will [make 3?

25. Find the value of

(*a*) Rs. $5\frac{5}{6}$ + Rs. $4\frac{3}{8}$ — Re. $1\frac{7}{12}$.

(*b*) $13\frac{5}{8}$ tons — $8\frac{15}{16}$ cwts. + $27\frac{63}{64}$ lbs.

(*c*) $15\frac{5}{8}$ gm. + $16\frac{1}{2}$ kg. — $90\frac{7}{8}$ cg.

26. A owns $\frac{1}{5}$ of a ship, B $\frac{1}{16}$ and C $\frac{1}{32}$ of it. The rest is owned by D. What fraction of the ship belongs to D.

27. A pipe fills half a cistern in an hour, a second pipe fills $\frac{1}{3}$ of the cistern in an hour, and a third empties $\frac{1}{4}$ of it in an hour. If all the three are open at the same time, what part of it will be full in an hour?

28. One-third of the number of coins in a purse are rupees, $\frac{1}{4}$ half-rupees and the rest quarter-rupees. What ratio does the number of quarter-rupees bear to the total number of coins?

29. $\frac{1}{4}$ of the number of books in a library are English, $\frac{1}{5}$ Mathematics, $\frac{1}{6}$ History and Geography and the rest miscellaneous. What fraction of the whole Library is the collection of miscellaneous books? What must be the least number of books in the library?

30. If in the last question the number of miscellaneous books in the Library is 460, find the total number of books in the Library.

Exercise—Symbolical.

Simplify (1) $\frac{p}{q} + \frac{r}{s}$. (2) $\frac{a}{b} + \frac{c}{d} + \frac{e}{f}$. (3) $\frac{p}{q} - \frac{r}{s}$.

(4) $\frac{a}{b} + \frac{p}{q} - \frac{c}{d}$. (5) $\frac{a}{b} - \frac{p}{q} - \frac{c}{d}$ (6) $\frac{x}{y} + \frac{a}{b} - \frac{p}{q}$.

§ 143. Conversion of vulgar fractions into decimals.

Ex. 1. To convert $\frac{2}{5}$ into decimals.

$\frac{2}{5}$ means (Art. 133) one-fifth of two units and is therefore equal to $\frac{1}{5}$ of 20 tenths, *i.e.*, 4 tenths, *i.e.*, ·4.

Ex. 2. Convert $\frac{11}{16}$ into a decimal.

$$\begin{aligned}
\tfrac{11}{16} &= \tfrac{1}{16} \text{ of 11 units.} \\
&= \tfrac{1}{16} \text{ of 110 tenths} \\
&= 6 \text{ tenths and } \tfrac{1}{16} \text{ of 14 tenths } (\because 110 = 6 \times 16 + 14). \\
&= {\cdot}6 + \tfrac{1}{16} \text{ of 140 hundredths} \\
&= {\cdot}6 + 8 \text{ hundredths} + \tfrac{1}{16} \text{ of 12 hundredths} \\
&= {\cdot}6 + {\cdot}08 + \tfrac{1}{16} \text{ of 120 thousandths} \\
&= {\cdot}68 + 7 \text{ thousands} + \tfrac{1}{16} \text{ of 8 thousandths} \\
&= {\cdot}68 + {\cdot}007 + \tfrac{1}{16} \text{ of 80 ten-thousandths} \\
&= {\cdot}687 + 5 \text{ ten-thousandths} \\
&= {\cdot}6875.
\end{aligned}$$

The same is shown in a shortened from on the right.

$$\begin{array}{r|l} 16 & 11{\cdot}0000\ldots\ldots \\ \hline & 0{\cdot}6875\ldots\ldots \end{array}$$

From the two examples worked out, it will be seen tha to convert a vulgar fraction to decimals, we have only to divide the numerator by the denominator by the ordinary methods used in decimals. The meaning of a fraction also tells us that the numerator is to be divided by the denominator.

Ex. 3. Convert $\frac{1}{7}$ into a decimal.

$$\begin{array}{r|l} 7 & 1{\cdot}00000\ldots\ldots \\ \hline & {\cdot}142857142857\ldots\ldots \end{array}$$

The figures 142857 in the quotient recur any number of times and so the quotient is written $\cdot\dot{1}4285\dot{7}$ to show that in this decimal all the figures commencing from 1 and ending with 7 recur.

When the denominator contains any power of 10 as a factor we have merely to remove the decimal point to the left to as many places as there are ciphers in the divisor in accordance with the rules of division of decimals by powers of ten.

Ex. 4. Express $\dfrac{14891}{1000000}$ as a decimal.

Remove the decimal point 6 places to the left. The required decimal is ·014891.

Ex. 5. Convert $\dfrac{101}{900}$ into a vulgar fraction.

First divide 101 by 100 and then the result by 9.

$$101 \div 100 = 1·01 \text{ and } 1·01 \div 9 = ·1122222... = ·11\dot{2}.$$

NOTE.—A mixed number is that which contains an integer and a fraction, *e.g.*, $3\frac{1}{8}$. In converting such a number into a decimal, the integer may be retained as it is and the fraction alone need be converted into a decimal. Thus $3\frac{1}{8} = 3·125$.

Exercise XVI (g).

Convert the following vulgar fractions into decimals :—

1. $\frac{5}{6}$. **2.** $\frac{7}{8}$. **3.** $\frac{88}{90}$. **4.** $\frac{70}{50}$.

5. $\frac{39}{70}$. **6.** $\frac{19}{400}$. **7.** $\frac{18}{16}$. **8.** $\frac{19}{64}$.

9. $\frac{1}{1431}$. **10.** $\frac{8}{1426}$. **11.** $\frac{9}{17325}$. **12.** $\frac{3}{12000}$.

Convert each of the following to 3 places of decimals :—

13. $3\frac{10}{221}$. **14.** $8\frac{19}{385}$ **15.** $10\frac{11}{468}$.

Convert each of the following to 4 places of decimals :—

16. $\frac{5}{11}$. **17.** $\frac{16}{17}$. **18.** $\frac{8}{18}$.

§ 144. Conversion of decimals into vulgar fractions.

Convert into a vulgar fraction :—(*a*) ·53 (*b*) ·5378.

Method I. (*a*) ·53 means 5 tenths + 3 hundredths

$$\text{or } \tfrac{5}{10} + \tfrac{3}{100} = \frac{50+3}{100} = \tfrac{53}{100}$$

(*b*) $·5378 = \tfrac{5}{10} + \tfrac{3}{100} + \tfrac{7}{1000} + \tfrac{8}{10000}$

$$= \frac{5000 + 300 + 70 + 8}{10000}$$

$$= \tfrac{5378}{10000}.$$

Method II. (*a*) $\cdot 53$ means 53 hundredths or $\frac{53}{100}$ (at once) $\cdot 5378 = 5378$ ten-thousandths $= \frac{5378}{10000}$.

Method III. (*a*) $\cdot 53 \times 100 = 53 \therefore \cdot 53 = \frac{53}{100}$.

(*b*) $\cdot 5378 \times 10000 = 5378 \therefore \cdot 5378 = \frac{5378}{10000}$.

We see that in each case the equivalent vulgar fraction has for its numerator the number consisting of the figures in the decimal (in the same order) and for its denominator unity followed by as many ciphers as there are figures in the decimal or the power of 10 corresponding to the number of decimal figures. Then the fraction should be reduced to its lowest terms.

The student will do well to remember the fractional equivalents of the following decimals :—

	$\cdot 125 = \frac{1}{8}$
$\cdot 5 = \frac{1}{2}$	$\cdot 375 = \frac{3}{8}$
$\cdot 25 = \frac{1}{4}$	$\cdot 625 = \frac{5}{8}$
$\cdot 75 = \frac{3}{4}$	$\cdot 875 = \frac{7}{8}$

Exercise—Oral.

1. Express as vulgar fractions the following decimals :—

(*a*) $\cdot 3$. (*b*) $\cdot 7$. (*c*) $\cdot 25$. (*d*) $\cdot 325$. (*e*) $\cdot 007$, (*f*) $\cdot 06$.
(*g*) $43 \cdot 1$ (*h*) $4 \cdot 31$. (*k*) $2 \cdot 07$. (*l*) $23 \cdot 75$.

2. Convert the following decimals into equivalent vulgar fractions in their lowest terms :—

(*a*) $\cdot 4$. (*b*) $\cdot 8$. (*c*) $\cdot 24$. (*d*) $\cdot 75$. (*e*) $\cdot 125$. (*f*) $\cdot 875$.
(*g*) $3 \cdot 75$. (*h*) $\cdot 72$. (*k*) $2 \cdot 08$. (*l*) $4 \cdot 105$.

3. Express the following as vulgar fractions in their lowest terms :—

(1) $\cdot 56$. (2) $\cdot 875$. (3) $\cdot 4759$. (4) $6 \cdot 4278$. (5) $\cdot 078395$.

§ 145. Multiplication of a vulgar fraction by an integer.

4 times $\frac{1}{4}$ anna = 1 anna ; 4 times $\frac{1}{2}$ anna = 2 annas.
i e., $\frac{1}{4} \times 4 = 1$ and $\frac{1}{2} \times 4 = 2$.

Example 1.—Multiply $\frac{3}{5}$ by 3.

To multiply $\frac{3}{5}$ by 3 means that $\frac{3}{5}$ must be repeated 3 times and added.

$$\therefore \quad \frac{3}{5} \times 3 = \frac{3}{5} + \frac{3}{5} + \frac{3}{5} = \frac{3+3+3}{5} = \frac{3 \times 3}{5} = \frac{9}{5} = 1\frac{4}{5}.$$

We thus see that multiplying a fraction by an integer the numerator must be multiplied by the integer.

If $\frac{a}{b}$ denotes a fraction and c an integer. $\frac{a}{b} \times c = \frac{a \times c}{b}$ $= \frac{ac}{b}$.

Prove graphically that $\frac{3}{5} \times 3 = \frac{9}{5}$.

Let ABCD (Fig. 146) represent a unit. Then AEFD represents 3 units. Let AEFD be divided into 15 equal parts by 3 vertical and 5 horizontal lines. The unit, *viz.*, ABCD, contains 5 such parts. And 3 such parts (or the portion shaded in the figure) therefore represent $\frac{3}{5}$ of a unit. So $\frac{3}{5} \times 3$ will mean the sum of 3 portions each equal to the shaded portion, *i.e.*, it equals AEKL which contains 9 parts, *i.e.*, it = ABCD + 4 parts = $1\frac{4}{5}$ (for each part = $\frac{1}{5}$ ABCD).

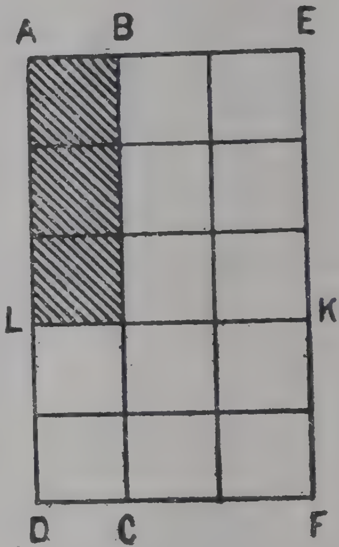


FIG. 146.

Example 2. Multiply $\frac{7}{20}$ by 5 : $\frac{7}{20} \times 5 = \frac{7 \times 5}{20} = \frac{7}{4}$ (since the numerator and the denominator of the fraction can be divided by 5 without altering its value.)

Note : $\frac{a}{bc} \times c = \frac{a}{b}$. Thus the multiplication of a fraction by an integer may be effected by dividing the denominator and leaving the numerator unchanged.

Exercise—Oral.

1. Multiply $\frac{5}{12}$ by 3, 4, 5, 8 ; $\frac{9}{16}$ by 4, 8, 12, 14, 16, 18.

2. Find the value of $\frac{1}{4} \times 130$, $\frac{7}{12} \times 38$.

3. Find the value of $5\frac{2}{5} \times 3$, $6\frac{1}{3} \times 4$, $8\frac{1}{8} \times 6$. (Multiply the integers and fractions separately and add the results.)

Exercise—Graphical or Practical.

1. Take 1 inch as the unit of length. Draw a line 3 inches long, divide it so as to show $\frac{3}{4} \times 3 = 2\frac{1}{4}$.

2. In the same way draw lines of suitable lengths to prove that
(*a*) $\frac{7}{8} \times 2 = 1\frac{6}{8}$. (*b*) $\frac{5}{6} \times 3 = 2\frac{3}{6}$.

3. Find graphically (using rectangles) the result of (*a*) $\frac{1}{6} \times 4$, (*b*) $\frac{8}{16} \times 6$, (*c*) $\frac{11}{30} \times 5$.

4. Find graphically the values of the following :—
(*a*) $\frac{1}{2} \times 7$, (*b*) $8 \times \frac{3}{4}$, (*c*) $4 \times \frac{3}{5}$, (*d*) $\frac{5}{12} \times 9$.

5. Draw lines on squared paper and show that
(*a*) $3\frac{7}{10} \times 3 = 11\frac{1}{10}$; (*b*) $1\frac{7}{8} \times 5 = 9\frac{3}{8}$.

The student is recommended to construct and commit to memory multiplication tables for each of the fractions $\frac{1}{2}$, $\frac{1}{4}$, $\frac{3}{4}$, $\frac{1}{8}$, $\frac{1}{16}$ as shown below :

$$\left.\begin{array}{l} 1 \times \frac{1}{2} = \frac{1}{2} \\ 2 \times \frac{1}{2} = 1 \\ 3 \times \frac{1}{2} = 1\frac{1}{2} \\ \cdots\cdots \;\; \cdots\cdots\cdots\cdots \\ 16 \times \frac{1}{2} = 8 \end{array}\right\} \text{from 1 to 16.}$$

§ 146. Division of a vulgar fraction by an integer.

Divide $\frac{1}{4}$ viss of coffee among 5 persons Now $\frac{1}{4}$ viss = 10 plms. ∴ each person gets 2 plms, or $\frac{1}{20}$ of a viss ∴ $\frac{1}{4}$ viss $\div$ 5 = $\frac{1}{20}$ viss. ∴ $\frac{1}{4} \div 5 = \dfrac{1}{4 \times 5}$.

Graphical. Divide $\frac{3}{5}$ by 4.

Let ABCD (Fig. 147) represent a unit. It is divided into 5 equal strips by vertical lines, each strip being thus a fifth of ABCD. AEFD contains 3 such strips and represents $\frac{3}{5}$ of ABCD. Now let the figure be divided into 4 equal parts by horizontal lines. AEFD is also thus divided into 4 equal parts, a *fourth* of it being AGHE. So, AGHE represents the result of $\frac{3}{5} \div 4$.

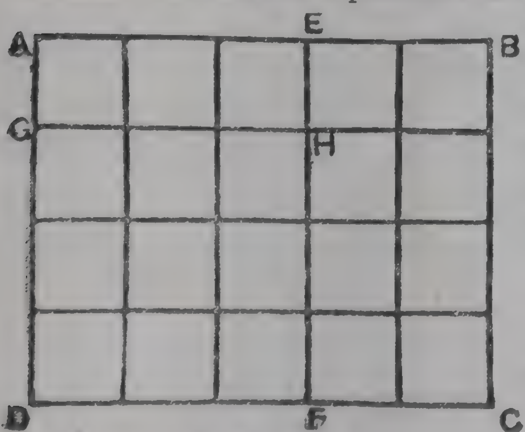


FIG. 147.

Again, the whole figure is divided into 20 equal parts; each being *one-twentieth* of ABCD. AGHE contains 3 such parts and is therefore 3 twentieths or $\frac{3}{20}$ of ABCD.

$$\therefore \frac{3}{5} \div 4 = \frac{3}{20}.$$

Thus $\frac{2}{5} \div 4 = \frac{3}{20}$, *i.e.*, $\dfrac{3}{5 \times 4}$.

Thus in dividing a fraction by an integer, you simply multiply the denominator by the integer.

Symbolically $\frac{a}{b} \div c = \frac{a}{b \times c} = \frac{a}{bc}$.

Exercise—Oral.

What is the value of

1. Re. $\frac{1}{4} \div 4$. **2.** £$\frac{1}{2} \div 8$. **3.** Re. $\frac{9}{8} \div 5$.

4. $3\frac{1}{4}$ cm. $\div 5$. **5.** $4\frac{1}{12} \div 7$. **6.** $12\frac{1}{2} \div 5$.

7. Find the value of (*a*) $4\frac{1}{2}$ annas $\div \frac{1}{2}$ anna. (*b*) $1\frac{1}{4}$ in. $\div \frac{1}{4}$ in., (*c*) $8\frac{1}{2}$ feet $\div 4\frac{1}{4}$ in., (*d*) $10 \div \frac{5}{2}$, (*e*) $12\frac{1}{4} \div 1\frac{3}{4}$, (*f*) $16\frac{1}{2} \div 4\frac{1}{8}$.

Exercise—Practical.

1. Show by paper folding that

(*a*) $\frac{1}{4} \div 4 = \frac{1}{16}$. (*b*) $\frac{1}{5} \div 4 = \frac{1}{20}$. (*c*) $\frac{3}{4} \div 9 = \frac{1}{12}$.

2. Draw convenient lines on squared paper to find the results of dividing.

(*a*) $3\frac{1}{2}$ by 7. (*b*) $3\frac{1}{8}$ by 5. (*c*) $4\frac{2}{7}$ by 6.

3. Find graphically the quotient (1) when $3\frac{3}{4}$ is divided by 3, (2) when $6\frac{2}{5}$ is divided by 8.

4. Draw the following rectangles on squared paper and find the result of division in each case :—

(*a*) 4 cm. by 3 cm.; $\frac{5}{6} \div 2$. (*b*) 8 cm. by 4 cm.; $3\frac{1}{2} \div 6$.

5. Draw convenient rectangles to show that (1) $\frac{3}{5} \div 3 = \frac{1}{5}$, (2) $\frac{12}{10} \div 4 = \frac{3}{10}$, (3) $\frac{5}{6} \div 3 = \frac{5}{18}$.

§ 147. Multiplication by a fraction.

According to the definition of multiplication given in Art. (49) multiplication becomes meaningless unless the multiplier be a whole number. By the following course of reasoning, we can however assign a meaning to multiplication by a fraction :—

E.g., $\frac{2}{3} \times \frac{4}{5}$. Here if we multiply by 4 we get $\frac{8}{3}$; but in this multiplication we have used a multiplier, *viz* 4 which is 5 times as great as it ought to be. ∴ to get the correct

result we must divide this result by 5. Thus $\frac{2}{3} \times \frac{4}{5}$

$$= \frac{2 \times 4}{3} \div 5 = \frac{8}{15}.$$

Or we may adopt a course of reasoning similar to the one adopted in the multiplication by a decimal (*vide* Art. 74).

Or thus :—We have seen in Art. 62 that in a series of multiplications and divisions the order in which the operations are conducted does not alter the final result. Also $\frac{2}{3}$ means $2 \div 3$ and $\frac{4}{5}$ means $4 \div 5$; assuming the above law even in cases where the division is not exact, we have $\frac{2}{3} \times \frac{4}{5} = (2 \div 3) \times (4 \div 5)$

$$= 2 \times 4 \div 3 \div 5 \text{ by the above law}$$
$$= (2 \times 4) \div (3 \times 5)$$
$$= \frac{8}{15} \quad \{ \because a \div b \div c = a \div (bc) \text{ page } 139. \}$$

Rule :—To multiply one fraction by another, take the product of the numerators for the numerator and the product of the denominators for the denominator.

Exercise—Oral.

Find the product of :—

1. $\frac{1}{2} \times \frac{1}{3}$. **2.** $\frac{3}{4} \times \frac{5}{6}$. **3.** $\frac{1}{8} \times 2\frac{3}{8}$. **4.** $8\frac{1}{3} \times 5$.

5. $2\frac{1}{4} \times 1\frac{1}{2}$ **6.** $3\frac{1}{5} \times 4\frac{1}{3}$.

Reduce all mixed numbers to improper fractions and then multiply; or the multiplication may be better effected thus: $2\frac{1}{4} \times 1\frac{1}{2} = (2 + \frac{1}{4})(1 + \frac{1}{2}) = 2 \times 1 + 2 \times \frac{1}{2} + \frac{1}{4} \times 1 + \frac{1}{4} \times \frac{1}{2} = 2 + 1 + \frac{1}{4} + \frac{1}{8} = 3\frac{3}{8}$.

The student is recommended to construct and commit to memory tables of multiplying one fraction by another. Thus

$\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$	$\frac{1}{4} \times \frac{1}{8} = \frac{1}{32}$
$\frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$	$\frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$
$\frac{1}{2} \times \frac{1}{8} = \frac{1}{16}$	$\frac{1}{4} \times \frac{3}{4} = \frac{3}{16}$
$\frac{1}{2} \times \frac{3}{4} = \frac{3}{8}$	$\frac{1}{8} \times \frac{3}{4} = \frac{3}{32}$
	$\frac{3}{4} \times \frac{3}{4} = \frac{9}{16}$

These tables and those recomended in Art. (145) are very useful in finding mentally the product of mixed numbers. Thus the product of $5\frac{1}{2} \times 3\frac{3}{4}$ is mentally found thus

5×3 ; 15 ; $5 \times \frac{3}{4}$; $3\frac{3}{4}$ (together) $18\frac{3}{4}$
$3 \times \frac{1}{2}$; $1\frac{1}{2}$, (altogether) $20\frac{1}{4}$
$\frac{3}{4} \times \frac{1}{2}$; $\frac{3}{8}$, (answer) $20\frac{5}{8}$.

Exercise—Oral.

1. Find the value of

(*a*) $2\frac{3}{4} \times 1\frac{1}{2}$. (*b*) $3\frac{1}{2} \times 2\frac{1}{4}$. (*c*) $5\frac{1}{4} \times 4\frac{3}{4}$.

2. If rice sells at $4\frac{1}{4}$ measures per rupee, what quantity do you get for $3\frac{1}{3}$ Rs.?

3. Ghee sells at Re. $1\frac{3}{4}$ a viss. What is the price of $2\frac{1}{2}$ viss of ghee?

Exercise—Graphical.

1. Take a line 4 in. long and divide it into 16 equal parts and show that $\frac{1}{4} \times \frac{3}{4} = \frac{3}{16}$.

2. Similarly by taking a line = 7 cm. show that $\frac{3}{5} \times \frac{5}{7} = \frac{3}{7}$ and by taking a line 6 inches long show that $\frac{5}{8} \times \frac{5}{6} = \frac{25}{48}$.

3. Draw on squared paper a rectangle

(*a*) 2 in. by 1 in. and show that $\frac{1}{5} \times \frac{3}{4} = \frac{3}{20}$.

(*b*) 4 cm. by 3 cm. and show that $\frac{5}{6} \times \frac{7}{8} = \frac{35}{48}$.

4. Draw rectangles on squared paper to show that

$1\frac{1}{2} \times 1\frac{1}{2} = 2\frac{1}{4}$; $2\frac{1}{2} + 2\frac{1}{4} = 5\frac{5}{8}$.

Exercise XVI (h).

Multiply

1. $\frac{15}{22}$ by $\frac{5}{8}$; $\frac{8}{7}$; $\frac{11}{13}$. **2.** $\frac{16}{35}$ by $2\frac{1}{2}$; $3\frac{1}{4}$; $16\frac{8}{9}$

3. $\frac{64}{100}$ by $8\frac{2}{3}$; $8\frac{1}{13}$; $7\frac{6}{7}$. **4.** $74\frac{2}{9}$ by $8\frac{1}{3}$; $16\frac{1}{2}$; $15\frac{41}{100}$.

Multiply together

5. $10\frac{2}{3} \times 5\frac{1}{8} \times 6\frac{1}{4}$. **6.** $4\frac{1}{3} \times 9\frac{5}{6} \times 101\frac{3}{5}$.

§ 148. Fraction of a fraction.

What is $\frac{2}{3}$ of $\frac{4}{5}$?

$\frac{2}{3}$ of a thing means that the thing is divided into 3 equal parts out of which 2 parts are taken. Similarly $\frac{2}{3}$ of $\frac{4}{5}$ means that $\frac{4}{5}$ is

divided into 3 parts and 2 of such parts are taken ; $\frac{4}{5} \div 3 = \frac{4}{15}$ and two such parts give $\frac{4}{15} \times 2$, *i.e.*, $\frac{8}{15}$. Thus $\frac{2}{3}$ of $\frac{4}{5} = \frac{2 \times 4}{3 \times 5}$.

We have already seen that $\frac{2}{3} \times \frac{4}{5} = \frac{2 \times 4}{3 \times 5}$.

$\therefore$ $\frac{2}{3}$ of $\frac{4}{5}$ is equal to $\frac{2}{3} \times \frac{4}{5}$.

$\therefore$ the word '*of*' is often used instead of the sign $\times$.

Exercise XVI (i)

Find the value of

1. $\frac{8}{9}$ of $7\frac{1}{5}$, 2. $\frac{1}{2}$ of $\frac{1}{3}$ of $8\frac{3}{5}$.

Multiply

3. $\frac{2}{3}$ of $\frac{3}{4}$ of 110 by $10\frac{1}{4}$. 4. $3\frac{5}{8}$ of $4\frac{5}{6}$ by $8\frac{2}{3}$ of $3\frac{1}{8}$.

5. $9\frac{5}{6}$ of $4\frac{10}{11}$ by $6\frac{2}{5}$ of $1\frac{4}{17}$.

Find the value of

6. $\frac{2}{3}$ of Rs. 10-5-4 + $\frac{5}{6}$ of Re. 1-2-6 − $\frac{7}{8}$ of Rs. 3-8-8.

7. $\frac{1}{4}$ of a guinea + £$\frac{1}{6}$ − $\frac{2}{5}$ of a crown + 15 of £ 1 5*s*.

8. $\frac{3}{7}$ of 4*s*. 1*d*. + $\frac{5}{8}$ of £1 8*s*. − $\frac{10}{11}$ of £ 10 0*s*. 9*d*.

9. $\frac{3}{5}$ of 16 mds. − 2 viss + $6\frac{2}{3}$ of 2 viss 13 pal. − $5\frac{1}{20}$ of $6\frac{1}{2}$ tolas + $\frac{1}{8}$ candy.

10. $\frac{1}{5}$ of $\frac{18}{15}$ ton + $\frac{2}{9}$ of $\frac{3}{8}$ qr. + $2\frac{4}{9}$ of $\frac{3}{8}$ lb.

11. 85 of Rs. 3-5-4 + 275 of Rs. 6-8-9 − ·125 of Rs. 756.

12. 3·85 tons + 4·65 cwts. − 3·295 qrs.

§ 149. Division by a fraction.

Example 2.—Divide $\frac{3}{4}$ by $\frac{6}{7}$.

According to our definition of division, this means how many times $\frac{6}{7}$ is contained in $\frac{3}{4}$, *i.e.*, what multiple is $\frac{3}{4}$ of $\frac{6}{7}$, *i.e.*, $\frac{3 \times 7}{4 \times 7}$ of $\frac{6 \times 4}{7 \times 4}$, *i.e.*, $\frac{21}{28}$ of $\frac{24}{28}$. If $\frac{24}{28}$ is divided into 24 parts each is $\frac{1}{28}$ and if 21 such parts are taken we have $\frac{21}{28}$. Thus $\frac{21}{28}$ is $\frac{21}{24}$ of $\frac{24}{28}$, *i.e.*, $\frac{3}{4}$ is $\frac{21}{24}$ of $\frac{6}{7}$, *i.e.*, $\frac{3}{4} \div \frac{6}{7} = \frac{21}{24}$. The process may be shown thus :

$$\frac{3}{4} \div \frac{6}{7} = \frac{3 \times 7}{4 \times 7} \div \frac{6 \times 4}{7 \times 4} = 3 \times 7 \div 6 \times 4 = \frac{21}{24}.$$

Or thus :— You have seen, in the chapter on multiplication and division (*vide* Exercise 3, page 140), that

$$a \div (b \div c) = a \times c \div b.$$

If this is extended to cases where the division is not exact we have $\frac{3}{4} \div \frac{6}{7} = \frac{3}{4} \div (6 \div 7) = \frac{3}{4} \times 7 \div 6 = \frac{3}{4} \times \frac{7}{6} = \frac{21}{24}$.

Or thus :—As division is the inverse of multiplication, the question means. ‘By what should $\frac{6}{7}$ be multiplied in order that the product may be $\frac{3}{4}$? ’

$$\frac{6}{7} = \frac{6\times4}{7\times4} = \frac{24}{28} \text{ and } \frac{3}{4} = \frac{3\times7}{4\times7} = \frac{21}{28}$$

and as before we see that $\frac{24}{28} \times \frac{21}{24} = \frac{21}{28}$. $\therefore$ $\frac{21}{24}$ is the result of divided $\frac{3}{4}$ by $\frac{6}{7}$.

Generally any fraction $\frac{a}{b}$ may be divided by another fraction $\frac{c}{d}$.

Thus $\frac{a}{b} \div \frac{c}{d} = \frac{a\times d}{b\times d} \div \frac{c\times b}{d\times b} = a \times d \div c \times b = \frac{ad}{bc}$. Hence the rule :—

To divide a quantity by a fraction interchange the numerator and the denominator of the fraction and then multiply.

Example 2.—Divide $\frac{3}{5}$ by $\frac{2}{3}$ (Graphical).

Let ABCD (Fig. 148) represent the unit. Let it be divided by vertical and horizontal lines, as in the figure, into 5×3 or 15 squares. AEFD contains 3 out of the 5 vertical strips into which ABCD is divided and is therefore $= \frac{3}{5}$ of ABCD.

Similarly, AGHB which contains 2 of the 3 horizontal strips represents $\frac{2}{3}$.

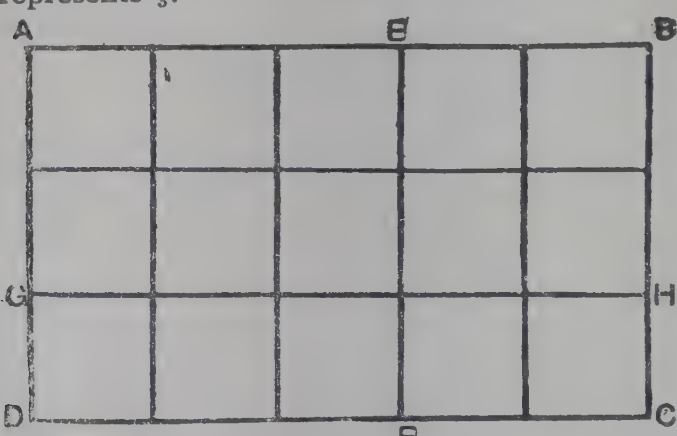


FIG. 148.

Now AEFD contains 9 squares and AGHB contains 10 squares.

It is required to divide $\frac{3}{5}$ by $\frac{2}{3}$; *i.e.*, we must find how much of AGHB is contained in AEFD; or we must find out how much of 10 squares is contained in 9 squares. The result is $\frac{9}{10}$.

1. Draw a rectangle on squared paper 3 in. by 3 in. and dividing it into 36 smaller rectangles show that $\frac{1}{4} \div \frac{1}{9} = \frac{9}{4}$; $\frac{3}{4} \div \frac{2}{9} = \frac{27}{8}$.

2. Draw a rectangle 5 cm. by 6 cm., divide it into 240 smaller rectangles and obtain the quotient of (*a*) $\frac{7}{8} \div \frac{2}{5}$ (*b*) $\frac{5}{6} \div \frac{2}{3}$ (*c*) $\frac{3}{16} \div$

3. Find graphically by drawing suitable rectangles the following quotients $\frac{8}{7} \div \frac{3}{8}$, $\frac{5}{12} \div \frac{5}{8}$.

4. Draw two rectangles each 8 cm. by 7 cm. regarding each of these as a unit, rule them of so as to represent the fractions $\frac{3}{8}$, and $\frac{6}{7}$ and from your figures compare these fractions, *i.e.*, find what multiple $\frac{6}{7}$ is of $\frac{3}{8}$ or find $\frac{6}{7} \div \frac{3}{8}$.

Exercise XVI (j).

1. Divide $\frac{18}{25}$ by 5 ; 7 ; 8 ; 9; verify by taking £1 5*s*. as the unit.

2. Divide $\frac{13}{42}$ by 7 ; 3 ; 15 ; 20 ; verify by taking Rs. 2·10 as the unit.

3. Divide $\frac{161}{2240}$ by 8 ; 10; 12 ; verify by taking 1 ton as the unit.

Divide

4. 2 by $\frac{3}{8}$, $\frac{4}{5}$, $\frac{6}{7}$, $\frac{8}{7}$.

5. $3\frac{1}{5}$ by $\frac{4}{7}$, $\frac{6}{11}$, $\frac{5}{21}$, $\frac{8}{25}$.

6. $\frac{10}{13}$ by $6\frac{1}{2}$, $2\frac{1}{3}$.

7. $1\frac{37}{108} \div 1\frac{9}{20}$.

8. How many lb. are there in (*a*) $\frac{4}{5}$ of a cwt. (*b*) $\frac{1}{3}$ of a cwt. ? What multiple is $\frac{4}{5}$ of a cwt. of $\frac{1}{3}$ of a cwt. ?

9. What multiple is $\frac{5}{6}$ viss of $\frac{1}{9}$ viss ?

10. How many times £$\frac{72}{44}$ should be repeated to make £$\frac{4}{11}$.

11. How often may $8\frac{1}{4}$ qrs. be taken from $4\frac{1}{3}$ tons and what will be the remainder ?

Exercise XVI (k).

Find the value of £73-7*s*.-6*d*. $\div \frac{11}{16}$.

$$£73\text{-}7\text{-}6 \div \frac{11}{16} = \frac{£73\text{-}7\text{-}6 \times 16}{11} = \frac{£1174\text{-}0\text{-}0}{11}$$

$$= £106\text{-}14\text{-}6\tfrac{6}{11} \text{ or } £106\text{-}14\text{-}7 \text{ nearly.}$$

Rough check. £73-7-6 divided by 11 gives us £6-13 nearly. This when multiplied by 16 gives us £106 nearly.

1. Find the value of

(*a*) £73-9*s*.-6*d*. $\div \frac{11}{16}$. (*b*) 5 m. 6 fur. 20 yds. $\div \frac{8}{5}$.

(*c*) Rs. 839-5-11 $\div 64\frac{2}{5}$. (*d*) 218 tons 2 qrs. 14lb. $\div 18\frac{9}{10}$.

2. If $\frac{7}{8}$ viss costs Rs. 5-3-4, what is the cost of a viss ?

3. If $\frac{5}{6}$ of an acre is worth Rs. 430-8-11, what is the cost of an acre ?

4. In $3\frac{1}{2}$ hours a train travels a distance of 110 miles. How many miles does it travel per hour ?

5. Express £2 2*s*. 6*d*. as a vulgar fraction of £5 17*s*. 6*d*. and also as a decimal.

6. When Rs. 16-8-0 has been paid out of a bill of Rs.240-12-0, what fractiou of the bill remains unpaid ? Express the answer in decimals also.

7. On a map a rectangle $15\frac{1}{2}''$ by $8\frac{3}{4}''$ represents an area of $46\frac{1}{3}$ yds by $26\frac{1}{4}$ yds. What fraction is the plan of the area it represents ?

8. Divide the sum of $8\frac{4}{15}$ and $9\frac{131}{162}$ by the product $2\frac{17}{20}$ and $3\frac{5}{12}$.

9. What is the difference between the product of $8\frac{1}{2}$ and $6\frac{3}{4}$ and the quotient got by dividing $106\frac{1}{4}$ by $1\frac{7}{25}$?

10. The length of a rod is 8·375 in. After expansion it is 9·3825. What fraction is the increase in the length of the rod of the original length of the rod ?

11. A labourer earns on the week days Rs. $\frac{3}{4}$, Re. $\frac{5}{8}$, Rs. $1\frac{1}{2}$, Re. $\frac{7}{8}$, Rs. $2\frac{5}{6}$, Rs. $3\frac{2}{3}$. Find his average earnings for the week.

*§ 150. To express a given number as a fraction with another given number for its denominator.

RULE.—Express it first as a fraction with unity for its denominator ; then multiply both the numerator and the denominator of the fraction by the second number.

Example.—Turn $17\frac{1}{2}$ into an equal fraction with (*a*) 5, (*b*) 6 for its denominator.

$$17\tfrac{1}{2} = \frac{17\frac{1}{2}}{1} = (a)\ \frac{17\frac{1}{2}\times 5}{5} = \frac{87\frac{1}{2}}{5}\quad (b)\ \frac{17\frac{1}{2}\times 6}{6} = \frac{105}{6}.$$

*§ **151.** By dividing the numerator and the denominator of a fraction and thus changing its form without altering its value, we can find two fractions with untiy for their numerators and integers for their denominators between which the given fraction lies.

Example.—Show that $\frac{19}{367}$ lies between $\frac{1}{19}$ and $\frac{1}{20}$.

Divide both numerator and denominator by 19. Thus $\dfrac{19}{367} = \dfrac{1}{19\frac{6}{19}}$ which is, of course, greater than $\frac{1}{20}$ and less than $\frac{1}{19}$.

§ 152. Simplification of fractions. In the simplification of fractions, the following conventions (*i.e.*, rules that are agreed upon for the sake of convenience) are to be borne in mind:—

(1) Multiplications and divisions should be performed before additions and subtractions.

(2) In performing multiplications and divisions as well as additions and subtractions, the order must be from left to right.

(3) The word **of** has the force of a bracket.

Thus $\quad \frac{2}{3} \div \frac{4}{5} \times \frac{3}{7} = \frac{2}{3} \times \frac{5}{4} \times \frac{3}{7} = \frac{5}{14}$.

Here if the operations were performed from right to left, the result would be

$$\frac{2}{3} \div \frac{4 \times 3}{5 \times 7} = \frac{2}{3} \times \frac{5 \times 7}{4 \times 3} = \frac{35}{18}$$

a result altogether different from the first. Hence the necessity for a convention.

Again $\frac{2}{3} \div \frac{4}{5}$ of $\frac{3}{7}$ means $\frac{2}{3} \div (\frac{4}{5} \times \frac{3}{7}) = \frac{2}{3} \div \dfrac{4 \times 3}{5 \times 7} = \frac{35}{18}$

Note therefore that with this convention about **of**, it is not exactly equivalent to the sign '×'.

Caution.—In simplifying a complex fraction, it should be simplified as a whole and not worked out in parts.

Example. Simplify

$$\frac{2}{3} \div \frac{5}{6} \text{ of } \frac{2}{15} \times \frac{3}{4} - \frac{4}{9} \times \frac{7}{12} \text{ of } \frac{9}{14} + \frac{8}{5} \div \frac{4}{25}.$$

The fraction $= \frac{2}{3} \div (\frac{5}{6} \times \frac{2}{15}) \times \frac{3}{4} - \frac{4}{9} \times (\frac{7}{12} \times \frac{9}{14}) + \frac{8}{5} \times \frac{25}{4}$.

$= \frac{2}{3} \div \frac{1}{9} \times \frac{3}{4} - \frac{4}{9} \times \frac{3}{8} + 10$

$= \frac{2}{3} \times 9 \times \frac{3}{4} - \frac{1}{6} + 10$

$= \frac{9}{2} - \frac{1}{6} + 10$, *i.e.*, $(4\frac{1}{2} - \frac{1}{6} + 10)$

$= 14 + \frac{1}{2} - \frac{1}{6} = 14\frac{1}{3}$.

Exercise XVI (l).

Simplify:—

1. $\frac{27}{36} \div (1\frac{12}{34}$ of $\frac{11}{33})$.

2. $\left(\frac{18}{104} \div \dfrac{1}{8\frac{2}{3}}\right)$ of $\frac{5}{12}$.

3. $1\frac{1}{28}$ of $\left[\frac{33}{144} \div \left(1\frac{53}{69} \text{ of } 1\frac{7}{16}\right)\right]$.

4. $\left(8\frac{16}{25} \text{ of } \frac{5}{27}\right) \div \left(1\frac{15}{84} \text{ of } 1\frac{10}{11}\right)$.

5. $\left(1\frac{43}{92} \text{ of } 1\frac{19}{27}\right) \div \left(1\frac{19}{256} \text{ of } 2\frac{18}{55}\right)$.

6. $\dfrac{\frac{3}{5}\left(2\frac{1}{2}+3\frac{1}{3}\right)}{\frac{5}{7}\left(3\frac{1}{3}-2\frac{1}{2}\right)}$.   7. $\dfrac{\frac{8}{13}+\frac{4}{11}}{\frac{3}{13}\times\frac{2}{11}}$.

8. $\dfrac{8\frac{2}{9} \text{ of } \left(\frac{11}{20}+\frac{13}{15}\right)}{8\frac{2}{9} \text{ of } \frac{11}{20}+\frac{13}{15}}$.

9. $\dfrac{\left(\frac{1}{2}+\frac{1}{3}\right)}{\frac{1}{2}-\frac{1}{3}} \text{ of } \dfrac{\frac{1}{3}+\frac{1}{4}}{\frac{1}{3}-\frac{1}{4}} \div \dfrac{\frac{5}{6}+\frac{8}{9}}{\frac{8}{9}-\frac{5}{6}}$

10. $6\frac{1}{2} + 8\frac{1}{3} \times 1\frac{3}{5} - 7\frac{1}{2}$.   11. $3\frac{3}{4} \div 1\frac{7}{8} \times \frac{11}{15} \times \frac{8}{11}$.

12. $\dfrac{2{\cdot}25 - \frac{3}{16}}{2{\cdot}25 + \frac{3}{16}} + \frac{7}{6} \text{ of } 1\frac{1}{14} - \dfrac{22{\cdot}45}{30}$.

13. $\dfrac{\frac{1}{2}+\frac{1}{3}+\frac{1}{5}+\frac{1}{6}}{\frac{1}{2}-\frac{1}{3}+\frac{1}{5}+\frac{1}{6}}$.   14. $8 - \dfrac{1}{7+\dfrac{5}{4+\dfrac{3}{1+\frac{1}{2}}}}$.

15. $\dfrac{1+\frac{1}{5}}{1-\frac{1}{5}} \div \dfrac{3+\frac{3}{5}}{2-\frac{3}{5}} + \dfrac{1-\frac{6}{7}}{4\frac{1}{2}+3\frac{1}{8}} \times \left(\dfrac{1+\frac{1}{3}}{1-\frac{1}{3}}\right)^2$.

16. $\dfrac{7}{5-\frac{8}{8}} + \dfrac{6}{8+\frac{1}{3}} \times \dfrac{1}{1+\dfrac{1}{2+\dfrac{3}{4+\frac{2}{7}}}}$.

Exercise—XVI (m).

1. If I lose a purse containing $\frac{3}{8}$ of my pocket money, what fraction have I left? If this be Rs. 2, how much had I at first and what sum was in the purse?

2. In a campaign, if an army loses $\frac{1}{12}$ of its numbers in battle and $\frac{1}{6}$ by sickness, what fraction survives?

3. A draper sells first $5\frac{3}{4}$ yds. and then $6\frac{2}{3}$ yds. of silk and finds that he has $8\frac{5}{6}$ yds. left. How much had he at first?

4. A person has to cycle a certain distance in the course of a day. He rides $\frac{1}{4}$ of the whole journey after breakfast, $\frac{1}{3}$ after lunch, and $\frac{1}{6}$ after dinner. If the whole distance is 24 miles, what distance has he yet to ride?

5. A can do a piece of work in 5 days. What fraction of it can he do in 1 day?

B does it in 6 days. What fraction of it can he do in 1 day?

If A and B work together what fraction will they do in 1 day?

6. If $\frac{5}{6}$ of a pound of tobacco cost 4 as., what is the cost of $\frac{2}{5}$ lb.; $\frac{5}{8}$ lb.?

7. A ball of thread contains 90 ft. How many pieces of $6\frac{1}{4}$ ft. can be cut from it and what will be left?

8. A man leaves $\frac{1}{2}$ of his property to his eldest son, the rest to be equally divided among the remaining four children. The total property is worth Rs. 20,000. Find the share of each.

9. The annual rainfall in a certain place is 40 inches. If in the month of October 9·6 inches are recorded, what is the ratio of this to the annual rainfall?

10. One train describes, at a uniform rate, 30 miles in 1 hr. 15 min. and another 50 miles in 1 hr. and 30 min. Compare the speeds of the trains.

11. Given that 5 cm. = 2 inches nearly, find the approximate ratio of a metre to a yard.

12. A room measures 30 ft. by 18 ft. in the centre of which there is a carpet 6 ft. by $4\frac{1}{2}$ ft. and the remainder is covered with mats. Find the ratio of the area of the carpet to that covered by the mats.

13. Draw a line to represent a mile. Construct a scale to read furlongs by dividing the line.

14. Show that (*a*) $\dfrac{2+7}{5+9}$ is greater than $\frac{2}{5}$ and less than $\frac{7}{9}$.

(*b*) $\dfrac{7+8}{16+19}$ is greater than $\frac{7}{16}$ and less than $\frac{8}{19}$.

(*c*) $\dfrac{3+6+7}{8+11+13}$ lies between the greatest and the least of the fractions $\frac{3}{8}$, $\frac{6}{11}$, and $\frac{7}{13}$.

15. Reduce the following fractions to decimals without division:

$\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{3}{8}$, $\frac{1}{25}$, $\frac{1}{125}$, $\frac{1}{40}$, $\frac{1}{80}$, $\frac{1}{500}$, $\frac{3}{800}$.

16. Show that ·625 = ·62$\frac{1}{2}$ = ·6$\frac{1}{4}$ and hence convert the following decimals into vulgar fractions:

(*a*) ·875 (*b*) ·375 (*c*) ·5625 (*d*) ·6875.

CHAPTER XVII.

PERCENTAGE.

§ **153**. Suppose for a certain examination, one school sent up 225 candidates out of which 90 came out successful, and another school sent up 275 candidates out of which 121 came out successful. To compare the results of the schools, *i.e.*, to find which has done better at the examination, it is convenient to find in each case the 'pass' per 100. In the first case out of 225 the number passed is 90. $\therefore$ out of 100 the number passed is $\frac{90}{225} \times 100 = 40$. In the second case out of 275, the number passed is 121. $\therefore$ out of 100, the number passed is $\frac{121}{275} \times 100 = 44$.

$\therefore$ we conclude that the second school has done better. The 'pass' in the first case is said to be 40 for every hundred or 40 **per cent.** and that in the second case as 44 for every 100 or 44 **per cent.**

20 per cent. of a class thus means 20 out of every 100, *i.e.*, 1 out of every 5 or $\frac{1}{5}$ of the class. Similarly 25 per cent. of the population of a town means $\frac{25}{100}$ or $\frac{1}{4}$ of the population. A percentage thus represents a fraction in which the numerator alone is specified and the denominator is understood to be 100.

per cent. is symbolically written '%.'

Ex. 1. Convert $50\frac{1}{2}$ per cent. into a fraction.

$$50\tfrac{1}{2}\% = \frac{50\frac{1}{2}}{100} = \frac{101 \times 1}{2 \times 100} = \frac{101}{200}.$$

Ex. 2. Convert $\frac{5}{6}$ into a percentage.

$\frac{5}{6}$ means $\frac{5}{6}$ of a unit.

$= \frac{5}{6} \times 100$ of every 100

$= 83\frac{1}{3}$ of every 100

$= 83\frac{1}{3}$ per cent.

Exercise—Oral.

1. Convert the following fractions into percentages :—

$\frac{1}{4}, \frac{1}{6}, \frac{1}{8}, \frac{1}{10}, \frac{1}{12}, \frac{1}{15}, \frac{1}{16}$.

2. Convert the following percentages into fractions :—50 °/₀, 40 °/₀, $12\frac{1}{2}$ °/₀, $6\frac{1}{4}$ °/₀, x °/₀, r °/₀.

3. A school contains 400 boys—

(*a*) 30 in every 100 read Vernacular only. How many boys are there of this sort ?

(*b*) 50 in every 100 read both English and Vernacular only. How many are there of this sort ?

(*c*) The rest read English only. How many are there of this sort ?

4. What is 5 per cent. of Rs. 60 ; Rs. 80 ; Rs. 100 ; Rs. 150 ; Rs. x ?

5. What is 4 per cent. of Rs. 50 ; Rs. 75 ; Rs. 125 ; Rs. 250 Re. 1 ; Rs. x ?

Exercise XVII (a).

1. The population of a certain village was 1200 ; ·5 per cent. died in a year. Find the number of deaths in the year.

2. Express as fractions and also as decimals the following rates per cent. :—

(*a*) $50\frac{1}{4}$. (*b*) 100. (*c*) 125. (*d*) $\frac{1}{2}$.

(*e*) $68\frac{1}{4}$. (*f*) 99·8. (*g*) 98·35. (*h*) 250.

3. Express the following fractions as percentages :—

(*a*) $\frac{1}{125}$. (*b*) $\frac{1}{48}$. (*c*) $1\frac{1}{66}$. (*d*) $\frac{1}{x}$.

4. Express in metres and decimals of a metre 5 °/₀ of 100 m ; 80 m. ; 5 m. ; 17 m.

§ 154. Graphical representation of percentage.

Ex. 1. Express, graphically, 30 per cent. as a fraction.

Take a line AB a decimetre long as the unit, mark AC equal to 30 millimetres. Then AC = 30 per cent. of the unit. With your dividers, see how many lengths each equal to AC you can step off along AB. You thus find there are $3\frac{1}{3}$ times AC in AB. ∴ the fraction $\frac{AC}{AB} = \frac{1}{3\frac{1}{3}}$ or $\frac{3}{10}$.

Ex. 2. Express $\frac{1}{6}$ as a percentage.

Take AB a decimetre long as the unit, divide it into six equal parts, at C, D, E... AC = $\frac{1}{6}$ unit and contains $16\frac{2}{3}$ mm. But the unit contains 100 mm. ∴ $\frac{1}{6} = 16\frac{2}{3}$ %.

Exercise XVII (b).

1. Express $\frac{1}{4}$, $\frac{1}{6}$, $\frac{7}{12}$ as percentages graphically.

2. Express the following percentages as fractions graphically : 64 per cent., $12\frac{1}{2}$ per cent., $3\frac{1}{3}$ per cent., 35 per cent., 40 per cent.

3. What percentage is

(*a*) 3 as. 6p. of Rs. 10 ? (*b*) 14 as. 8p. of Rs. 15 ?
(*c*) £12-3-6 of £75 ? (*d*) 318 m. 7dm. 8cm. of 410 km. ?
(*e*) 1 qr. of 1 ton 2 cwts. ? (*f*) 8 mds. 2 viss of 10 can. ?

4. How much per cent. is

(*a*) 16 of 240 ? (*b*) $37\frac{1}{2}$ of 150 ?
(*c*) 3·375 of $19\frac{2}{7}$? (*d*) 4·2875 of 40 ?

5. A gramophone costs Rs. 125, and every month records are purchased for Rs. 7-8-0. How much per cent. is this of the cost of this gramophone ?

6. Out of 141 students that went up for Matriculation from a certain school, 39 were successful. What is the percentage of success ?

7. The receipts of a certain college are Rs. 50,680. The expenses amount to Rs. 48,725. What per cent. is the balance of the total receipts for the year ?

8. In a school of 582 boys 533 are present in the forenoon and 542 in the afternoon. What is the percentage of attendance (1) in the morning, (2) in the evening?

9. In an examination, out of a maximum of 550, a boy gets 341 marks. What percentage of the maximum does he get?

10. A boy gets 33 $^0/_0$ of the maximum. He fails for want of 6 marks and the minimum is 35 $^0/_0$. What is the maximum? How many marks does he get, and what is the minimum for a pass?

11. In the VI Form consisting of 200 boys, 113 are Brahmans, 70 non-Brahmans, and the rest Christians. Find the percentage of each.

12. Of what sum of money is

(*a*) Rs. 8—10 $^0/_0$? (*b*) £71-7-6—25 $^0/_0$?

13. Of what length is

(*a*) 3 miles 2 fur.—18 $^0/_0$? (*b*) 8 metres and 3 cm.—21 $^0/_0$?

14. Of what weight is

(*a*) 7 cwt. 4 lb. 33⅓ $^0/_0$?

(*b*) 18 mds. 4 seers, 15 $^0/_0$?

15. 37½ $^0/_0$ of the maximum is 40. What is the maximum?

16. If 5 $^0/_0$ of the candidates that applied for an examination were absent from it and the number that attended the examination was 7695, find the number that applied for the examination.

17. In a school of 600 boys, 25 per cent. were examined in Mathematics, 60 per cent. in History, 12½ per cent. in Vernacular and the rest in Botany. How many were examined in each subject?

18. In a class of 150 boys the guardians of 40 per cent. of the boys have an income of above Rs. 2,000; those of 44 per cent. have an income between Rs. 2,000 and Rs. 250; those of the rest below Rs. 250. Find the number of guardians coming under each of the above classes.

19. 75·54 $^0/_0$ of atmospheric air being nitrogen, 23·33 $^0/_0$ being oxygen, and the rest carbonic acid, how much of each gas is there in 500 cubic feet of air?

20. A merchant orders 80 tins of ghee from Salem at the rate of Rs. 15 per tin. 5 per cent. of the tins are leaky and

only three-fourths full. If each full tin contains $12\frac{1}{2}$ viss and each viss is sold at Re. 1-12-0, find his gain.

PRACTICE.

§ 155. If a person buys 15 viss of ghee in a bazaar at Re. 1-12 per viss, he does not ordinarly find the price of 15 viss by multiplying Re. 1-12 by 15; but he often calculates thus:—The cost of 15 viss at Re. 1 is Rs. 15 and the cost at 12 as. or Re. $\frac{3}{4}$ is $\frac{3}{4}$ of 15 or Rs. $11\frac{1}{4}$, total cost is Rs. $26\frac{1}{4}$. This method of finding the cost of a certain number of articles by breaking up the price of each article into component parts each of which is a simple fraction of one of the preceding component parts is found to be convenient and is consequently largely employed *in practice*; and the method itself has come to be known as **practice.**

Example 1. Find the cost of 1120 articles at 13 as. 6 pies each

		Rs.
Price of 1120 articles at Re. 1 each	=	1120
Price at 8 as. or $\frac{1}{2}$ Re.	=	560
,, 4 ,, $\frac{1}{2}$ of 8 as.	=	280
,, 1 ,, $\frac{1}{4}$ of 4 ,,	=	70
,, 6 pies or $\frac{1}{2}$ of 1 anna	=	35
,, 13 as. 6 pies	=	945

Note.—8 as. or Re. $\frac{1}{2}$ divides Re. 1 exactly twice, 4 as. divides 8 as. exactly. 8 as. is said to be an **aliquot** part of Re. 1; 4 as. is also an aliquot part of 8 as. **Aliquot parts** of any quantity are such as will divide it an exact number of times.

Exercise—Oral.

1. What fraction of Re. 1 is 1 a.; 2 as.; 4 as.; 8 as.; 1 a. 4 p.; 2 as. 8 p.; and 5 as. 4 p.?

2. What fraction of 1 anna is 1p.; 2 p.; 3 p.; 4p.; 6 p.; $1\frac{1}{2}$p.?

3. What fraction of 1 shilling is $1d.$; $2d.$; $3d.$; $4d.$; $1\frac{1}{2}d.$?

4. What fraction of a £ is $1s.$; $2s.$; $4s.$; $5s.$ $10s.$; $2s.\ 6d.$; $3s.\ 4d.$; $1s.\ 3d.$; $1s.\ 4d.$; $1s.\ 8d.$; $6s.\ 8d.$?

5. Find the value of each of the following aliquot parts, *viz.*, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{6}$, $\frac{1}{12}$ of (1) 2 as. 6 p. (2) 1 a. 6 p. (3) 3 as. (4) 6 as.

6. Express as aliquot parts (1) 1 a. 6 p. of 6 as., (2) 2as. 3. p. of 9 as., (3) 3 as. 4. p. of 10 as., (4) 1*s*. 4*d*. of 5*s*., (5) 3*s*. 2*d*. of 9*s*. 6*d*.

Example 2.—Divide 14 as. 11 pies into convenient aliquot parts.

8 as. = $\frac{1}{2}$ of Re. 1.
4 as. = $\frac{1}{2}$ of 8 as.
2 as. = $\frac{1}{2}$ of 4 as.
6 pies = $\frac{1}{4}$ of 2 as.
3 pies = $\frac{1}{2}$ of 6 ps.
2 pies = $\frac{1}{3}$ of 6 ps.

Exercise—Oral.

Break into convenient aliquot parts (1) 6 as. 9 pies (2) 8*s*. 11*d*. (3) 15 as. 5 p. (4) 14*s*. 7*d*. (5) 9*s*. 10$\frac{1}{2}$*d*.

Example 3.—Find the cost of constructing 2560$\frac{1}{4}$ miles of railway at £577-9-6 per mile.

The cost at £1 each mile is £2560$\frac{1}{4}$ or £2560-5-0.

				£	*s.*	*d.*
The cost at £1		each mile	=	2560	5	0
						8
,,	£8	,,	=	20482	0	0
						8
,,	£64	,,	=	163856	0	0
						9
,,	£576	,,	=	1474704	0	0
,,	£1	,,	=	2560	5	0
,,	£577	,,	=	1477264	5	0
,,	5*s*. or $\frac{1}{4}$ of £1	,,	=	640	1	3
,,	2*s* 6*d*. or $\frac{1}{2}$ of 5*s*.	,,	=	320	0	7$\frac{1}{2}$
,,	2*s*. or $\frac{1}{10}$ of £1	,,	=	256	0	6
,,	£577-9-6	,,	=	1478480	7	4$\frac{1}{2}$

Note.—In this method of *practice*, multiplication is generally done by factors. Sometimes the multplier may not be resolvable into factors as in the above example. In that case, by means of convenient additions and subtractions the multiplier may be split up into factors and parts as we have done; 577 being put into 576+1, *i.e.*, 8×8×9+1. The examples may be worked in decimals also.

The student may note that, in the above example, the working may be simplified by using the method of subtraction thus :

As before the cost at £577 can be determined and the subsequent work will stand thus :—

			£	s.	d.
The cost at £577	each mile	=	1477264	0	0
,,	10s. or $\frac{1}{2}$ of £ ,,	=	1280	2	6
,,	£577-10-0 ,,	=	1478544	2	6
,,	6d. or $\frac{1}{20}$ or 10s. ,,	=	64	0	$1\frac{1}{2}$
,,	£577-9-6 ,,	=	1478480	2	$4\frac{1}{2}$

Example 4.—Find the cost of 841 tons 6 cwts. 3 qrs. of wood at Rs. 15-12-6 per ton.

		Rs.	A.	P.
The cost of	1 ton =	15	12	6
				3
,,	3 ,, =	47	5	6
				5
,,	15 ,, =	236	11	6
				7
,,	105 ,, =	1 657	0	6
				8
,,	840 ,, =	13,256	4	0
,,	1 ,, =	15	12	6
,,	841 ,, =	13,272	0	6
The ccst of	5 cwts. or $\frac{1}{4}$ of 1 ton =	3	15	$1\frac{1}{2}$
,,	1 cwt. or $\frac{1}{5}$ of 5 cwt. =	0	12	$7\frac{1}{2}$
,,	2 qrs. or $\frac{1}{2}$ of 1 cwt. =	0	6	$3\frac{3}{4}$
,,	1 qr. or $\frac{1}{2}$ 2 qrs. =	0	3	$1\frac{7}{8}$
,,	841 tons 6 cwt. 3 qrs. =	13,277	5	$8\frac{5}{8}$

Exercise XVII (c).

Calculate the value of the following :—

1. 1976 articles (*a*) 5 as. 6 p. (*b*) 7 as. 11 p. each.
2. 8364 articles at (*a*) 8*s*. 9*d*. (*b*) 13*s*. 5*d*. each.
3. 2032 articles at (*a*) Rs. 26-8-4 (*b*) Rs. 97-15-11 each.
4. 7582 articles at (*a*) £ 15 18*s*. 9*d*. (*b*) £37 18*s*. 6*d*. each, using only one aliquot part in each case and then check your result by using two.
5. Find the cost of $357\frac{3}{4}$ yards of silk at Rs. 2-4-9 a yard.

INVOICES.

§ 156. When purchases are made at a shop, the seller of the goods sends with the goods a list of articles bought, with their prices set down. Such a list is called an *invoice or*

a bill. Usually the goods are not paid for, when they are sold or supplied. At the end of a certain period *an account* is sent to the buyer showing all the goods bought by him together with the charges.

The following is a specimen of a bill or invoice :—

Invoice No. 292. VIENNA, *November 27th, 1908.*
Indent No. As per your letter *VII/1, Kandlgasse 11.*
dated June 25th, 1908.

MESSRS. N. PERUMAL CHETTY & CO.,
Madras.

Bought of JOS. JAFF & SOHN,
Vienna.

Copy.

The undermentioned goods sent by S.S.......................
from Antwerp *to* Madras *upon your order and for your account and risk against our 30 D/S Draft* D/P.

No claim will be entertained, unless written application be made to our agents within 10 days after arrival of the steamer.

J. J. & S.	*Roping Twines.*		
1536	Made in Belgium 2 cases each containing 100 bundles		
	2 cases 200 bundles No. 39.		
Cr. W. kg.	each bundle $2\frac{3}{4}$ lbs. 550 lbs. at 8*d.*	£18- 6-8	
171 each. No. 3-5	3 cases each containing 100 bundles		
	3 cases 300 bundles No. 40		
Cr. W. kg.	each bundle $2\frac{3}{4}$ lbs. 825 lbs. at $8\frac{1}{8}$*d.* ...	£27-18-7	
171 each.	12 balls per bundle ...	£46-5-3	
	Bank postage ...	£ -2-	
			£46-7-3

Cif. Madras Harbour.
JOS. JAFF & SOHN.

Exercise XVII (d).

1. Make out in proper form an invoice for the following goods :—2 Page's selected bats at Rs. 6 each ; 2 Duke's Match balls at Rs. 4-12-0 each ; 3 Eclipse composition balls at Rs. 2 each ; 3 Nicholson's composition balls at Rs. 3-8-0 each ; 6 Ayres' champion tennis balls at Rs. 10 per dozen.

2. Make out in proper form an invoice for the following goods :—15 Kashmere bats at Rs. 6-12-0 (credit) each ; 5 Excellent balls at 6-12-0 each ; 10 Sialkot balls at Rs. 2-4-0 each ; 15 Hockey balls at Rs. 4 each ; 24 Lawn Tennis champion balls at Rs. 11-4-0 per dozen.

Make out invoices for, and find the total cost of the following purchases and insert any names and dates in the proper places :—

3. 75,000 bricks at Rs. 4-8-0 per thousand ; 50 casks of Portland cement (Engine Brand) at Rs. 7 per cask ; 20 tons of teakwood at Rs. 10-5-0 per ton ; 5 tons 10 cwt. of galvanized iron sheets at Rs. 11-8-0 per cwt.

4. 6 doz. slates at 3 for half a rupee ; 6 gross of pencils @ 1 a. 9 p. per dozen ; 4 dozen boxes of crayon @ 7 as. per box ; 5 bottles of Stephens' ink @ 12 as. per bottle ; 2000 Note envelopes @ 2 as. per 100 ; 2000 Note paper (cream white) at 3 as. per quire.

5. 128 copies of Algebra @ 7*s*. 6*d*. each ; 200 copies of Dictionary @ 5*s*. each ; 20 copies of Grammer @ 3*s*. 6*d*. each ; 235 copies of Novels @ 3*s*. each.

6. $18\frac{1}{2}$ yds. of Tassur silk @ Rs. 2 per yd. ; 10 pieces of 1703 mull @ Rs. 7-2-0 per piece ; 30 packets of 6 each (Rally Brothers banyans)@ 12 as. each ; 5 pieces of Cawn. twill @ Rs. 10 per piece ; 5 pieces of Glasgow mull @ Rs. 7-3-0 each ; 15 Ivory collars @ 6 as. each and half a dozen silk neck-ties @ Rs. 6 per dozen.

MISCELLANEOUS EXAMPLES.

1. A person performs $\frac{1}{2}$ his journey by rail, $\frac{1}{3}$ by motor-car and the rest, a distance of 15 miles, walking. Find the length of the journey.

Let x miles be the length of the journey. The distance walked in miles $= x - \frac{1}{2}x$ (by rail) $- \frac{1}{3}x$ (by motor-car) $= x\left(1-\frac{1}{2}-\frac{1}{3}\right) = \frac{1}{6}x$.

By the question this is 15 miles.

$\therefore \frac{1}{6}x = 15.$

$\therefore x = 90.$ $\therefore$ the length of journey is 90 miles.

2. A man buys plantains with $\frac{1}{4}$ of the money he has with him; then buys mangoes with $\frac{1}{6}$ of the remainder and 6 annas more. He finds he has Rs. 2-2-0 still left. What money had he at first?

Let x annas be the sum of money he had at first.

He buys plantains for $\frac{1}{4}x$ annas.

$\therefore$ the remainder is $\frac{3}{4}x$ annas. He buys mangoes for ($\frac{1}{6}$ of $\frac{3}{4}$ of $x + 6$) annas, *i.e.*, ($\frac{1}{8}x + 6$) as.

$\therefore$ the remainder in annas is $\frac{3}{4}x - (\frac{1}{8}x + 6) = \frac{3}{4}x - \frac{1}{8}x - 6 = \frac{5}{8}x - 6.$

This by the question = Rs. 2-2 or 34 annas.

$\therefore \frac{5}{8}x - 6 = 34.$ $\therefore \frac{5}{8}x = 34 + 6 = 40.$ $\therefore x = 64.$ The money he had at first was 64 as. or Rs. 4.

3. A can do a piece of work in 5 days and B in 6 days. How long will it take them both together to finish the work?

In 1 day A does $\frac{1}{5}$ of the work.

In 1 day B does $\frac{1}{6}$ "

$\therefore$ In 1 day A and B together do $\frac{1}{5} + \frac{1}{6}$ or $\frac{11}{30}$ of the work.

$\frac{11}{30}$ of the work is finished in 1 day.

$\therefore$ the whole work is finished in $\frac{30}{11}$ or $2\frac{8}{11}$ days.

Exercise XVII (e).

1. If $\frac{3}{4}$ of a bag of rice cost Rs. 7-8-4, what is the cost of the whole bag? And if it contains 42 measures what is the price of each measure correct to a pie?

2. If $\frac{9}{20}$ of a ton of coal costs Rs. 15-8-9, what is the cost of 1 ton 1 cwt.?

3. If 1 metre = 39·4 inches approximately, express 23640 yds. in kilometres.

4. If 1 kilogram = $2\frac{3}{5}$ lbs. (av.), express 3 ton 5 cwt. 2 qrs. in kilogrammes.

5. A boy spends $\frac{1}{5}$ of his pocket money in the first week of the term and $\frac{3}{8}$ of what remains in the second week. What fraction is left ? If this is Rs. 10, what is his pocket money ?

6. $\frac{3}{10}$ of a certain journey is performed by train, $\frac{5}{24}$ by jutka and the rest by horse. What fraction of the whole journey is performed by horse ?

7. A man leaves $\frac{1}{2}$ of his property to his widow and the rest is to be divided equally among his four children. If the widow gets Rs. 10,500 more than each of the children, what is the value of his property ?

8. The distance between the boiling point and the freezing point in the Fahrenheit thermometer is divided into 180 equal parts and into 100 equal parts on the Centigrade. What fraction of the whole space is the difference between the length of one division on the former and that of one division on the latter ?

9. From a cask of wine $\frac{1}{5}$ is drawn off and 3 gallons of water added. Again $\frac{1}{3}$ of this new mixture is drawn off and the cask then contains 18 gallons. Find the original contents of the cask in gallons.

10. *A* can do a piece of work in 4 days. He works at it for 3 days and then leaves it. The work is then taken up by B who can do the whole work in 8 days. How long does B take to finish the work ?

11. A can do a piece of work in 9 days of 8 hours each. B can do the same work in 8 days of 6 hours each. What fraction of the whole work is finished in 4 days if both A and B work together ?

12. If I take steps of 27 inches, how many do I take in walking (1) a furlong (2) a mile ?

13. Of the land under cultivation in the Madras Presidency $\frac{15}{32}$ produces rice and $\frac{1}{3}$ other crops. The rest is pasture. What fraction is the pasture of the whole area under cultivation ?

14. I go out on a holiday trip with a certain sum in my pocket, I spend $\frac{1}{6}$ of it on a railway ticket, $\frac{1}{4}$ of what remains on a coach, $\frac{1}{6}$ of what remains on plantains and coffee, $\frac{2}{5}$ of what now remains on a ticket home. I find I have Rs. 2 left. What was the price of the refreshments I had, and with what sum did I start ?

15. Find the cost of laying a road of $68\frac{3}{8}$ miles at £27-15-6 a mile.

16. A family buys in a month 73 marakkals 3 measures and 2 ollocks of milk at 5 as. a measure; calculate the cost of milk bought in the month.

17. Find the cost of 1246 tons 8 cwt. 3 qrs. at Rs.9-8-6 per ton.

18. Find the cost of 35 ac. 5 ro. 27 sq. po. at (1) Rs.24-15-0; (2) Rs. 39-14-6 per acre.

19. Find the cost of 18450 bricks at Rs. 5-8-0 per 1000.

20. Find the cost of 980 envelopes at 4 as. 6 p. per hundred.

21. Find the cost of 23 lbs. 7 oz. of silver at Re. 1-4-6 per oz.

22. Find the value of 875 Madras Bank shares at Rs. 1098-8-0 each.

23. Find the cost of carrying 834 tons 13 cwt. for 80 miles at 2 as. 6 pies per ton per mile.

24. Find the income-tax on an assessed income of Rs. 1834 at the rate of 4 pies in the rupee.

25. A field of 28 ac. 3 ro. 40 sq. po. yields 738 measures of ground-nut per acre. Calculate the value of the ground-nut at Rs. 15 per kalam.

26. If a coal-field yields 12,250 tons of coal per acre, find the weight of the estimated coal in a taking of 250 acres 3 roods.

27. A pipe A can fill a cistern in 3 hours and it can be emptied by a pipe B in 6 hours. Suppose the cistern is half full and both the pipes are open. When will the cistern be filled?

28. Gunpowder is composed of $\frac{3}{5}$ths sulphur, $\frac{3}{20}$ths charcoal and $\frac{1}{4}$th nitre. How much of each material is required to make up 2 tons?

29. A cup is made of silver weighing 100 rupees; the cost of making it is $\frac{1}{16}$ of the value of silver. If each rupee weight of silver is worth 12 as. 3 p., what is the value of the cup?

30. 5 times a number and a sixth part of it make 1984. What is the number?

31. The seventh part of a number exceeds the eleventh part of it by 88. Find the number.

32. The difference of two numbers is 120. Their sum is $2\frac{2}{3}$ times the smaller; what are the numbers?

33. A train starts full of passengers. At the first station it drops one-third of these and takes in 65 more. At the next it drops a fifth of the new total and takes in 15 more. On reaching the next station there are found to be 475 left; how many started?

34. The population of a country is 13,846,000. The men form $\frac{2}{5}$ of the whole population and their number is $\frac{40}{41}$ of the women; find the number of women.

35. Divide Rs. 25-13-6 between A and B so that A may get $2\frac{1}{2}$ times as much as B.

36. Divide Rs. 4500 among A, B and C so that A may get $\frac{1}{3}$ as much as B, B $\frac{1}{4}$ as much as C.

37. In a cricket match a side of 11 men made a certain number of runs; one obtained $\frac{1}{10}$ of the whole number, two others each made $\frac{1}{8}$ and each of three others $\frac{1}{25}$ whilst the rest made 212 runs between them. What was the total score?

38. At an election $\frac{1}{9}$ of the voters did not vote; one candidate received the votes of $\frac{29}{54}$ths of the constituency and beat his opponent by 20 votes; how many voters were there, and how many votes did each candidate get?

39. In a book on Arithmetic an example was printed thus :—

Add together $\dfrac{1}{7\frac{1}{3}}$, $\dfrac{1}{9\frac{5}{8}}$, $\dfrac{1}{\quad}$, $\dfrac{1}{6\frac{7}{8}}$ the denominator of one fraction being accidentally omitted. The answer given at the end of the book was $\frac{11}{56}$. Required the missing denominator.

40. $\frac{1}{4}$ of the money allotted for a marriage was spent on jewels; $\frac{1}{3}$ of the remainder on cloths, $\frac{11}{16}$ of the then remainder in feeding the poor and relatives and the rest consisted of sundry expenses which amounted to Rs. 1,200. How many were fed if each individual was entertained at a cost of 4 as.?

REVISION PAPERS—II Series.

1.

1. A person buys 4 cartfuls of bricks. The first cart contains 750 bricks, the second 870, the third 640 and the fourth 520. If the price of 1000 bricks is Rs. 4 6as., how much does he pay on the whole for the bricks?

2. A retail dealer buys vegetables at 14 as. a maund and retails at 2 as. per viss. What weight of vegetables more would a person get for Re. 1 5 as. from the wholesale dealer than from the retail dealer?

3. If $a=13$, $b=11$ and $c=143$, find the value of $\frac{6}{7a}+\frac{3}{4b}+\frac{5}{c}$

4. AB is 2 in. long. Through a point C 2·5 inches distant from AB draw CD parallel and equal to AB. Examine if AC and BD are parallel. State the test you apply.

5. Two circles whose radii are 4 cm. and 6 cm. with their centres P and Q 8 cm. apart intersect at A and B. Bisect AB at C. Show that PC and QC are in the same straight line.

6. How many lakhs of rupees (to two decimal places) are there in 847289280 rupees.

7. A cistern is 1 m. 70 cm. long, 1 m. wide and 70 cm. deep. How many hectolitres of water will it hold?

8. Find the value of $\frac{1}{281 \cdot 2}$ to four places of decimals. Before you commence your work, state how many ciphers there will be before the first significant figure.

2.

1. Simplify $\frac{1 \cdot 2 \times 3 \cdot 14 - 3 \cdot 7 \div 2 \cdot 5}{\cdot 45 \div \cdot 9 \times 3 \cdot 2}$.

2. If a ton of fuel costs Rs. 15-4-0, what is the cost of 3 loads of fuel each load weighing 56 lbs.?

3. Assuming that a year = 360 days, divide 16 years by 120 and multiply the result by 19. Give the answer in years, months and days.

4. Find the weight of an iron bar 14 ft. long, 3 in. wide, $\frac{3}{4}$ in. thick, taking the weight per cubic foot to be 480 lbs. If the weight

of a waggon filled with such bars is 14 ton. 3 cwt. 22 lb., exclusive of the weight of the waggon, find the number of bars in the waggon.

5. Draw any angle ABC. Bisect the angle B by BD. Through any point D in the bisector draw parallels to BA and BC respectively meeting BC and BA in P and Q. What kind of figure is BPDQ ?

6. Find the H.C.F. of 17550 and 36075 by means of factors and also their L.C.M.

7. Express 3 acres 44 sq. yds. as (1) a vulgar fraction (2) as a decimal of 120 acres.

8. A man takes y hours to go from A to B travelling at the rate of x miles an hour and q hours to go from B to C travelling at the rate of p miles an hour. Find the distance from A to C.

3.

1. Multiply ·008934 by ·7625 and divide the product by ·0834 to 3 places of decimals. Before beginning to work give an approximate answer to the question.

2. Find the cost of fencing a tennis court 40 ft. long, 30 ft. wide, leaving a margin 10 ft. wide all round, with wire at the rate of 1 a. 4 p. per yard.

3. Simplify $\dfrac{8\frac{1}{2}+7\frac{5}{9}\div 5\frac{2}{3}}{7\frac{1}{24}-8\frac{1}{2}\text{ of }\frac{2}{3}}$.

4. Two men who can build a wall in 10 and 12 days respectively work for $4\frac{3}{4}$ days at it. What fraction of it remains to be done ?

5. Two sides of a triangle are 50 ft. and 75 ft. and they include an angle of 120°. Find the lenghth of the other side and also measure the remaining angles.

6. If $m = 4$, $n = 3$, and $a = 2$, $b = -1$, find the value of

$$(m+n)(a+b)^2 + (m-n)(a-b)^2.$$

7. A milkman buys milk at 4 as. a measure and adds quarter of an ollock of water to every ollock of milk. If he sells the mixture at 7 pies per ollock, what does he gain in the rupee ? Express his gain as a percentage.

8. A machine can be got from a place A (40 miles distant) for Rs. 10,050 or B (56 miles distant) for Rs. 9,850. The machine

weighs 2 ton. 3 cwt. and 21 lb. If the cost of carriage per lb. per mile is $1\frac{1}{4}$ pies, which will be the cheaper—to get the machine from A or from B, and by how much ?

4.

1. A ghee merchant sells the following quantities of ghee in the months noted below for the following prices :—

	Can.	Mds.	Viss	Average Price.
July ...	30	4	2	Rs. 16 per maund.
August ...	43	17	4	,, 15 ,,
September ...	89	11	5	,, 14 ,,
October ...	64	14	6	,, 12 ,,
November ...	83	10	7	,, 11 ,,
December ...	73	5	6	,, 10 ,,

Find (1) the average quantity sold per month, (2) the total receipt from the sale of ghee during the half-year, and (3) the average price per maund for the half-year.

2. An estate is occupied by 3 tenants, the first holding $\frac{2}{9}$ of it, the second $\frac{3}{7}$ of it and the 3rd the remainder, *viz.*, 26 ac. 75 cents. What is the area of the whole estate ?

3. (*a*) Find the value of ·48765 of £6.

(*b*) Simplify $\dfrac{4\frac{8}{9} \text{ of } \frac{3}{11} + 8\frac{4}{9} \div 3\frac{4}{5}}{6\frac{2}{3} - 1\frac{5}{6} \text{ of } \frac{15}{33}}$.

4. The angles of a triangle are 50°, 60°, 70°. The longest side is 100 ft. Find the length of the shortest side by careful drawing and measurement.

5. Solve the equation $5(x+6) + 6(8x+3) = 8(7x+1) + 2(8x+1)$.

6. The angle A of a triangle ABC is $\frac{2}{3}$ of the angle B which is $2\frac{1}{2}$ times as large as C. Find the magnitude of the angles A, B and C.

7. A man buys 77 horses for Rs. 15,400. He sells 35 at 6% profit on the cost price and 25 at 9% profit ; he loses 4 by disease and sells the rest at cost price. How much does he gain ?

8. Draw a diagram of a box 4 ft. long, 2 ft. wide and 1 ft. 9 in. high. Write down the area of (i) the top (ii) the front (iii) either end (iv) the total area of the outside.

5.

1. In each of the following results find whether the decimal point is in the right place. Give reasons for your answer and show all the working.

i. $800 \div 376 = 21{\cdot}27$. ii. $({\cdot}00125)^2 = {\cdot}00015625$.

2. If the cost of a thousand cocoanuts is Rs. 37-8-0, find the cost of 75 cocoanuts. Make use of the fact that $750 = \frac{3}{4}$ of 1000 and check your answer by deducing the price of 75.

3. The sides of a triangle are 3·5, 4·8 and 5·9 respectively. Find the three angles; also find the length of the prependicular drawn on the side 3·5 from the opposite angle.

4. A reservoir supplies water to the inhabitants of a town m in number. Each inhabitant gets n gallons of water per day for his use. Find the quantity of water consumed in a month by the town.

5. 1 cubic foot of copper is beaten into a thin plate 40 ft. long and 23 ft. wide. Find the thickness of the plate to three places of decimals. How many such plates will give an inch thickness?

6. Two pedestrians start at the same time from two towns 32 miles apart, and each walks at a uniform rate towards the other town and they meet in 3 hours. The faster walks 2 miles an hour more than the slower. Find their rates.

7. A rectangular field is 14 ch. 20 links long and 9 ch. 75 links wide. Find its area in acres and cents.

8. Find a number between 6,000 and 7,000 divisible by 4, 8, 13, 21 and 24.

6.

1. Simplify $(\frac{1}{4}+\frac{1}{5}-\frac{1}{8}) \times (2\frac{2}{3}-\frac{4}{21}) \times (3\frac{2}{7}-7\frac{29}{161}) \div 5\frac{7}{17}$.

2. How many times can a vessel holding ·063825 gal. be filled from a cistern containing 983·465 gal.? What is left over?

3. The following rates for the construction of a wall are given in a schedule :

Each cubic foot of the wall (including labour) Rs. 0-4-0 ;

Plastering 100 sq. ft. of the wall (including labour) Rs. 3-0-0.

What is the cost of building a wall 40 ft. long, 10 ft. high and $1\frac{1}{2}$ ft. thick, according to the schedule rates ?

4. In a triangle one side is 12·4 ft. and the two adjacent angles are 48° and 56°. Find the length of the side opposite the former angle.

5. There are three balls of which the one weighs one-fourth as much again as the second, and the second one-fifth as much as the last, the three together weigh 2 qr. 9 lb. 7 oz. How much do they each weigh ?

6. In a certain municipality the total collections from taxes on houses, including house, water and lighting taxes amount to Rs. a at the rate of Rs. b per house in a certain year. In the next year c rupees are collected under the same item at the rate of d rupees per house, the number of houses remaining the same. What is the equation connecting a, b, c and d ?

7. A rectangular piece of paper is measured in (1) inches and (2) centimetres with the following results : Length—4·37 inches ; 11·09 cm. Breadth—2·79 inches ; 7·08 cm. Find the area of the paper in (1) square inches and (2) square centimetres and hence find, true to the second decimal place, the number of square centimetres in a square inoh.

8. Assuming that the Earth and Jupiter are in a line with the sun (on the same side of the sun) on a certain day, what time will it take them to come to the same position again, their periods of revolution round the sun being 365·256 and 4332·585 days respectively ?

7.

1. The table below gives the average daily duration of daylight at Latitude 13°N for each of the twelve months, with the exception of September.

The mean or average duration for the months January to September inclusive is 12 hours 14 minutes.

(*a*) What is the duration during September ? (*b*) What is the average duration from September to December inclusive ?

AVERAGE DURATION OF DAYLIGHT.			
Month.	Latitude 13°N.	Month.	Latitude 13°N.
January	11 h. 9m.	July	12 h. 52m.
February	11 h. 37m.	August	12 h. 35m.
March	11 h. 52m.	September	......
April	12 h. 20m.	October	11 h. 42m.
May	12 h. 43m.	November	11 h. 18m.
June	12 h. 54m.	December	11 h. 5m.

2. Find the greatest and the least of the fractions, $\frac{8}{13}$, $\frac{12}{23}$, $\frac{31}{55}$, $\frac{43}{88}$ (1) by reducing them to a common denominator, (2) by reducing them to decimals and (3) by reducing them to percentages.

3. State clearly why the answers suggested to the following sums are obviously wrong :

(*a*) 8483×909 *Ans.* 7721·47.

(*b*) Simplify

$$\frac{\frac{1}{5} \text{ of } 1 \text{ Kilometre} - 70 \text{ metres} + 480 \text{ cm.}}{\frac{1}{3} \text{ of } 45 \text{ decm.} + \frac{1}{5} \text{ of } 9 \text{ metres.}}$$ *Ans.* 480 cm.

4. Two roads intersect at an angle of 60°. A town is 2 miles distant from one road and 3 miles distant from the other road. Show by means of a figure that there can be four positions of the town. Also find the distance of any one of them from each of the remaining three.

5. The side of an equilateral triangle is 12 inches. Find the length of the perpendicular drawn from the vertex to the base.

6. There are 4 consecutive integers. The product of the 2 odd integers is equal to that of the 2 even integers increased by 63. Find the numbers.

7. Rangoon candles cost 14 as., whereas Belmont 15 as. 6 ps. per packet of 24. The former burns for 7 hours while the latter does for 6 hours. If a student reads daily 3 hours using candle light, what amount of money can he save in a year by buying the former instead of the latter?

8. A box is 3 ft. long. 2 ft. 6 in. wide and 2 ft. high, (inner measurements). If its height were increased by 8 in., by how much would (1) the surface (2) the volume of the inside be increased?

8.

1. If Re. I=1*s*. $4\frac{1}{2}$*d*., find the value in English money of a lakh of rupees. Check your answer by converting the English money back into rupees.

2. How many revolutions does the second hand of a watch make more than the minute hand in a fortnight? Express this difference in right angles.

3. A room is 30 ft. 1 in. long, 23 ft. 9 in. broad aud 17 ft. 5 in. high. The room is to be stored with cubical packets. What is the least number of such packets and the size of each packet?

4. Divide Rs. 8,475 among A, B, and C such that A may get half as much as B, B half as much again as C.

5. Two places A and B are 15 miles apart. A place C is equidistant from A and B and 8 miles distant from the road leading from A to B. Find the distance of C from A. At what angle do the roads branching from C to A and B intersect?

6. A train starts from a station A to a station B a distance of d miles and performs the journey in t hours. If its rate is r miles per hour, find the relation between d, t and r.

7. The circumference of a circle is 3·1416 times its diameter. If the circumference of a circular lake is 40 miles, find its radius to 4 significant figures.

8. A writing table 3 ft. 6 in. by 5 ft. 6 inches is covered with leather at 10 as. per sq. foot with the exception of an edge $2\frac{1}{2}$ in. wide all round it. Find the cost of the leather to the nearest pie.

9.

1. If 4 metres = 157 inches, find the number of metres in a mile. Give your result to the nearest metre.

2. Add $9\frac{1}{2}$ cwt. to 8·875 qrs. and reduce the result to the decimal of a ton.

3. Find the value of 780 cwt. 2 qr. 26 lb. of sugar at Rs. 12-7-6 per cwt.

4. A man spent $\frac{1}{3}$ of his income on board and lodging, $\frac{1}{5}$ on dress and $\frac{1}{10}$ on other things. If he still had Rs. 21-4-0 left, what was his income to the nearest rupee ?

5. I have two rectangular sheets of paper, one 1 ft. 8 in by 1 ft. 6 in. and the other 1 ft. 6 in. by 1 ft. Suppose they were cut up in a suitable manner and the pieces fitted together (without waste) and a square formed. Find the length of a side of the square.

6. Simplify to three places of decimals

$$\frac{\cdot875\times\cdot073-\cdot629\times\cdot083}{\cdot025\times\cdot03625}.$$

7. Find the area of the hall shown in the accompanying figure, the dimensions being given in feet.

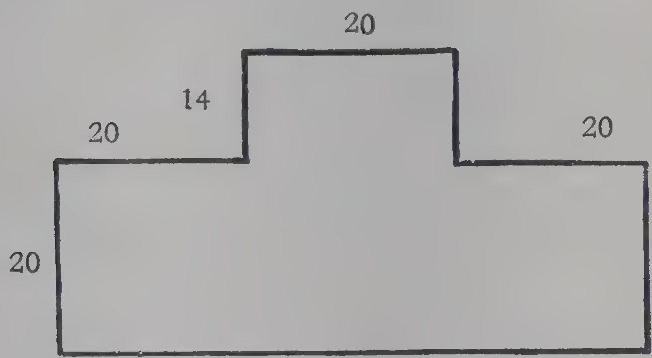


FIG. 149.

8. Draw an equilateral triangle ABC. On the sides AB, BC, CA describe equilateral triangles externally. If A′, B′ and C′ are the vertices of the equilateral triangles opposite to A, B and C, show by measurement that AA′ = BB′ = CC′.

10.

1. If 454 grammes = 1 lb. Av., express 7645 kg. in tons, etc.

2. A rectangular brick-kiln measures 32 ft. in length, 16 ft. in breadth and 12 ft. 3 in. in height. How many bricks measuring 10″ by 5″ by $4\frac{1}{2}$″ will it contain without gaps between them ?

3. The floor of a room 20 ft. 6 in. long and 14 ft. 8 in. wide is covered with carpet 27 in. wide. What area of carpet is wasted if the strips are laid parallel to the length? What is the area of the waste if the strips are laid the other way?

4. Construct a right-angled triangle given that 40° is one of the acute angles and that the hypotenuse is 60 ft. long; and measure the other sides.

5. If there are a beggars each receiving b annas, I find I have 8 annas wanting to pay them; but if there are c beggars each receiving d annas, I find I have 8 annas to spare; what is the equation connecting a, b, c and d?

6. Show that 3 is not a root of the equation $8(x+3)+9(x-2) = 15(x-6)$ and find the root of the above equation.

7. A bankrupt's debts are Rs, 89,350 and the amount for distribution among the creditors is Rs. 42,875-8-0. Find how much a creditor for Rs. 3,050 will receive.

8. Find the cost of 93 kg. 375 gr. of butter at 4 fr. 28 c. per kg.

CHAPTER XVIII.

CONGRUENCE OF TRIANGLES.

§ 157. Two triangles are said to be **congruent** when the sides and angles of one are respectively equal to the sides and angles of the other, *i.e.*, when the two triangles are *equal in all respects.* The following exercise lead to the different sets of conditions for the congruence of two triangles.

Ex. 1.—Make a $\triangle$ ABC having AB = 2·3 in., BC = 1·9 in. and CA = 1·7 in. Make another $\triangle$ A′B′C′ with the same measurements, *i.e.*, have A′B′ = 2·3 in., B′C′ = 1·9 in., and C′A′ = 1·7 in. Measure the angles of the 2$\triangle$s and show that the triangles are equal in all respects.

Ex. 2.—Cut out the $\triangle$ ABC in Exercise 1, place it over the other so that B falls on B′ and BC on B′C′ and see if the $\triangle$s coincide. [$\because$ BC = B′C′, C will fall on C′ and the $\triangle$ ABC will be found to coincide with $\triangle$ A′B′C′.]

From these exercises we conclude :—

If two triangles have the three sides of the one equal to the three sides of the other, each to each, the triangles are congruent ... ... (1)

Ex. 3.—Make a $\triangle$ ABC having AB = 2·3 in., BC = 1·9 in. and $\angle$ B = 36°. Make another $\triangle$ A′B′C′ with the same measurements, *i.e.*, having A′B′ = 2·3 in., B′C′ = 1·9 in, and $\angle$ B′ = 36°. Measure the remaining sides and angles of the two $\triangle$s and show that they are equal in all respects.

Ex. 4.—Cut out the $\triangle$ ABC in Exercise 3, place it over the other so that B falls on B′ and BC on B′C′ and see if the $\triangle$s coincide. [$\because$ $\angle$ B = $\angle$ B′, BA will fall on B′A′ and $\because$ BC = B′C′, C will fall on C′ and $\because$ BA = B′A′, A will fall on A′ and the $\triangle$ ABC will thus be found to coincide with the $\triangle$ A′B′C′.]

From these exercises we conclude :—

If two triangles have two sides of the one equal to two sides of the other and the contained angles also equal, then the two △s are congruent ... ...(2)

Ex. 5.—Construct a △ ABC having AB = 5·4 cm. and angles A = 42° and B 84°. Make another △ A′B′C′ with the same measurements, *i.e.*, having A′B′ = 5·4 cm. and angles A′ and B′ = 42° and 84° respectively. Measure the remaining sides and angles and show that they are equal in all respects.

Ex. 6.—Cut out the △ ABC in Exercise 5, place it over the other so that A falls on A′ and AB on A′B′ and see if the △s coincide. [∵ AB = A′B′, B will fall on B′; and ∵ ∠A = ∠A′, AC will fall on A′C′ and ∵ ∠B = B′, BC will fall on B′C′ and the △ ABC will thus be found to concide with the △ A′B′C′.]

Hence we learn :—

If two triangles have two angles of the one equal to two angles of the other and the sides adjacent to the equal angles equal, then the two triangles are congruent ... ...(3)

Ex. 7.—Construct a △ ABC having AB = 3·7 cm. and A = 64°, C = 100°. Make another △ A′B′C′ with the same measurements, *i.e.*, having A′B′ = 3·7 cm. and ∠A′ = 64° and ∠C′ = 100°. Measure the remaining sides and angles and show that they are equal in all respects.

Ex. 8.—Cut out the triangle ABC in Ex. 7 and place it over the other so that A falls on A′ and AB on A′B′ and see if the △s coincide. [Then the △ ABC will be found to coincide with the △ A′B′C′. The student is recommended to show by reasoning, as in the previous cases, that the △s coincide.]

Hence we learn :—

If two triangles have two angles of the one equal to two angles of the other and the sides opposite to a pair of equal angles equal, the two △s are congruent. ...(4)

Ex. 9.—Make a right-angled △ ABC in which the angle ACB is a right angle, the hypotenuse AB = 4·5 cm. and AC = 2·4 cm. Make another right-angled △ A′B′C′ having ∠C′ a right-

angle and the hypotenuse $A'B' = 4\cdot5$ cm. and $A'C' = 2\cdot4$ cm. and show by measurement that the two triangles are congruent.

Ex. 10.—Cut out the triangle ABC in Ex. 9 and place it over the other so that C falls on C′ and CA on C′A′ and see if the △s coincide. [Then the △ ABC will be found to coincide exactly with the △A′B′C′. The reasoning is left as an exercise to the student].

Hence we learn :—

If two right-angled △s have their hypotenuses equal and a side of the one equal to a side of the other, the triangles are congruent. ...... (5)

Ex. 11.—Make a triangle ABC having $BC = 5\cdot8$ cm., $CA = 7\cdot9$ cm. and $\angle C = 60^0$; make another △ A′B′C′ having $B'C' = 5\cdot8$ cm. $C'A' = 7\cdot9$ cm. and $\angle C' = 50°$. Measure AB and A′B′. Which is the greater ?

Ex. 12.—Repeat the experiment with different values for C and C′.

Hence we learn :—

(1) If two triangles have two sides of the one equal to two sides of the other each to each and the contained angles unequal, the triangle which has the greater contained angle has the greater third side....(6)

From a similar exercise we learn :—

(2) If two triangles have two sides of the one equal to two sides of the other, each to each, and the third sides unequal, the triangle which has the greater third side has the greater contained angle. ...... (7)

§ **153.** Some of these and other propositions about triangles can be inferred from the following experiments :

Take AB, AC two narrow strips of cardboard. Join them at A with a paper fastener, and pierce small holes at B and C so that a thin piece of string can pass through. Knot one end of a string, pass it through C and the free end through B (Fig. 150). Now attach a small weight to the free end to keep the string tight. You thus get a △

ABC with two sides of fixed length ; the other side and angles you can alter at will.

Exp. 1. Now begin by placing AB along AC (Fig. 151); keep AC fixed in position and rotate AB as

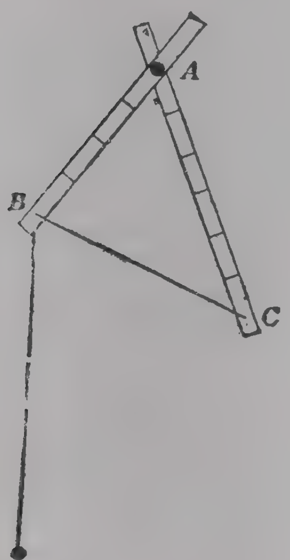


FIG. 150.

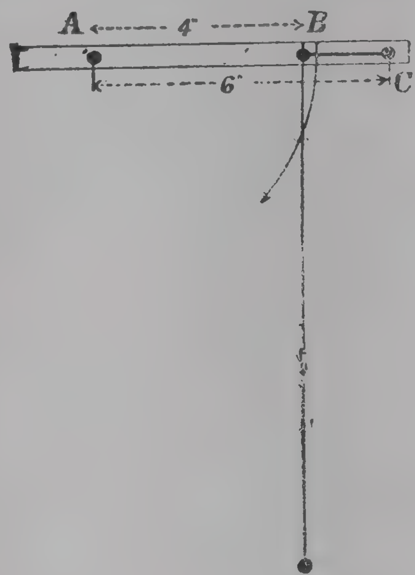


FIG. 151.

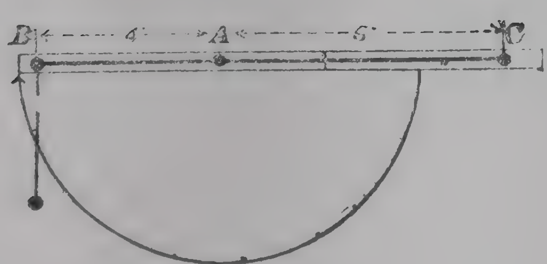


FIG. 152.

indicated by the arrow head until it comes in a line with AC as in (Fig. 152).

(Fig. 150) is an intermediate position.

As you rotate AB notice the length of the string between B and C and also the size of ∠A in the various positions.

Assuming AB and AC to be 4 in. and 6 in. respectively fill up the following :—

(1) As AB turns from the first to its last position, the length BC continually increases from......to.....

(2) At the same time $\angle$A increases from. . .to. . . .

If AB and AC be 8″ and 5″, between what limits does BC lie?

Exp. 2. How many positions can AB have in Exp. 1? How many triangles can you have with two sides fixed in length?

Now fix on some $\angle$ (say, 60°) for A and turn AB as before, and observe how often this angle of 60° occurs at A.

How many different triangles can you have with two sides fixed and an included angle of 60°?

Repeat the experiment with 90° taken for A, 45° taken for A and so on.

How many different $\triangle$s can you have with two given sides and a given included angle?

If, therefore, you had two $\triangle$s having two sides and the included angle in one equal to two sides and the included angle in the other, what further could you say about the triangles?

Also if you had two $\triangle$s having two sides of the one equal to two sides of the other and the included $\angle$ of the first *greater* than that of the second, what could you say about their bases or third sides?

Exp. 3. Place AB along AC as in (Fig. 151); fix on some length (say, 5″) for BC and turn AB as before and observe how often this length of 5″ occurs for BC.

How many different $\triangle$s can you have with two sides 4″ and 6″ and the base 5″?

Repeat the experiment with 8″ for BC, 7″ for BC, etc. How many different $\triangle$s can you have with two given sides and a given base?

If, therefore, you had two $\triangle$s having the three sides of the one equal to the three sides of the other, what further could you say about the $\triangle$s?

Also if you had two $\triangle$s having two sides of the one equal to two sides of the other and the base of the first *greater* than the base of the second, what could you say about the *vertical angles*, (*i.e.*, angles opposite the bases)?

Exercise XVIII (a).

1. Two sides of a $\triangle$ are 9 cm. and 6 cm. respectively. Find the limits between which the third side should be.

2. ABC is an isosceles triangle having AB=AC. From B and C draw BD and CE $\perp$ to AC and AB respectively, measure BD and CE and show that BD=CE. Prove this result using 'Congruence of $\triangle$s.'

3. Using the same property prove that the perpendiculars from the vertices of an equilateral triangle on the opposite sides are equal to one another.

4. Draw any angle ABC. Bisect the $\angle$ B; and from any point D on the bisector, draw DE, DF $\perp$ to BA, BC respectively. Show by measurement that DE=DF. Also prove this result using 'Congruence of $\triangle$s.' Since D is *any* point on the bisector, learn that "**every point on the bisector of an angle is equally distant from the arms of the angle**.

5. Two straight roads are inclined at an angle of 40°. A man walks in the field between them so as always to be equidistant from the two roads. What is his locus? Draw a figure to illustrate your answer.

6. Draw a $\triangle$ ABC. Bisect $\angle$s B and C by BI and CI meeting at I. From I draw ID, IE, IF $\perp$ to BC, CA, AB respectively. Show by measurement that ID=IE=IF and that AI bisects $\angle$ A. Prove these results using 'Congruence of $\triangle$s.' Hence learn that **the bisectors of the angles of a triangle are concurrent,** i.e., **meet in a point**.

7. Draw a triangle ABC. Bisect the sides BC, CA, at right angles by DO, EO meeting at O. Show by measurement that AO=BO=CO. Prove this result using 'Congruence of $\triangle$s.'

8. If, in exercise 7, F be the middle point of AB, show that OF is $\perp$ to AB. Hence learn that **the perpendicular bisec-**

tors of the sides of a △ are concurrent. The point of concurrence is the circumcentre of the △. (*Vide* page 299).

9. Draw any $\|^m$ ABCD. Show by measurement that AB = CD and BC=AD. Prove these results by drawing a diagonal and using 'Congruence of △s.'

10. If, the diagonals of the ∥ *m* in exercise 9, intersect at E, show that the diagonals bisect each other.

11. ABCD is a parallelogram and O the point of intersection of the diagonals. Show that the triangles ABO and CDO are equal in all respects. What properties of parallelograms and what proposition on congruence of triangles do you use?

12. Draw any △ABC. Bisect AB at D and draw DE ∥ to BC. Show by measurement that AE=EC. Prove this result by drawing EF ∥ AB and using 'Congruence of △s.' Similarly prove the proposition given in bold type on page 329.

13. In exercise 12, show by reasoning or measurement that EF bisects BC and DE = ½ BC.

14. The following method for measuring the breadth of a

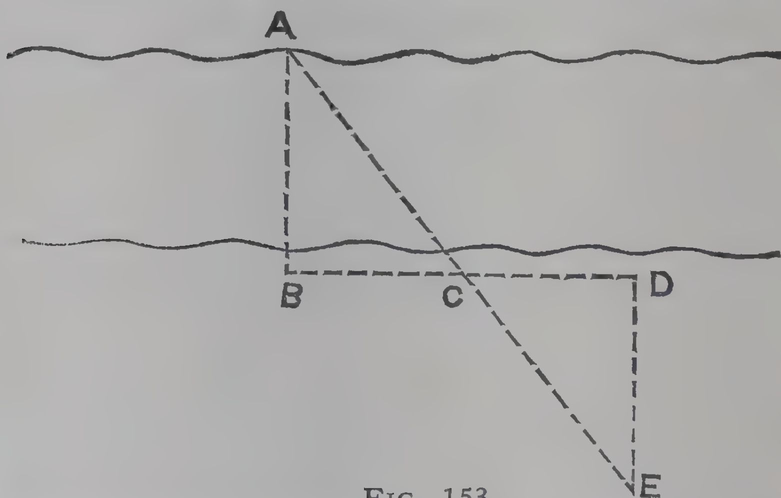


FIG. 153.

river without crossing it is given: "Suppose there is some object on the edge of a bank at A. From B exactly opposite to A, measure two equal lengths BC, CD along a line at right angles to the direction of BA. From D walk up to E at right angles to BD until you see C and A in the same line. Then the breadth of the river AB = DE." What proposition on the congruence of △s do you use?

15. A line AB cannot be directly measured. To find the distance AB, take any point O which is accessible from both A and B. Produce AO and BO to C and D so that AO = OC and BO = OD. Join CD. Then CD = AB. What proposition on congruence of triangles do you apply?

16. Two ladders of equal length are placed with their feet resting on the ground at the same distance from the wall. Show that they reach the same height on the wall. What proposition on congruence of triangles do you use?

§ **159. To construct an angle** (with ruler and compasses only) **at a given point** (D) **in a given line** (DE) **equal to a given angle** (BAC).

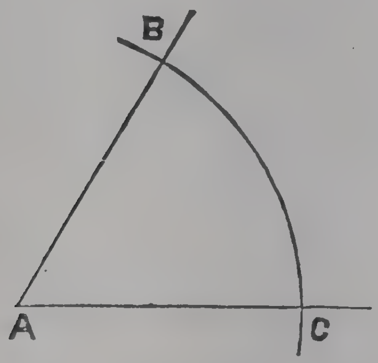


FIG. 183.

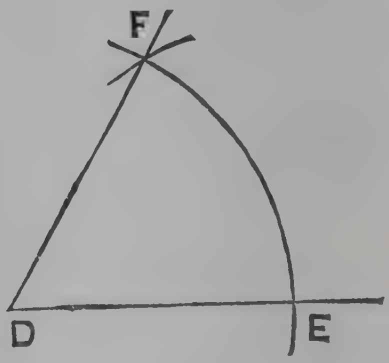


FIG. 184.

With centre A and any length as radius draw a circle cutting the arms of the angle BAC at B and C (Fig. 183).

With centre D and the same radius draw a circle cutting DE at E.

Alter your compasses so as to step off the distance CB, then with E as centre and this length as radius describe a circle cutting the second circle at F. Join DF.

Now measure both the angles CAB and EDF and you will find them to be equal.

Prove this using 'Congruence of △s.'

§ **160. Through a given point** (C) **to draw** (with ruler and compasses only) **a line parallel to a given straight line** (AB).

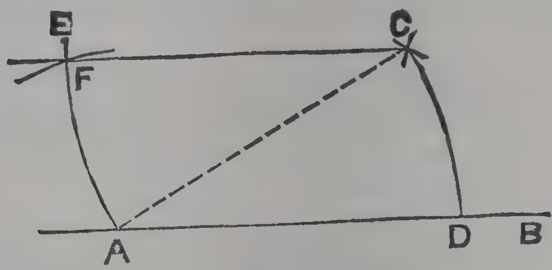


FIG. 185.

With A (any point in AB) as centre and AC as radius draw an arc cutting AB at D.

With C as centre and with the same radius CA draw the arc AE. Take the distance CD in your compasses and with A as centre draw an arc cutting the arc AE at F. Join CF. Now test whether CF is || to AB.

Prove, using 'Congruence of △s' that the alternate ∠s CAD and ACF are equal and hence that AD is || to CF.

Exercise XVIII (b).

1. Draw an angle of 78° with your protractor. With ruler and compasses only, construct an equal angle; and test your drawing by measuring with the protractor.

2. Repeat Exercise 1, with different angles.

3. Draw an angle of 32°. With ruler and oompasses only, construct another angle *four* times the angle you have constructed. Test with the protractor.

4. Draw a circle of any radius with centre C; step off this radius from A to B on the circumference. Join CA, CB. How many degrees are there in the angle ACB? Give reasons for your answer and test by measurement.

5. Draw a right angle ACB. Making use of the construction in Exercise 4, divide the right angle into three equal parts.

6. Take two points A and B 5 cm. apart. Through A draw any line and through B draw a line parallel to the first.

7. Draw a line PQ 2 in. in length. With your protractor draw PR making an angle of 58° with PQ. Through Q draw a line || to PR (with ruler and compa sses only.)

8. Repeat Exercise 7 making PQ 8 cm. long and the angle QPR equal to 63°.

9. Draw a straight line AB of length 3 in. Find a point C 3 in. distant from each of the points A and B (using only the ruler and compasses).

10. Draw a line AB of length 5 cm. Bisect it at C and draw AX, BY, CZ || to any line (using the ruler and compasses only). Now draw any line across the parallels cutting them at L, M and N. Measure and compare LN and NM.

11. On a straight line 5 cm. long describe an equilateral triangle.

12. Draw the triangle in the previous example using only your protractor and ruler.

§ **161. Chords of a circle.**—Describe a circle of radius 2 inches and taking, in your compasses, a distance of one inch, step off round the circumference and join the points successively. The lengths you have been stepping off are each equal to one inch. From the centre draw perpendiculars on these chords, and measure the perpendicular distances. What do you notice about the lengths of these perpendiculars?

Repeat the experiment with a circle whose radius is 6·4 cm. and with chords each 3 cm. long. What do you infer?

Hence learn that **if chords of a circle are equal their perpendicular distances from the centre are also equal.** Prove this result using 'Congruence of △s.'

Again describe a circle of 3 inches radius. Draw several radii. At points on these radii distant one inch from the centre draw chords at right angles to those radii. Measure the chords and compare their lengths. Repeat the experiment with a circle whose radius is 8 cm. and chords each 3 cm. distant from the centre. What do you notice about the lengths of the chords so drawn? Hence learn that **if chords of a circle are equally distant from the centre they are equal.**

Draw any two chords AB and CD of unequal lengths in a circle. Draw perpendiculars from the centre. Measure and fill up the following table :

	Length of the chord.	The perpendicular distance from the centre.
Greater chord.		
Smaller chord.		

Which has the greater perpendicular distance, the greater chord or the lesser chord ? Repeat the experiment with half a dozen pairs of unequal chords filling up the tabular form in each case. Examine in each case as to which chord has the greater perpendicular distance. Hence learn that **if one chord of a circle is greater than another the perpendicular distance of the centre from the greater chord is less than that from the smaller chord ;** *conversely* **if the perpendicular distance of one chord from the centre is less than that of another chord, i.e., if one chord is nearer the centre than another, the length of the nearer chord is greater than the other.**

The pupil may be led to these truths by placing a pencil across a circle in the position of a diameter, and making them observe the length of the part of the pencil intercepted by the circumference as the pencil is moved away from the centre.

Also in the case of the equal chords in a circle, measure the angles which they subtend at the centre. You will find they are equal. Prove this using 'Congruence of $\triangle$s.' Hence learn that **equal chords in a circle subtend equal angles at the centre** and *conversely* **equal angles at the centre of a circle subtend equal chords.**

§ 162. Regular hexagons and octagons.

With O as centre and any length as radius describe a circle; then starting from a point A, step off lengths equal to the radius round the circumference. You will find that after stepping off *six* lengths you come back to the starting point. Join these points successively. The figure you get is called a **hexagon** because it has *six* sides.

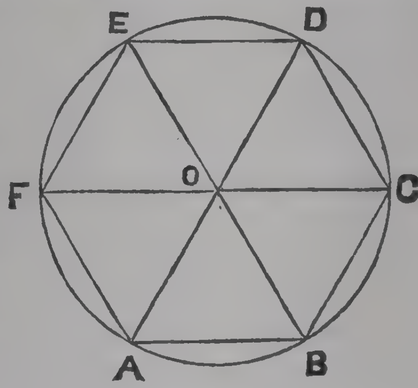


FIG. 186.

Join OA, OB, OC, OD, OE, OF. Measure the six angles at O you will find they are equal each being 60°.

Give a reason why they should be equal, and show that each of them should be $\frac{1}{6}$ of 4 right angles, *i.e.* 60°.

What kind of a triangle is AOB? And what is the size of ∠OAB? Test your answer by measurement. What is the size of the angle BAF (or any angle of this hexagon)? Thus it will be found that *this hexagon* has all its sides equal and all its angles equal and is hence said to be **regular.**

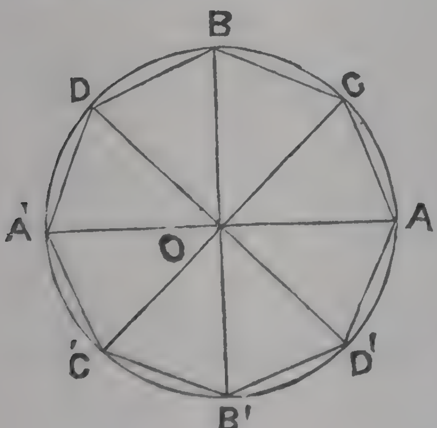


FIG. 187.

With O as centre and any length as radius describe a circle. Draw two diameters AOA′, BOB′ cutting at right-angles. Draw two diameters COC′, DOD′ bisecting the angles AOB, AOB′. Join successively the points AC, CB, BD, &c. The figure you get is called an **octagon**

because it has *eight* sides. Measure the lengths of the sides. You will find they are all equal. Prove this by using 'Congruence of $\triangle$s.'

What is the magnitude of each of the angles AOC COB, &c., at O? Hence what is the magnitude of each of the angles OAC, OCA, OCB, OBC, &c.? Give your answer also in degrees. What is the magnitude of each of the angles ACB, CBD, &c. In this *octagon*, you will find that the sides are all equal and the angles too all equal. Hence it is said to be **regular.**

Exercise XVIII (c).

1. Describe a circle of radius 5 cm. Place within it two chords each 6 cm. long. Draw $\perp$s to the chords from the centre and measure their lengths.

2. Repeat Exercise 1 with chords 8 cm. long.

3. In a circle of radius 5 cm. place two chords one 7 cm. long and another 5 cm. long. Compare their distances from the centre.

4. Describe a circle of radius 2 in. and in it describe a hexagon as in Art. (162). Measure its angles. What is the length of each side?

5. On a given straight line of length 4 cm., show how to describe a regular hexagon.

6. Repeat Exercise 5 on a line $1\frac{1}{2}$ in. long.

7. Describe any regular hexagon and join the angular points alternately. What kind of $\triangle$ do you get?

8. Describe a circle of radius 3 in. and in it describe a regular octagon. Measure the angles of the octagon (express the measure in degrees).

9. On a given straight line of length 1·5 in., show how to describe a regular octagon. Repeat the exercise on a line 3 cm. long.

10. Describe any regular octagon. Join alternately the angular points. What kind of a figure do you get?

Exercise XVIII (d).

1. Construct a triangle ABC having $a=b=2{\cdot}5''$ and $c=1$ inch. Measure the perpendicular AF drawn from A on BC. Take any point P in AB. Draw PD and PE $\perp$ to AC and CB respectively. Find PD + PE and show that AF = PD + PE.

2. Take any other point P′ in AB and draw P′D′ and P′E′ $\perp$ to AC and CB respectively and find P′D′ + P′E′.

3. Repeat the experiment with half a dozen points in AB. What do you infer?

4. Take the point P in AB produced. As before from P draw PD and PE $\perp$ to AC and BC respectively. Find PD $\sim$ PE.

5. Repeat experiment 4 with half a dozen points in AB produced. What inference do you draw?

6. Construct an equilateral triangle ABC having $a=b=c$ = 5 cm. Take any point P in the $\triangle$. From P draw perpendiculars PD, PE, PF to the sides BC, CA and AB respectively. Draw AQ perpendicular to BC. Measure also PD + PE + PF and show that AQ = PD + PE + PF.

7. Two ships start from a port and sail at the rate of 15 and 20 knots per hour respectively in two directions inclined to one another at an angle of 30°. How far will they be apart at the end of 6 hours? (Take 1 cm. = 15 knots).

8. There is a tower in the centre of a circular tank whose radius is known to be 100 yards. From the middle of the tower the bearings of two points on the bank are 40° S of W and 10° N of W. Find the distance between the points.

9. 3 persons A, B and C start from the same place and walk in the directions N, N-E and E respectively. At the end of 2 hours they are in a straight line. If A and B walk at 4 and 2 miles an hour, find C's rate.

10. A man wants to find the distance between two objects A and B which cannot be directly measured. He selects a place P from which PA, PB and the angle APB can be measured. If PA = 2 furlongs, PB = 4 furlongs and APB = 60°, find approximately the distance from A to B.

11. A man observes a tower in the direction of 30° E of N. After walking 6 miles due East he finds the tower to be North-West of him. Find the distance of the tower from the two places of observation.

12. A, B, C are three forts such that AB subtends an angle of 30° at C and BC subtends an angle of 60° at A. If AB = 1 mile, find the distance of A and B from C.

13. A place A is due South-East of B and at a distance of 3 miles from it, C is a place due East of B and N-E of A. Find the distance of C from A and B.

14. Draw a triangle ABC, bisect the sides BC, CA, AB at D, E, F. Join AD, BE, CF. (These are called the medians of the Δ ; **Medians** are lines drawn from the vertices to the middle points of the opposite sides). What do you notice about AD, BE and CF ?

15. Repeat experiment 14 with half a dozen triangles. What do you infer ?

Hence learn that **the medians of a triangle are concurrent and the point of concurrence is called the centroid of the triangle.**

16. Draw a triangle ABC and from A, B, C draw AD, BE and CF perpendicular to the opposite sides. What do you notice about AD, BE and CF ?

17. Repeat the experiment with half a dozen triangles. What do you infer ?

Hence learn that **the perpendiculars of a triangle are concurrent and the point of concurrence is called the orthocentre of the triangle.**

18. ABC is a triangle. AB and AC are produced to D and E. Bisect the exterior angles at B and C and the angle A. What do you notice about the bisectors ?

19. In experiment 18 bisect the exterior angles at A and C and the angle B. What do you notice about the bisectors ?

20. In experiment 18 bisect the exterior angles A and B and the angle at C. What do you notice about the bisectors of the angles ?

21. Repeat experiments 18, 19 and 20 with another △. What do you infer?

Hence learn that **the external bisectors of any two angles of a triangle and the internal bisector of the third angle are concurrent.**

22. Draw (1) a right-angled, (2) an obtuse-angled, and (3) an acute-angled triangle. Find the orthocentre in each case. Note the position of the orthocentre with respect to the △.

23. Repeat experiment 22 with 3 sets each consisting of 3 different kinds of triangles. What do you infer?

Hence learn that **the orthocentre falls within the △ in an acute-angled △, without the △ in an obtuse-angled △ and on the vertex in a right-angled triangle.**

24. Obtain (with ruler and compasses only) the angles of $157\frac{1}{2}°$, $270°$, $315°$, $300°$. All these angles should be made in one figure and should be examined with the protractor.

25. A hidden treasure is equidistant from two given trees 10 yds. apart. What do you know about its position? Suppose it is at a distance of 8 yards from another tree, draw a figure to show the exact position of the treasure.

26. If the hidden treasure is in a line with two other trees, draw a figure to show how you can determine the exact position of the treasure.

27. If a hidden treasure is equidistant from 3 given trees, how will you find its position?

28. Describe a circle of 6 cm. radius and place in it 5 chords each = 3 cm. Find the locus of the middle points of these chords.

29. Repeat the experiment with a circle 2·9 in. radius and 6 chords each = 2·9 in. What do you infer? Hence learn **that the locus is a concentric circle.**

30. AB, AC are two equal chords of a circle. Show that the bisector of the angle BAC is a diameter (1) by paper folding, (2) by reasoning.

CHAPTER XIX.

GRAPHS.

§ **163. Variables and constants.** A quantity which has not always the same value is called a **variable**; *e.g.*, the population of Madras, the strength of a school are variables; whereas a quantity which has always the same value is called a **constant**; *e.g.*, the number of annas in the rupee, the length denoted by a yard are constants.

It is often found that one quantity is related to another in such a way that if a change is made in the value of one, there is a corresponding change in the value of the other. For example, take the case of a man walking. In half an hour he walks a certain distance, in $\frac{3}{4}$ of an hour he walks another distance and so on. Thus the distance walked and the time taken for walking, are two quantities related to one another so that if you change one, the other is correspondingly changed.

When two quantities are so connected, the relation between them can be exhibited clearly on a diagram drawn on squared paper by representing by points, as shown below, pairs of corresponding values of the quantities and drawing a line through those points. Such a line is called **a graph**.

§ **164. Representation of pairs of numbers by points.** In articles (13) and (26) you have seen how numbers (positive or negative) are represented by lengths taken in one line; positive numbers by lengths taken in one direction and negative numbers by lengths taken in the opposite direction.

When pairs of numbers, such as the corresponding values of two quantities related to one another as described above, are to be so represented we proceed as follows :—

Take two lines XOX′, YOY′ intersecting at right angles at O as in Fig. (188); and choose a convenient length to represent the unit, say, $\frac{1}{10}$ of an inch as in Fig. (188); then measure off lengths from O along XX′ to represent the values of one variable (along OX for positive values and along OX′ for negative values) and along YY′ to represent the values of the other variable (along OY for positive values and along OY′ for negative values).

Now to represent any one pair of corresponding values, say, (+ 8, + 5); through the extremity of the length taken in XX′ to represent + 8, draw a line parallel to YY′, and through the extremity of the length taken in YY′ to represent + 5, draw a line parallel to XX′; the point P_1 (Fig. 188) where these two lines meet represents the pair of

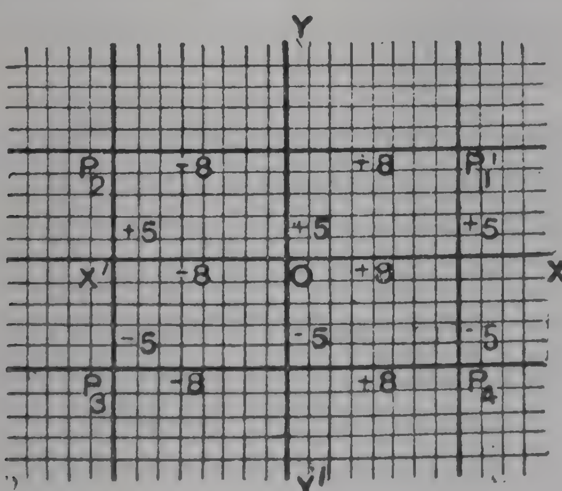


FIG. 188.

numbers (+ 8, + 5) and is generally called the point (+ 8, + 5).

Generally to represent *any pair* (a, b); measure off a length from O in XX′ to represent a (to the right of O

if a be $+^{ve}$ and to the left of O if a be $-^{ve}$) and through the extremity of that length draw a line parallel to YY′; and measure off a length from O in YY′ to represent b (upwards from O if b be $+^{ve}$ and downwards from O if b be $-^{ve}$) and through the extremity of that length draw a line parallel to XX′; the point P where these two lines meet represents the pair (a, b) and is generally called the point (a, b).

Thus the points P_2, P_3, P_4 of Fig. (188) respectively represents the pairs of numbers $(-8, +5)$, $(-8, -5)$, $(+8, -5)$.

In this method of representation note that *corresponding to every pair of numbers there is one point and only one.*

Also note that the pair (a, b) is different from the pair (b, a) and the points representing these are consequently different.

Further *it is not essential that the unit of length chosen for measurement along* YY′ *should be the same as that chosen for measurement along* XX′.

The lines XOX′, YOY′ are called **the lines of reference** or **the axes of co-ordinates**; XOX′ being called the X axis and YOY′ the Y axis. The point O is called the **origin**. The pair of numbers represented by a point or the corresponding lengths measured on the axes are called the **co-ordinates** of the point; the co-ordinate on the line XX′ being called the **X co-ordinate** or the **abscissa** of the point and that on the line YY′ the **Y co-ordinate** or the **ordinate** of the point. Thus in Fig. (188) the **abscissa** of the point P_2 is -8 and the ordinate is $+5$.

Generally the + sign is ommitted before a positive number, *i.e.*, when no sign is given before a number, it is taken as

positive. Thus the co-ordinates of P_1 (Fig. 188) are generally written (8, 5) those of P_2 (—8, 5) and those of P_4 (8,—5).

The process of marking a point by means of its co-ordinates is called **plotting the point.** Note that the co-ordinates are always given in a definite order, first the abscissa and then the ordinate.

Marking or describing the position of any place on a map is based on the same principle. The two lines on which we are to measure the distances are the Equator and the Meridian of Greenwich and the distances themselves are called the *longitude* and the *latitude.*

Example 1.—Plot the points (1) (3,5), (2) (—1, 2), (3) (—3, —4), (4) (4, —5) and find the distance between the first two.

Take $\frac{1}{10}''$ to represent the unit.

(1) Take ON 3 units along OX to the right, and OM 5 units upwards along OY and draw NP and MP parallel to OY and OX respectively. The point P where these meet is the point (3, 5). Since NP is equal and parallel to OM, we reach the same point P if we first measure ON 3 units along OX, and thence NP, 5 units upwards and at right angles to XX′. Also NP is called the **ordinate** of the point P.

(2) Measure 1 unit along OX′ since the abscissa —1 is negative and thence 2 units upwards and at right angles to XX′. The resulting point Q is the point(—1,2).

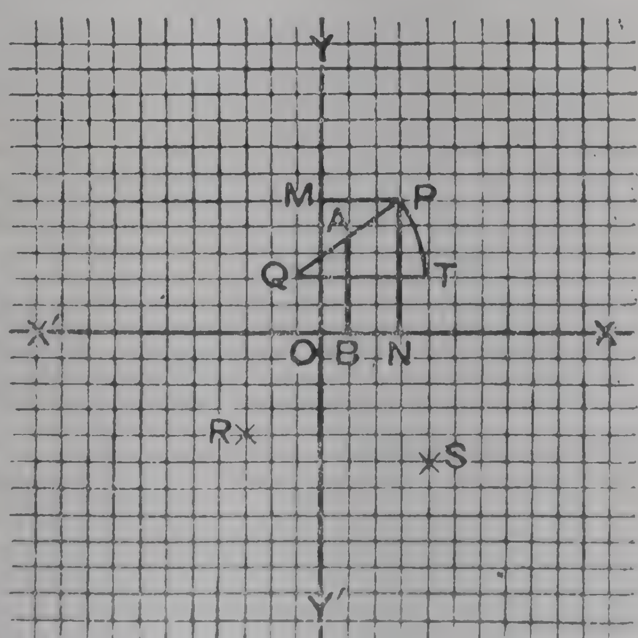


FIG. 189.

(3) Measure 3 units along OX′ since the abscissa -3 is negative and thence 4 units *downwards* and at right angles to XX′ since the ordinate -4 is negative. The resulting point R is the point $(-3, -4)$.

(4) Similar constructions will lead to the point S which represents $(4, -5)$.

To find the distance between Q and P, draw an arc of a circle with centre Q and radius QP. Let this arc cut the horizontal line through Q in T. Then QT = QP. But QT = 5 divisions or 5 units. $\therefore$ QP is also 5 units.

Also given any point P, you can find its co-ordinates referred to the lines XOX′, YOY′ intersecting at right-angles at O.

From P draw PN perpendicular to XOX′. Then ON is its abscissa and NP is its ordinate.

ON is $+^{ve}$ or $-^{ve}$ according as N falls on OX or OX′, *i.e.*, to the right or to the left of O.

NP is $+^{ve}$ or $-^{ve}$ according as it is above or below the line XX′.

The lines XOX′, YOY′ divide the paper into four parts called **quadrants.** The part included between OX and OY is called the *first* quadrant; that between OY and OX′ the *second* quadrant; that between OX′ and OY′ the *third* quadrant and that between OY′ and OX the *fourth* quadrant.

Thus the abscissa of any point is positive if the point be in the first or the fourth quadrant and negative if in the 2nd or the 3rd quadrant; and its ordinate is positive if it be in the 1st or the 2nd quadrant and negative if in the 3rd or the 4th quadrant.

Example 2.—Find the co-ordinates of the middle point of the line joining the points (1) and (2) of Example 1.

Let A be the middle point, draw AB perpendicular to XX′. Then OB is the abscissa and AB is the ordinate. OB=1 division and is to the right of O. ∴ the abscissa=1. AB=3·5 divisions and is above XX′. ∴ the ordinate=3·5.

Exercise XIX (a).

1. Plot the following pairs of points on squared paper and draw the line joining each pair :

(*a*) (3, 3), (6, 2). (*b*) (7, 2), (– 3, 5).
(*c*) (8, 4), (5, –3). (*d*) (2, –2), (–6, –8).
(*e*) (0, 10), (10, 0.) (*f*) (10, 12), (–3, 5).

2. Two roads meet at right angles. Draw a figure showing the positions of the following :—

(1) A tower distant 100 yards from the first road and 80 yards from the second.

(2) A well, distant 60 feet from the first and 90 feet from the second.

3. Find the lengths of the lines joining the following pairs of points.

(*a*) (4, 3), (6, 8). (*b*) (8, 9), (1, 2).
(*c*) (0, 0), (5, 6). (*d*) (–10, 3), (3, –10).
(*e*) (8, 4), (3, 6). (*f*) (4·5, 5), (–6·5, 7).

4. Plot the following points and find their distances from the origin :

(*a*) (6, 3). (*b*) (3, 8). (*c*) (–4, –9). (*d*) (–5, 6).
(*e*) (8, –10). (*f*) (–6·5, –4·5).

5. Show that the distance between the first pair of points = the distance between the second pair.

(*a*) (8, 10), (6, 9) ; (10, 8), (9, 6).
(*b*) (0, 8), (6, 4) ; (8, 0), (4, 6).
(*c*) (10, 7), (8, 4) ; (12, 9), (10, 6).

6. (*a*) Show that the points (8, 4) and (–8 –4) are equidistant from the origin, and also the points (9, 12) (–9, –12).

(*b*) Find the co-ordinates of the points of intersection of the straight lines joining :

(1) (1, 2), (9, 11) and (−2, 1), (−7, −11).

(2) (0, 0), (6, 8) and (0, 2) and (3, 0).

7. Find the length of the perpendicular drawn from (5, 6) on the line joining the points (8, 12) and (14, 16).

8. Show that the following points are in the same line :

(*a*) (0, 0), (8, 4), (−8, −4). (*b*) (0, 2), (2, 8), (5, 17).

9. Show that the points (5, 5), (5, −5), (−5, 5) and (−5, −5) are the angular points of a square.

10. Show that the points (8, 6), (−8, 6), (8, −6), (−8, −6) are the angular points of a rectangle.

11. Find the co-ordinates of the points of intersection of the diagonals in Ex. (9) and Ex. (10).

12. (*a*) Plot the points (0,5), (5,0), (4,3), (3,4) and show that they lie on the circumference of a circle whose centre is the origin and measure its radius.

(*b*) Show that −3,10 ; 10,−3 ; 2, 9 ; 9, 2 lie on the circumference of a circle whose centre is −3, −3. Find the radius of the circle.

§ 165. Statistical graphs.

Example 1.—A man starts on an 8 hours' walking match. The following are the distances in miles he traverses at the end of successive hours :

$$5, 10, 14\tfrac{1}{2}, 19, 23, 26\tfrac{1}{2}, 29, 31.$$

Draw a graph showing the relation between the distance walked and the time taken to walk that distance.

Take two lines OX, OY. Represent the times by lengths measured along OX and the distances by lengths measured along OY. Let one small division or $\frac{1}{10}''$ along OX represent quarter of an hour and $\frac{1}{10}''$ along OY a distance of one mile. Just when he started, the time lapsed is zero and the distance walked is also zero. The successive pairs of values are (0 hour, 0 mile), (1 hr. 5 mls.), (2 hrs., 10 mls.), (3 hrs., $14\frac{1}{2}$ mls.), &c , and the corresponding points will be (0, 0), (4, 5), (8, 10), (12, $14\frac{1}{2}$), as in the figure.

A line freely drawn through these points is the required

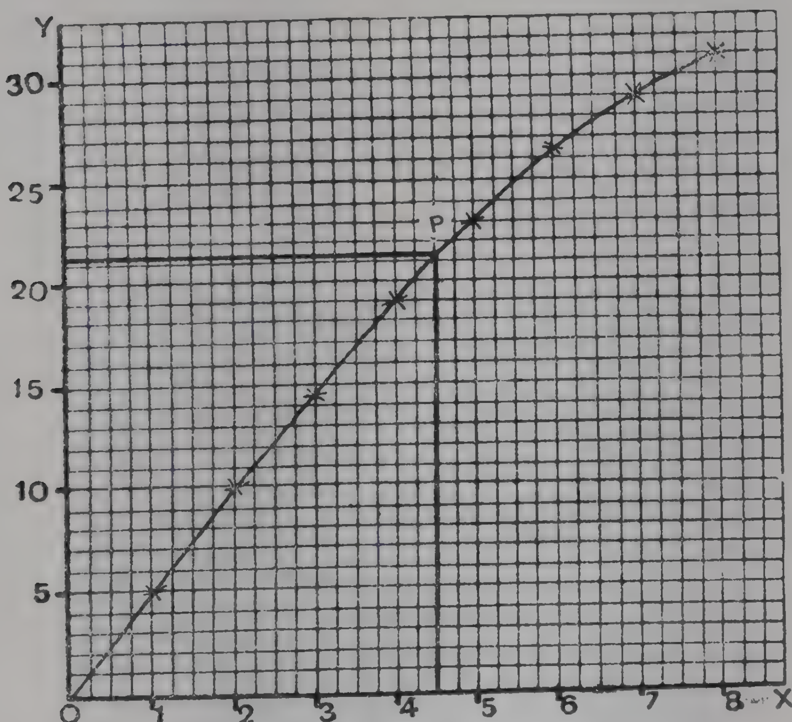


FIG. 161.

graph. The graph shows how the distance is steadily increasing with the time, and also from the slope of the curve you can see that he walks fast in the beginning, and gradually his rate of walking decreases. Also you can approximately find the distance walked in any time; (say, $4\frac{1}{2}$ hours). Take the length along OX corresponding to $4\frac{1}{2}$ hours, *i.e.*, 18 divisions, and there erect a perpendicular meeting the graph at P. Draw the horizontal line through P and note where it meets OY. It meets OY at a point distant a little over 21 divisions from O. ∴ the corresponding distance walked is slightly over 21 miles.

This method of obtaining information from the graph at the intermediate stages for which data are not furnished is called **Interpolation.**

Ex. 2.—In a class at school there were present on Monday morning 39 pupils, afternoon 37; on Tuesday

morning 39, afternoon 40; on Wednesday morning 35, afternoon 38; on Thursday morning 37, afternoon 37; and on Friday morning 36, afternoon 32. Draw the graph of the attendance.

The attendance either in the morning or the evening may be denoted by a point; thus from the given data we may plot 10 points.

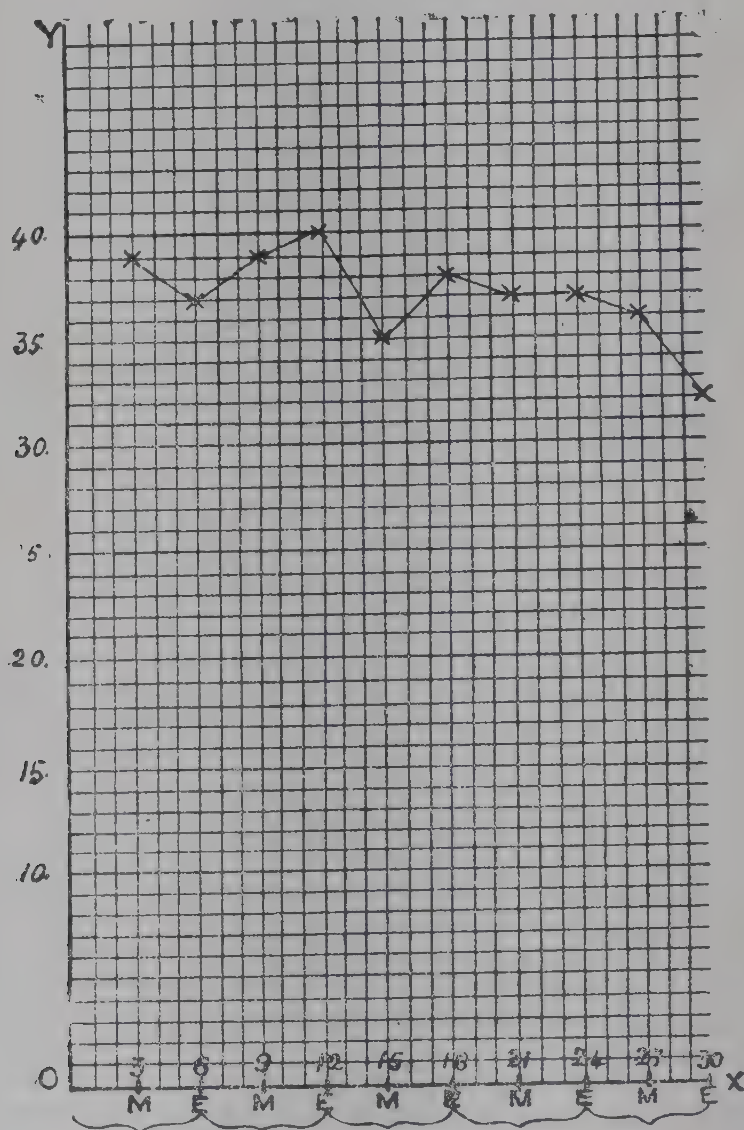


FIG. 162.

Draw the axes OX and OY at right angles. Represent the morning and evening on the X axis taking 3 divisions for every morning and 3 divisions for every evening. Represent the number present on the Y axis taking 1 division for each boy present. Then we get the following points :—

(1)	Monday Morning,	39 boys,	*i.e.*,	(3, 39.)
(2)	,, Evening,	37 ,,	*i.e.*,	(6, 37.)
(3)	Tuesday Morning,	39 ,,	*i.e.*,	(9, 39.)
(4)	,, Evening,	40 ,,	*i.e.*,	(12, 40.)
(5)	Wednesday Morning,	35 ,,	*i.e.*,	(15, 35.)
(6)	,, Evening,	38 ,,	*i.e.*,	(18, 38.)
(7)	Thursday Morning,	37 ,,	*i.e.*,	(21, 37.)
(8)	,, Evening,	37 ,,	*i.e.*,	(24, 37.)
(9)	Friday Morning,	36 ,,	*i.e.*,	(27, 36.)
(10)	,, Evening,	32 ,,	*i.e.*,	(30, 32.)

Join the points thus marked and the required graph is completed. The figure exhibits ascent and descent of slope corresponding to rise and fall in attendance respectively. Thus the graph shows the rise and fall in the attendance much more clearly than the mere numbers. The level indicates that there is neither rise nor fall (as on Thursday morning and evening).

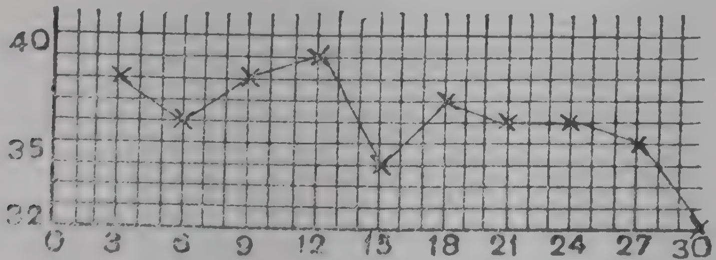


FIG. 163.

We see from the figure that no part of the graph lies below the line drawn through 32 on the Y axis which indicates

a waste of squared paper. To avoid this waste, at O on the Y axis, instead of beginning with zero for the number of boys we may start with the lowest number present, *viz.*, 32. The effect will be that the graph will be lowered as shown in (Fig. 163) and so much squared paper saved.

Ex. 3.—The following table gives the temperature of a fever patient at the several hours of observation on a certain day :—

Time ...	7 A.M.	7-30	12 NOON.	4 P.M.	8 P.M.	12 MID-NIGHT.	4 A.M.
Temperature.	104·6	104·8	105·4	105·6	104·4	104·1	104·6

Draw a graph showing the variation in the temperature.

Since nearly one whole day or 24 hrs. is to be represented you take one small division along OX to represent 1 hour

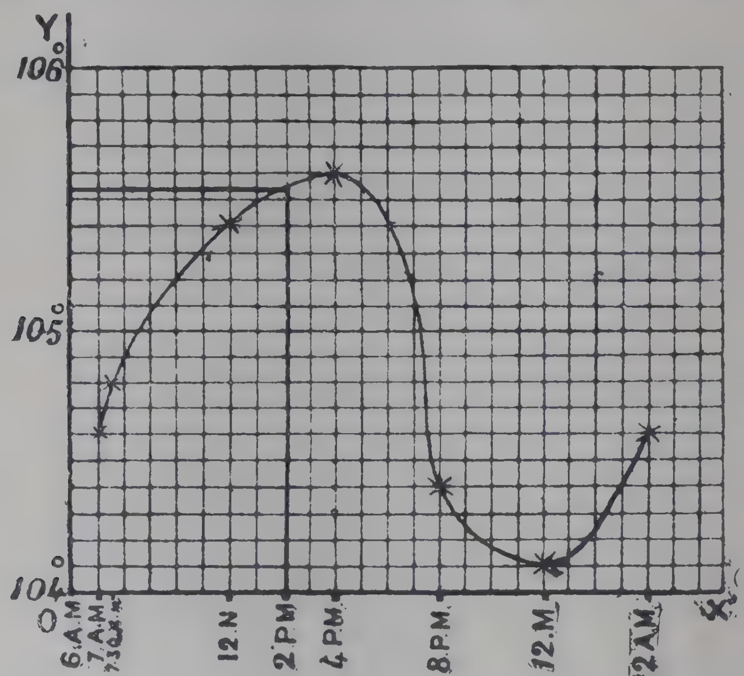


FIG. 164.

and since small changes in the temperature are to be clearly shown you take one small division along OY to represent $\cdot 1^\circ$. Since the temperature is never below 104 you take the origin as 104 for temperature and 6 A.M. for time. Then the successive points will be (1, 6), ($1\frac{1}{2}$, 8), (6, 14), (10, 16), (14, 4), (18, 1), (22, 6). Mark these points and join them freely by a curve.

The graph shows clearly the variation in the temperature, how it was gradually rising from 7 A.M. to 4 P.M., and afterwards falling down until 12 midnight. After 12 midnight it again began to rise. You have the maximum in the neighbourhood of 4 P.M., and the minimum in the neighbourhood of 12 midnight. We say '*in the neighbourhood of*' because from the data it is not clear that the maximum was exactly *at* 4 P.M. and the minimum *at* 12 midnight.

By *interpolation* you can find approximately the temperature at any time not given in the table (say, at 2 P.M.). Erect a perpendicular at the point on the X axis corresponding to 2 P.M.; you see it meets the curve at a point the horizontal through which meets the Y axis a little above the division corresponding to 105·5. Thus the temperature at 2 P.M. was approximately 105·5.

§ **166. Choice of scale-units.**—In all questions on graphs great care should be taken to see that proper units are employed.

To secure good results, the unit must be so chosen that the plotted points are at a convenient distance apart. As a rule, the diagrams should be as large as possible.

When the abscissa values range not from zero, but from 50 to 100, it is better to start with 50 at the origin. Similarly

for the ordinate values. In the previous example that was why, for the ordinates, we started with 104 at the origin.

Exercise XIX (b).

1. The relation between the weight and height of women is given by the following table: Draw the graph showing this relation and estimate the weight when the height is (1) $62\frac{1}{2}$ in. (2) $64\frac{1}{2}$ in.

Height in inches...	60	61	62	63	64	65	66
Weight in lbs. ...	100	$106\frac{1}{2}$	113	$119\frac{1}{2}$	131	138	149

2. The following table gives the height (in feet) fallen through by a body in a given time (in seconds): Draw the graph; and find (i) the height fallen through in 0·8 of a second (ii) the time for a fall through 20 feet :—

Time ...	$\frac{1}{2}$	$\frac{3}{4}$	1	$1\frac{1}{4}$	$1\frac{1}{2}$	$1\frac{3}{4}$	2
Height ...	4	9	16	25	36	49	64

3. The following table gives the maximum and minimum temperatures on 5 successive days. Draw graphs for (i) maximum temperature (ii) minimum temperature :—

	1st day.	2nd day.	3rd day.	4th day.	5th day.
Max. ...	83°	80°	87·7°	86·9°	79°
Min. ...	73·5°	73·7°	76·1°	75·2°	73°

4. The relation between the circumference of a circle and its diameter is given by the table :—

Diameter ...	1	4	7	10	13	16
Circumference ...	3·1	12·6	22	31	40·8	50·3

Draw the graph representing the relation. Find the diameter of a circle whose circumference is (i) 33 in. (ii) 27 in.

5. The following table gives the quantity of silk (in millions of lbs.) imported into the United Kingdom between 1891 and 1900 :—
Represent this graphically.

Year.	1891	1892	'93	'94	'95	'96	'97	'98	'99	1900
Import Silk.	3	2	2·6	1·8	2·1	2·3	2·2	2·5	2·6	2·1

6. The heights of an individual at different ages are given by the table :—

Age ...	2	4	10	12	14	17	20	35
Height in inches ...	34	40	50	55	61	68	68½	68¾

Draw a graph showing the growth of the individual; and find his probable height at (i) 16 years; (ii) 26 years of age.

7. The following is copied from the premia table of a Life Assurance Company for an assurance of Rs. 1000. Draw the graph representing the premia and find the annual premium to be paid by one who intends to insure his life and whose age next birthday is 23 :—

Age next Birthday.	20	25	30	35	40	45
Premium.	Rs. 21-8	Rs. 24	Rs.27-4	Rs. 31-4	Rs. 36-12	Rs.44

8. The number of Masters of Arts turned out in successive years by the Madras University between 1902 and 1907 is given by the table :—

Year	1902	1903	1904	1905	1906	1907
Number o M.A.'s ...	13	8	6	13	8	25

Represent this by means of a graph.

9. The velocity acquired by a body falling on account of the earth's gravity, from a height H ft., is V ft. per second. The following table gives the relation between H and V :—

H	1	4	9	16	25	36
V	8	16	24	32	40	48

Draw the graph showing the above relation; and estimate (i) H when V = 19; and (ii) V when H = 12.

10. The following table gives the areas of circles of given diameters: Draw a graph showing the relation between the diameter and the area of a circle, and find the area of one whose diameter is $6\frac{1}{4}$ ft:—

Diameter ...	4	5	6	7
Area of the circle ...	12·57	19·63	28·27	38·48

11. The greatest weight that a steel wire rope of girth G in can support is W tons and is given by the following table:—

G ...	1	2	3	4	5	6
W ...	$2\frac{1}{2}$	10	24	40	63	90

Represent this by a graph. Find W when G = 3·6. What minimum girth should a rope have in order that it may be employed to support a weight of 50 tons?

12. The prices of terrestrial globes of different sizes are as follow:—

Diameter in inches.	2	3	5	6	8	10	12	14
Price in Rupees ...	$5\frac{1}{8}$	7	14	$17\frac{3}{8}$	$24\frac{1}{2}$	$33\frac{1}{4}$	42	$53\frac{1}{2}$

Find graphically the prices of globes of diameters 7" and 11".

13. The following table gives, in pounds, the average weight of a boy at various ages:—

Age ...	0	2	4	6	8	10	12	14	16
Weight ...	6	$28\frac{1}{2}$	35	42	$51\frac{1}{2}$	63	$74\frac{1}{2}$	86	112

Represent this graphically and hence estimate the average weight of a boy at (1) 11 years, (2) 13 years.

14. The accompanying table records, in a case of malarial fever, the temperature (Fahrenheit) of the patient, taken morning and evening daily from the 10th to the 15th day. Represent this by means of a graph.

Date	10		11		12		13		14		15	
	M.	E.	M.	E.	M.	E.	M.	E.	M.	E.	M.	E.
Temp.	103	103·2	104	104·6	105	105·3	104	104·9	102	102·6	100	98

§ **167. Functions of x.** We have seen that we can always draw a graph when we are given a set of corresponding values of two quantities so related to one another that a change in one produces a corresponding change in the other.

If x and y be two such quantities y is generally called **a function** of x and if the relation between the two quantities is given in the form of an equation or formula, then giving particular values to one, the corresponding values of the other can be found and the graph plotted.

E.g., let $y = 2x + 3$ be the equation connecting the two quantities x and y.

We first construct a table of pairs of co-ordinates, *i.e.*, corresponding values of x and y, thus :

$x =$	-4	-2	0	2	4	6
$2x =$	-8	-4	0	4	8	12
y or $2x+3=$	-5	-1	3	7	11	15

We next plot the points $(-4, -5)$, $(-2, -1)$, $(0, 3)$,

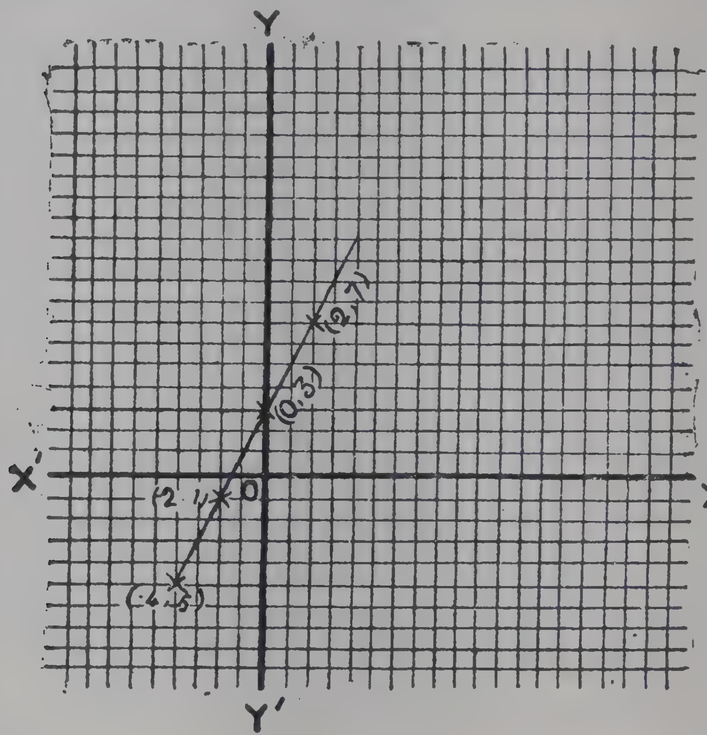


FIG. 165.

$(2, 7)$, etc., and then draw through them the line shown in the accompanying diagram

Exercise—Practical.

1. Copy the tabular form given below; fill in the blank spaces; plot the points whose co-ordinates you thus obtain and join them by straight lines. What do you notice? [Let one-tenth of an inch represent unity.]

When $x =$	-18	-20	-24	0	5	6
$2x =$						
$y=2x-8=$						

2. Use half an inch as unit, plot the points (8, —3) and (8, 7) and determine, from your figure, the co-ordinates of the middle point of the line joining them.

3. In each of the following examples, copy the table given and fill in the blank spaces : Plot the points obtained and join them by straight lines.

(*a*) $y = 6x+9$.

When $x =$	0	3	5	—6	—8
$6x =$					
$y = 6x+9 =$					

(*b*) $5y = 6x-3$.

$x =$	8	3	13	—2	18
$6x =$					
$5y = 6x-3 =$					
$y =$					

4. In the following examples give any integral values to x, find the corresponding values of y. Plot the points representing the corresponding values of x and y and join them by straight lines.

(1) $y = 2x-5$.
(2) $y = 2x+4$.
(3) $2y = 7x+2$.
(4) $3y = 9x+4$.
(5) $6x = 7y-4$.
(6) $10x = 3y+2$.

CHAPTER XX.

PROPORTION—DIRECT AND INVERSE.

§ **168.** We have already seen that if two quantities are of the same kind the ratio of the first to the second is the quotient obtained by dividing the first by the second whether that quotient be integral or fractional. The ratio of two concrete quantities is an abstract number.

If two ratios are equal, the four quantities involved therein are said to be in **proportion.** *E.g.*, if $\frac{a}{b} = \frac{c}{d}$, then a, b, c, d are said to be in proportion. a and d are called the *extremes* of the proportion and b and c are called the *means.*

The following are very important and the student must be quite familiar with them :—

1. If $\frac{a}{b} = \frac{c}{d}$; multiplying each of the equal ratios by bd we have $\frac{a}{b} \times bd = \frac{c}{d} \times bd$, *i.e.*, $\mathbf{ad = bc}$, *i.e.*, **the product of the extremes = the product of the means,** *e g.*, $\frac{4}{5} = \frac{12}{15}$. Multiplying by 15×5 we have $4 \times 15 = 12 \times 5$; evidently each $= 60$.

2. If $\frac{a}{b} = \frac{c}{d}$, then $\frac{b}{a} = \frac{d}{c}$. Divide unity by each of these fractions. Then $1 \div \frac{a}{b} = 1 \div \frac{c}{d}$, *i.e.*, $\frac{b}{a} = \frac{d}{c}$.

e.g., $\frac{13}{25} = \frac{52}{100}$. $\therefore \frac{25}{13} = \frac{100}{52}$.

3. If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a}{c} = \frac{b}{d}$. Multiply each of the ratios by $\frac{b}{c}$. $\therefore \frac{a}{b} \times \frac{b}{c} = \frac{c}{d} \times \frac{b}{c}$, *i.e.*, $\frac{a}{c} = \frac{b}{d}$, *e.g.*, $\frac{4}{9} = \frac{48}{108}$. $\therefore \frac{4}{48} = \frac{9}{108}$.

4. The **inverse or reciprocal** of any quantity is unity divided by that quantity, *e.g.*, the inverse or reciprocal of 2 is $\frac{1}{2}$; of $\frac{4}{5}$ is $\frac{5}{4}$; of a is $\frac{1}{a}$.

5. If $\frac{a}{b} = \frac{b}{c}$ or $a : b = b : c$, b is said to be the **mean proportional** between a and c.

Example 1.—Find the mean proportional between 2 and 8.

Let x be the mean proportional. Then, by definition $\frac{2}{x} = \frac{x}{8}$.

Multiplying both sides by $8x$,

$16 = x^2$ or $x^2 = 16$. $\therefore x = 4$.

6. If $\frac{a}{b} = \frac{b}{c}$ or $a : b = b : c$, c is said to be the *third proportional* to a and b.

Example 2.—Find the third proportional to 4 and 8.

Let x be the third proportional. By definition $\frac{4}{8} = \frac{8}{x}$.

$\therefore$ By cross multiplication $4x = 64$. $\therefore x = 16$.

§ 169. The idea of proportion may be illustrated graphically as follows :

Take two lines OX, OY at right angles at O and draw any line OP through O. From P_1, P_2, P_3, P_4, P_5, P_6 on the straight line OP draw any number of ordinates P_1N_1, P_2N_2......

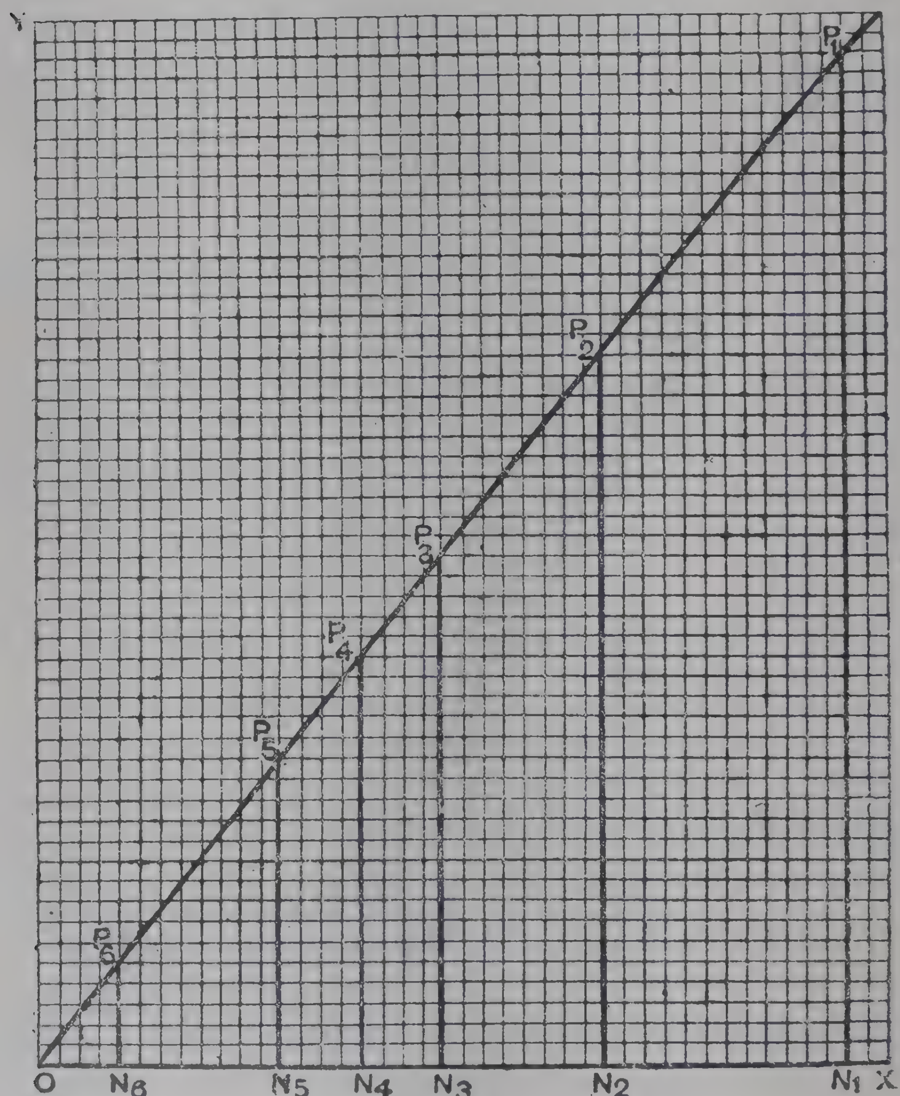


FIG. 166.

By inspection or measurement we see that

$$\left.\begin{aligned} \frac{P_1N_1}{ON_1} &= \tfrac{50}{40} = \tfrac{5}{4} \\ \frac{P_2N_2}{ON_2} &= \tfrac{35}{28} = \tfrac{5}{4} \end{aligned}\right\} \qquad \begin{aligned} \frac{P_3N_3}{ON_3} &= \tfrac{25}{20} = \tfrac{5}{4} \\ \frac{P_4N_4}{ON_4} &= \tfrac{20}{16} = \tfrac{5}{4}. \end{aligned}$$

$$\therefore \frac{P_1N_1}{ON_1} = \frac{P_2N_2}{ON_2} = \frac{P_3N_3}{ON_3} = \frac{P_4N_4}{ON_4}$$

and so on for other points. This is expressed by saying that for any point on a straight line drawn through the origin the ordinate is *proportional* to the abscissa.

Exercise—XX (a).

1. Express the following ratios as ratios of integers :—

(*a*) $\frac{4}{5} : \frac{10}{9}$; (*b*) $\frac{8}{9} : \frac{7}{6}$; (*c*) $\frac{115}{143} : \frac{112}{169}$.

2. Solve the following equations :—

(*a*) $\dfrac{x}{8}=\frac{6}{13}$; (*b*) $\dfrac{9}{x}=\frac{8}{13}$; (*c*) $\frac{7}{8}=\dfrac{x}{32}$; (*d*) $\dfrac{x}{\cdot 05}=\frac{4}{9}$; (*e*) $\dfrac{x}{a}=\dfrac{b}{c}$.

3. Find the fourth proportional to (*a*) 8, 5, 20; (*b*) 10, 8, 16; (*c*) 143, 12, 156.

4. Find the third proportional to (*a*) 5 and 9; (*b*) 7 and 8; (*c*) 9 and 15.

5. Find the mean proportional between (*a*) 4 and $20\frac{1}{4}$; (*b*) 6 and 24; (*c*) $\frac{1}{21}$ and $\frac{7}{3}$; (*d*) $\frac{1}{5}$ and $\frac{4}{45}$.

6. (*a*) If $8x=9y$, what is the ratio of $x : y$?

(*b*) If $9x+4y = 8y$, find the ratio of x to y.

(*c*) $5x+3y = 14y-6x$. What is the ratio of x to y?

7. Find the value of x in each of the following cases :—

(1) $x : 9=8 : 10$. (2) $6 : x=15 : 21$. (3) $8\frac{1}{3} : 9\frac{1}{2}=x : 15$.

8. Find the reciprocal of $9\frac{1}{2}$, $71\frac{1}{3}$, $87\frac{4}{9}$, $\dfrac{xa}{3y}$.

§ 170. Direct proportion.

Ex. 1.—If 15 articles cost Rs. 2-13 as., what is the cost of 23 articles?

We have seen that questions of this kind can be worked according to the unitary method; but here we shall work by 'proportion.'

If the number of articles is doubled the cost also is doubled; if the number of articles is trebled, the cost also is trebled, &c.; also if the number of articles is halved, the cost also is halved, &c.; also if there are no articles, the cost is zero. This is expressed by saying that the cost is *proportional* to the number of articles.

$\therefore$ since the cost of 2, 3, etc., articles is twice, three times, etc. the cost of 1 article, we see that the ratio between the cost of 15 articles and the cost of 23 articles is the same as the ratio between 15 and 23.

$$\therefore \quad \frac{15 \text{ articles}}{23 \text{ articles}} = \frac{\text{the cost of 15 articles}}{\text{the cost of 23 articles}},$$

$$\textit{i.e.,} \quad \frac{15}{23} = \frac{\text{Rs. 2-13 or 45 as.}}{x \text{ as.}}$$

if you suppose the cost of 23 articles to be x as.

then $15 \times x$ as. $= 23 \times 45$ as. (by cross multiplication).

$$\therefore \quad x = \frac{23 \times 45}{15} = 69. \quad \therefore \text{ the cost is 69 as. or Rs. 4-5-0.}$$

Graphical Representation.

This may be worked graphically as follows :—

Draw two lines XOX^1 and YOY^1 at right angles to each other. Let each sub-division on the Y axis represent 1 article and each sub-division on the X axis, 3 annas. (As the price increases much more rapidly than the number of articles it is best to take a small unit for the rapidly increasing variable). Since 0 articles cost 0 annas, plot the point (0, 0).

Also plot the point Q, corresponding to (45 as., 15 art.) (for 45 as. is the cost of 15 articles). Join OQ. Every point on this line has its ordinate proportional to the abscissa as explained before. And since the cost is proportional to the number of articles, the line OQ completely represents the relation between any number of articles and their cost. Now the value of 23 articles is easily got by drawing through the reading 23 on the Y axis a line parallel to the X axis and noting the point where it intersects OQ (or OQ produced), the abscissa of the point being the cost.

Here the point on OQ corresponding to 23 on the Y axis is P and its abscissa ON contains 23 small divisions; and since each such division represents 3 as., the cost is 23×3 or 69 as.

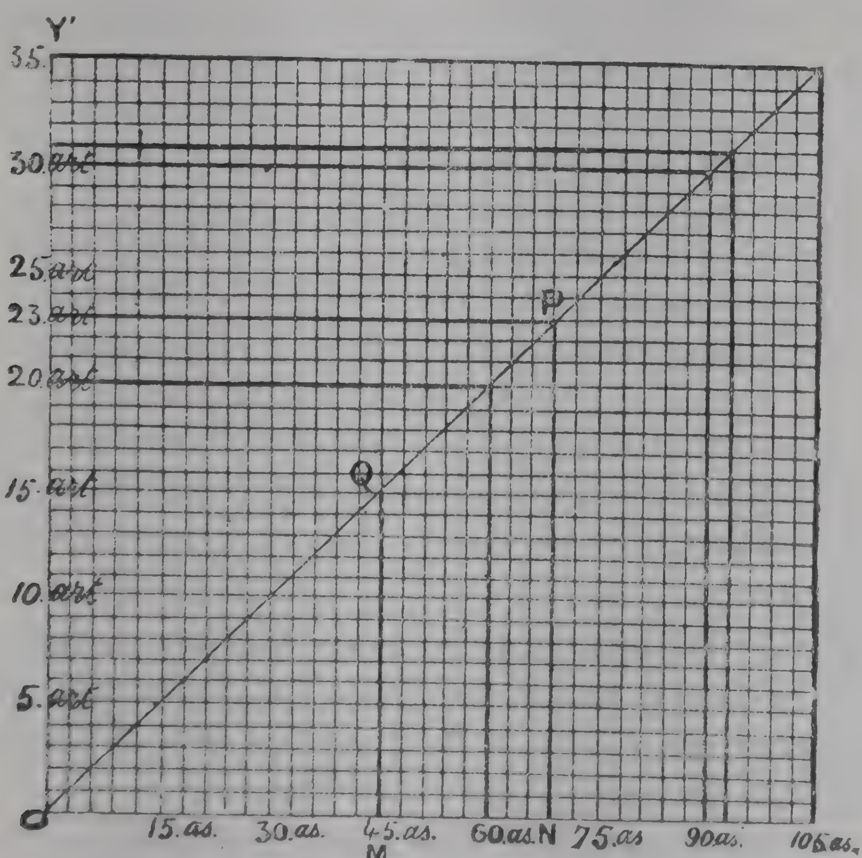


FIG. 167.

This graph not only gives the answer to the above question but contains the answers to several questions.

For instance the price of any number of articles can be determined from the same straight line graph ; *e.g.*, the costs of 30 articles, 31 articles and 20 articles are found to be respectively 90 as., 93 as. and 60 as. by drawing parallels through the reading 30, 31 and 20 on the Y axis, noting their points of intersection with the straight line graph and reading off the abscissæ of those points.

The same figure is also useful in finding the answer to the following question : "If 15 articles cost Rs. 2-13, how many articles can be purchased for (1) Rs. 6-9 ; (2) Rs. 5 ?" Now Rs. 6-9 = 105 as. Draw a line parallel to the Y axis through the reading 105 as. on the X axis, note the point of its intersection with the straight line graph and read off the ordinate of that point. It is 35. Therefore 35 is the required answer. Again Rs. 5 = 80 as. By a similar process the answer lies between 25 and 26 showing that 25 articles can be bought and also a fraction of an article ; but since a fraction of an article cannot be purchased it means some money will be left behind.

Such a graph will also be useful to a merchant, for with the aid of the graph he can without calculation find at once the price of any number of articles his customer purchases, or find how many articles can be purchased for a given sum of money. In this case the graph is called a *ready reckoner.*

Ex. 2. Draw a ready reckoner to show the price of goods at Re. 1, Rs. 2, Rs. 3,......Rs. 7 per viss. Read off the cost of 2 v. 15 p. at Rs. 7 per viss and of 3 v. 10 p. at Rs. 2-8 per viss.

Let each division on OX represent 5 palams, so that 8

divisions represent a viss, and 2 divisions on OY represent a rupee.

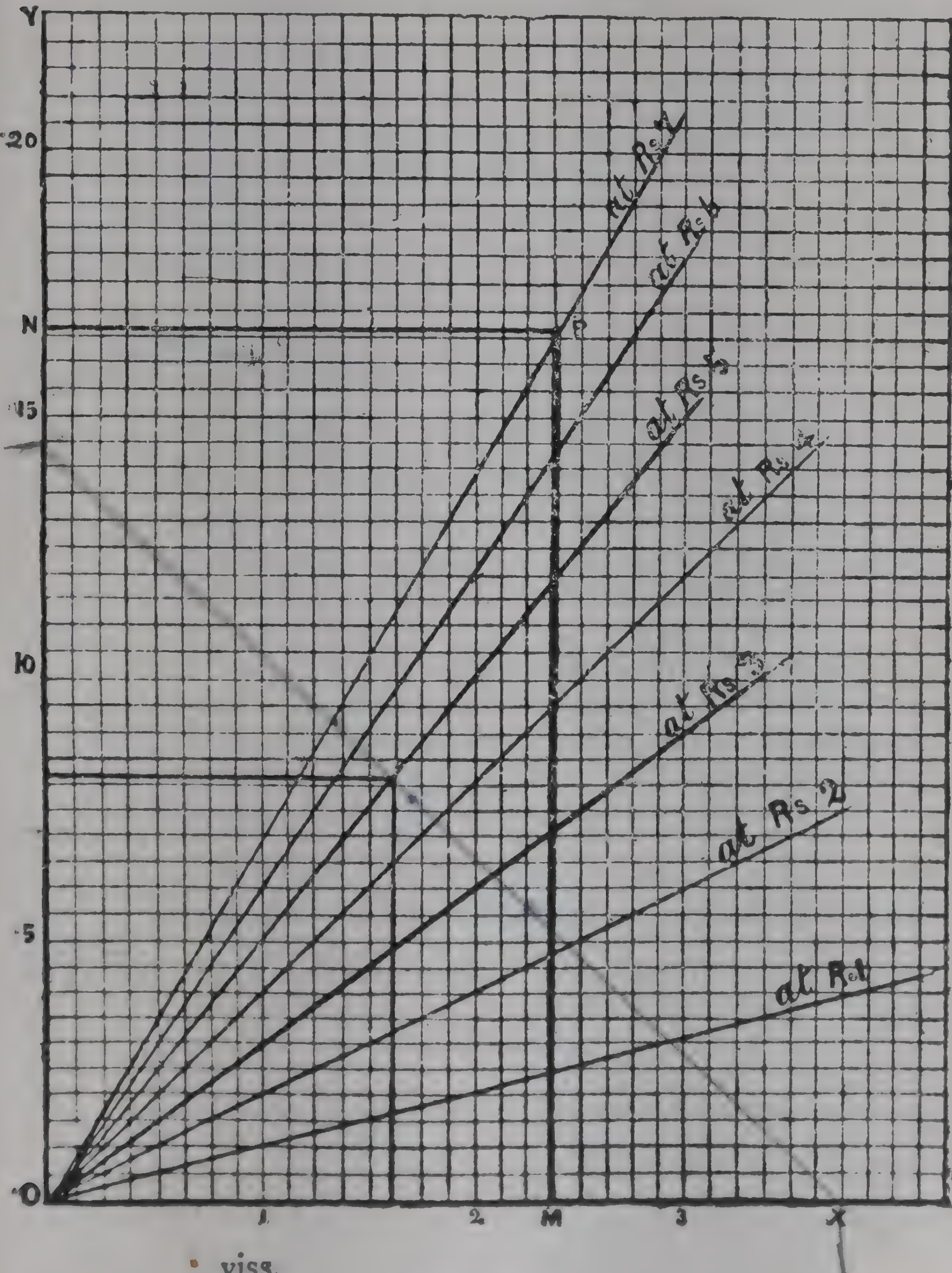


FIG. 168.

To construct the ready reckoner for price at Re. 1 per viss, plot the point (1 viss, Re. 1); join it to the origin (for when no viss is bought no rupee is charged). Similarly construct graphs for Rs. 2, Rs. 3, etc., per viss.

The price of 2 v. 15 p. at Rs. 7. per viss is found thus: Take the point on OX corresponding to 2 v. 15 p.; draw a perpendicular to OX at that point to meet the graph of Rs. 7 per viss at P. Draw PN parallel to OX, meeting OY at N. The price is given by ON, and is found to be Rs. 16-10.

Again, the price of 3 v. 10 p. at Rs. 2-8 per viss is the same as that of 1 v. 25 p. at Rs. 5 per viss; and proceeding as in the previous case, it is found to be Rs. 8-2.

Exercise—Graphical

1. Examine if the cost of articles is proportional to the number bought in the following case:—

No. of Cocoanuts	...	2,	8,	25,	100.
Cost in annas.	...	$1\frac{1}{2}$,	6,	$16\frac{2}{3}$,	64.

And test your result graphically.

2. The following table gives the number of miles walked by a person and the time (in hours) taken for walking:—

Distance walked.	4	10	$12\frac{1}{4}$	17	$19\frac{1}{2}$
Time taken	1	2	3	$4\frac{1}{2}$	$6\frac{1}{2}$

Examine if the distance walked is proportional to the time spent. Test it graphically.

3. The following table gives the railway fare corresponding to the length of the journey:—

Number of miles.	3	6	48	54
Railway fares charged in annas ...	$\frac{1}{2}$	1	8	9

Examine if the fare is proportional to the number of miles travelled. Test it graphically.

4. The following figures have been taken from the Census 1901:

	Madras.	Salem.	Tanjore.
Area in sq. miles ...	27	7530	3710.
Population ...	... 509,346	2,204,974	2,215,029.

Examine if the population is proportional to the area.

Exercise XX (b).

1. If the rent of 25 acres is Rs. 312-8 as., what is the rent of 65 acres?

2. If 1 gross of Guzerat pencils cost Rs. 4-8, what is the cost of 2 dozen?

3. If a river pours into the sea 25,600 cu. ft. of water in every 15 minutes, how much will it pour into the sea in a week?

4. If the annual fee income of a school is Rs. 22,000 when it has on its rolls 580 pupils, find what it will be when its strength is 640, assuming the fee income to be proportionate to the strength.

5. If the weight of a sheet of brass 6 ft. long and 4 ft. broad be 240 lb., what will be the area of a similar sheet of brass weighing 999 lb.?

6. If the weight of a piece of copper wire 6000 ft. long be 196·32 lb., what will be the weight of a piece of similar wire 275 ft. long?

7. On a map drawn to the scale of 1 inch to a mile the distance between two places is 4·3 in. What is the actual distance between the towns?

8. A cyclist rides at 10 miles an hour. Find the time at which he passes the following places (to the nearest minute if he leaves Madras at 8 A.M.):

Places.	Miles from Madras.
Saidapet ...	6
St. Thomas' Mount.	9
Pallavaram ...	13
Chingleput ...	36

9. If 5 third class tickets from Madras to Srirangam (260 miles) cost Rs. 15, what should be the fare for (1) 1 mile, (2) 80 miles

10. If a ship consumes 24 tons of coal in 12 hours, how many tons does it consume in a week at the same rate?

11. A railway is being made at the rate of 2500 m. per week of 6 days. How manv working days will it take to make 150 km. at the same rate?

12. Describe a $\triangle$ ABC having $BC=1''$, $\angle B=50°$ and $\angle C=60°$ and a $\triangle$ A′B′C′ having $B'C' = 2''$ and $\angle B' = 50°$ and $\angle C' = 60°$. Measure and compare the corresponding sides of the two $\triangle$s.

13. Describe two $\triangle$s having two of the angles in each respectively equal to two given angles and show by measurement that the corresponding sides are proportional.

14. Two triangles have their sides proportional. The sides of one are 3, 4, 5. (1) If the smallest side of the second triangle is $7\frac{1}{2}$ inches, find its other sides. (2) If the greatest side of the second triangle is $17\frac{1}{2}$ inches, find its other sides.

15. In a graph 14 divisions represent 1 ton, how many lbs. will $10\frac{1}{2}$ divisions represent?

16. Draw a graph connecting lbs. and Madras maunds (1 Madras maund = 25 lbs.) Find the numbei of (*a*) lbs. in $3\frac{1}{2}$ maunds, (*b*) the number of maunds in $112\frac{1}{2}$ lbs.

17. Change 25 rupees into £ *s. d.* given that 4*s.* = Rs. 3.

18. X, Y, Z and U are 4 points on a map. If the distance from X to Y is 20 miles, find the distance of XU, XZ and ZY.

*X

*U

Y* *Z

19. The published prices of 3 books are given below; the prices at which they are sold being given in brackets. Examine if the selling prices are proportional to the published prices. Re. 1-2-0 (0-15-0), Rs. 10-1 (8-9), Rs. 7-8-0 (6-4-0).

20. Measure the length of a page of your graph note-book (1) in inches (2) in centimetres. Find from this (1) the number of centimetres in an inch.

21. A vessel when full of water weighs 40 lbs. : when half full it weighs 25 lbs. How much does it weigh when quarter full?

22. In an examination there are p examiners and each examiner gets r answer books to correct. Find the number of answer books in the subject?

23. If a man earn Rs. b, how many men will earn Rs. c?

24. When it is 12 o'clock at Greenwich, it is 5-20 P.M. at Madras. What is the time at Madras when it is 6 o'clock at Greenwich? (Note that this and the next question are not on 'proportion.')

25. A train reaches the Central Station at 7-30 A.M. and is 10′ late. When would it have reached it if it had been 30′ late?

§ 171. Inverse proportion.—Consider the following example :—(1) If 18 men can do a work in 16 days, in how many days will 12 men do the same work?

First according to the unitary method.

18 men build a wall in 16 days.

1 man builds it in 16×18 „

12 men build „ $\dfrac{16 \times 18}{12}$ „

i.e., **24** days.

By the method of proportion.

It is clear that if 9 men are employed to do the whole work, they will take a longer time to build the wall. Since 9 men is half of '18 men' the number of days required for 9 men will be twice as many days as are required for 18 men. Similarly if the number of men be increased, *i.e.*, doubled, trebled, etc., the number of days will be proportionately diminished, *i.e.*, halved, reduced to one-third, etc. Also if the number of men be decreased, *i.e.*, reduced to $\frac{1}{2}$, $\frac{1}{3}$, etc., the number of days will be doubled, trebled, etc.

This is expressed by saying that the number of days taken to build a wall is **inversely proportional** to the number of men employed.

We have $\frac{12 \text{ men}}{18 \text{ men}} = \frac{16 \text{ days}}{x \text{ days}}$ (each is a proper fraction).

$\therefore \quad 12x = 288$ (by cross multiplication).

$x = 24.$

N.B.—The proportion may be put in the following form as well: $\frac{18}{12} = \frac{x}{16}$ (for each is an improper fraction). But $\frac{12}{18} = \frac{x}{16}$ is wrong, for x being greater than 16, $\frac{x}{16}$ is an improper fraction and cannot be equal to the proper fraction $\frac{12}{18}$.

Graphical Solution.

In Fig. (169) OX and OY are two perpendicular lines. On OX, one division represents one man and on OY, one division represents one day. Thus OM represents 18 men and ON 16 days. Let each small square represent the quantity of work done by a single man in a single day; then the figure OMPN will represent the total work, *i.e.*, 288 squares represent the total work.

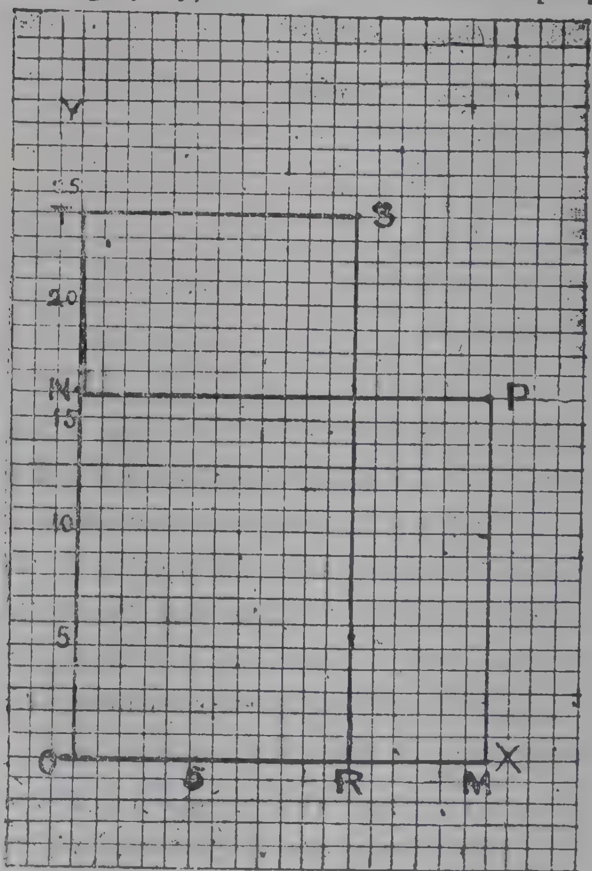


FIG. 169.

Again OR represents 12 men; and if the rectangle ORST be completed so as to be equal to OMPN, *i.e.*, so as to contain 288 squares, RS or OT will represent the number of days required by 12 men to do the work. OT contains 24 divisions; hence the number of days required is 24.

Exercise XX (c).

1. A train travelling at the rate of 20 miles an hour describes a certain distance in 15 hours. How many hours will a train, travelling at the rate of 30 miles an hour, take to travel the same distance? Test your result graphically.

2. The following table gives the cost in Rs. of rice consumed by a family in a month and the price of rice in the month:—

	1st mo.	2nd mo.	3rd mo.	4th mo.	5th mo.
Cost of rice consumed.	15	$13\frac{1}{2}$	$11\frac{1}{2}$	10	9
Price of rice in measures per rupee ...	4	$4\frac{1}{2}$	5	$5\frac{1}{2}$	6

Examine if the cost is inversely proportional to the price (per rupee).

3. Examine if the square of the diameter of each pipe is inversely proportional to the time it takes to fill a certain cistern.

Diameter in inches	1,	2,	3,	4.
Number of minutes taken for filling the cistern	30,	$7\frac{1}{2}$,	$3\frac{1}{3}$,	2.

4. A quantity of mercury is poured into test tubes of different bores. Examine if the height is inversely proportional to the area of the cross-section of the tube.

Area of the cross-section in sq. mm.	2,	5,	4,	6,	8,	9.
Height in mm. of the column of mercury	90,	36,	45,	30,	23,	20.

5. Two plots of ground are of equal area, one is 220 yds. by 140 yds.; the other is 340 yds. long. What is the width of the second plot of ground?

6. The three sides of a triangle are 10 in., 18 in., 20 in., and the corresponding perpendiculars are 18 in., 10 in., 9 in., show that a perpendicular is inversely proportional to the corresponding side.

7. If rails are 32 ft. long the number required to make a mile of railway is 330. How many are required if the rails are (1) 44 ft. long (2) 60 ft. long.

8. A shelf holds 60 books each 5 cm. thick; how many books 4 cm. thick can it hold?

9. The rice store of a hostel will last for a month of 30 days at the rate of 40 measures a day. How long will it last if 35 measures are consumed in a day?

10. If out of a certain quantity of gold, 9 pairs of solid bangles each weighing as much as 16 sovereigns can be made, find how many pairs of solid bangles each weighing as much as 12 sovereigns can be made out of the same quantity of gold.

11. 2,000 men have provisions for 120 days: they are joined by 400 men. How long will the provisions now last?

12. A given quantity of gold is beaten into a thin leaf 3" by $2\frac{1}{2}$" by $\frac{1}{16}$". If an equal quantity of gold is beaten into a leaf 4" by 2", what will be its thickness?

13. A reservoir which contains a year's supply of water to a town justifies a daily allowance of 1 gallon of water to each inhabitant. If the original population is increased by a fourth coming from plague-affected districts for health, find by how much the daily supply of water should be reduced to make the supply last for the whole year.

14. If with a certain sum of money x beggars can be paid at y pies each, how much could each beggar be paid with the same sum if there were only p beggars?

15. There are a rooms in a hostel and b students sleep in each room. If c students were to sleeep in each room ($c > b$ and divides ab), how many rooms would be occupied and how many would be free?

CHAPTER XXI.

INTEREST.

§ **172.** If A lives in B's house, A pays money to B for using his house. This money is house-rent. Similarly if A uses B's money, then A must pay B some money for using his money as in the case of rent. **The money paid for the use of money is called Interest**. Just as rent is calculated for a certain number of months at a certain rate so also interest is calculated for the time the money lent has been made use of, at a certain rate.

The person A may be an individual or a bank inviting deposits or any other company carrying on money transactions.

In questions of interest, there are four elements to be considered :—(1) the *principal* or the money lent; (2) the *rate* of interest, which is generally given as rate per cent ; (3) the *period* or *time* for which money is lent ; and (4) the *interest* paid at the end of the period. If any three of these elements are given, the fourth can be found out very easily. There is also another term used in such questions ; *viz*., the *amount* (= the principal + the interest).

Example 1.—Find the interest on Rs. 625 at 3 per cent. for 2 years.

Sol.—The interest on Rs. 100 for 1 year is Rs. 3

∴ ....1......1 yr....... „ $\frac{3}{100}$

∴ ....625......2 yrs.......... $\frac{3 \times 625 \times 2}{100}$ or

Rs. 37-8-a.

N.B.—Interest $= \dfrac{625 \times 3 \times 2}{100} = \dfrac{\text{Principal} \times \text{rate} \times \text{time}}{100}$.

Generally, if P stands for principal, T for time in years, and R for the rate per cent. and I for interest. $I = \dfrac{P \times T \times R}{100}$.

Example 2.—In what time will £500 amount to £575 at 6 per cent. ?

Sol.—The interest on £500 at 6 per cent. for 1 year = £30.

And we are asked to find in what time the interest will amount to (£575—£500) or £75.

Hence the period required is $75 \div 30$ or $2\frac{1}{2}$ yrs.

Example 3.—At what rate per cent. will Rs. 600 amount in 5 years to Rs. 720?

Sol.—The interest on Rs. 600 for 5 years is Rs. 120.

......................Rs. 600 for 1 year is $\dfrac{\text{Rs. } 120}{5}$ or Rs. 24.

......................Rs. 100 ....Rs. $24 \div 6 =$ Rs. 4.

Hence the rate required is 4 per cent.

Example 4.—What sum will amount to Rs. 700 in 3 years at 4 per cent. ?

Sol.—The interest on Rs. 100 for 3 years at 4 per cent. $= 3 \times 4$ or 12 Rs.

$\therefore$ Rs. (100+12) or Rs. 112 is the amount of Rs. 100.

$\therefore$ Rs. 700..................Rs. $700 \times \frac{100}{112}$

or Rs. 625.

Caution.—The student is warned against regarding the time as proportional to the amount. If the amount at the end of the first year be £108, the amount at the end of the second year will not be $2 \times$ £108.

When interest is calculated from one given day of the year to another, it is usual to include *one only of the days* named. The interest from the 10th of February to the 25th of February of the same year is interest for 15 days.

The rate per cent. is generally understood to be the rate *per cent. per annum.*

When the time is given in months and days, 12 months are reckoned to the year, and 30 days to the month.

When the time is given in weeks, 52 weeks are allowed to the year.

When the time is given in days, 365 days are allowed to the year.

The student is advised to perform multiplication, wherever practicable, by the method of practice. Thus if the rate of interest is given to be $3\frac{3}{4}$ per cent., to the interest calculated at 3 per cent., $\frac{1}{4}$ of itself may be added; and this sum will be interest at $3\frac{3}{4}$ per cent.

* Again, if interest is to be reckoned on a certain sum of money for a number of days, most often division will have to be made by 73; and the work may perhaps be shortened by remembering that

$\frac{1}{73}=\frac{1}{100}\times\frac{100}{73}=\frac{1}{100}\left(1+\frac{1}{3}+\frac{1}{30}+\frac{1}{300}-\frac{1}{7300}\right)$; or, putting

$1+\frac{1}{3}+\frac{1}{30}+\frac{1}{300}$ equal to x, $\frac{1}{73}=\frac{1}{100}\left(x-\frac{x}{10000}\right)$, approximately.

Example 5.—Find the amount at simple interest of £735 5*s*. 4*d*. at 3 per cent. *per annum* for 2 years, 7 months.

Sol.—Here, by our formula, we have to multiply £735 5*s*. 4*d*. by 3 and by $2\frac{7}{12}$ and divide the product by 100.

Now, $3\times2\frac{7}{12}=\frac{31}{4}$.

$$\begin{array}{rrrr} & £735 & 5s. & 4d. \\ & & & 31 \\ \hline 4) & £22,793 & 5s. & 4d. \\ \hline 100) & £5,698 & 6s. & 4d. \\ \hline & £56 & 19s. & 7{\cdot}96d. \end{array}$$

Thus the total interest correct to the nearest farthing is £56 19*s*. 8*d*. and the amount required is therefore £792 5*s*.

Or thus. The interest for 2 yrs. = £735- 5-4

$$\begin{array}{r} 6 \\ \hline 100)4411\text{-}12\text{-}0 \\ \hline 44\text{-}\ 2\text{-}4 \end{array}$$ to the nearest farthing.

interest for 6 months or $\frac{1}{4}$ of 2 yrs. = 11- 0-7

" for 1 month or $\frac{1}{6}$ of 6 mths. = 1-16-9

Total interest = 56-19-8

and the amount = 792- 5-0

Example 6.—Find the simple interest of £700 at 8 per cent. for 1 year, 1 month, 6 days.

Sol.—When the time is given in months and days, we allow 12 months to the year and 30 days to the month.

Thus 1 year, 1 month, 6 days = 1 year, $1\frac{1}{5}$ mo. = $1\frac{1}{10}$ yrs.

The interest on £700 for 1 year = 7×8 or £56.

Hence the interest required = £56 + £5·6 = £61-12*s*.

Example 7. Find the simple interest of £200 at 8 per cent. for 13 weeks.

Sol.—When the time is given in weeks, we allow 52 weeks to the year. Hence the interest required

$$= £200 \times 8 \times \tfrac{13}{52} \div 100 = £4. \quad \textit{Ans.}$$

**Example* 8.—Find the simple interest on Rs. 147 from 23rd April to 17th August at $7\frac{1}{2}$ per cent. per annum.

The interest = Rs. $\dfrac{147 \times 116 \times 7\frac{1}{2}}{365 \times 100} = \dfrac{147 \times {\cdot}58 \times 3}{73}$.

Now $147 \times {\cdot}58 \times 3 = 255{\cdot}78 = k$ (suppose).

$$\begin{array}{rr} k = & 255{\cdot}78 \\ \frac{1}{3}k = & 85{\cdot}26 \\ \frac{1}{30}k = & 8{\cdot}526 \\ \frac{1}{300}k = & 0{\cdot}853 \\ \hline & 350{\cdot}419 \\ \text{Deduct } \frac{1}{10000} \text{ of this} & {\cdot}035 \\ \hline & 350{\cdot}384 \end{array}$$

The interest is therefore equal to Rs. 3·504, *i.e.*, to Rs. 3-8-1 correct to a pie.

NOTE.—The interest on Rs. P for n days at r per cent. per annum $= \dfrac{P \times n \times r}{100 \times 365} = \dfrac{2Pnr}{73000}$. In making use of this formula, we need not retain any figure after the decimal in the product $2Pnr$.

Exercise XXI (a).

Find the simple interest on

1. £180 for 3 years at 5 per cent. per annum.
2. £576 for 6 years at $4\frac{1}{2}$ per cent. per anuum.
3. £675 for 2 years 4 months at 4 per cent.
4. £375 for 3 years 6 months at $3\frac{1}{3}$ per cent.
5. Rs. 312-8 as. for 2 years 146 days at 5 per cent.
6. Rs. 175 for 3 years 35 days at 6 per cent.
7. Rs. 624 for 4 years 12 weeks at $3\frac{3}{4}$ per cent.
8. £106 13*s*. 4*d*. from June 18, 1864 to Nov. 11, 1865 at 5 per cent.
9. Rs. 121 10 as. 8 ps. from Sept. 3, 1895 to April 4, 1896 at $6\frac{1}{4}$ per cent.
10. Rs. 243 5 as. 4 ps. from Aug. 17, 1899 to March 8, 1900 at $3\frac{3}{4}$ per cent.

Find also the amount in questions 1—10 given above.

In what time will

11. £350 amount to £400 at 5 per cent. ?
12. £2151 6*s*. 8*d*. amount to £2366 9*s*. 4*d*. at $2\frac{1}{2}$ per cent. ?
13. £88 double itself at 5 per cent. ?
14. Rs. 425 amount to Rs. 510 at 4 per cent. ?
15. Rs. 360 produce Rs. 90 interest at 5 per cent. ?
16. Rs. 312 8 as. produce Rs. 67 8 as. at 4 per cent. ?

At what rate per cent. per annum simple interest will

17. Rs. 320 amount to Rs. 360 in 5 years ?
18. £2630 amount to £3090-5*s*. in $3\frac{1}{2}$ years ?
19. Rs. 825 produce Rs. 143 interest in $3\frac{1}{4}$ years ?
20. £416 13*s*. 4*d*. produce £87-10*s*. interest in $4\frac{2}{3}$ years ?

What sum will produce as interest

21. £42 in $3\frac{1}{2}$ years at 3 per cent. ?
22. Rs. 112 8 as. in $4\frac{1}{2}$ years at $3\frac{1}{3}$ per cent. ?

What sum will amount to

23. £472 in 3 years at 6 per cent. ?
24. £287 10*s*. in 5 years at 3 per cent. ?
25. £78 7*s*. 6*d*. in 1 year 6 months at 3 per cent. ?
26. Rs. 106 4 as. in 3 years 4 months at $7\frac{1}{2}$ per cent. ?

In the Post Office Savings Bank, the regulations as to interest are governed by the following rule :—

"Interest is allowed, at the rate of 3 per cent. per annum on all deposits at call for each calendar month on the lowest balance at credit of an account on any date between the close of the fourth day and the end of the month, provided that interest shall be allowed only on sums of complete rupees and that it shall be calculated to the nearest pie. The interest calculated as above for each month will be added each year to the balance of each account."

Example.—The deposits and withdrawals of a person in the Post Office Savings Bank are as indicated by the following table :—

Date of deposit or withdrawal.	Amount of deposit or withdrawal in words.	Amount of deposit.	
			RS. A. P.
2—12—1910	Deposited Eight rupees	+	8—0—0
29—12—1910	Withdrawn Two rupees six annas	—	2—6—0
3—1—1911	Wtihdrawn Eight annas.	—	0—8—0
5—2—1911	Deposited Ten rupees only	+	10—0—0
5—3—1911	Withdrawn Five rupees only	—	5—0—0

Calculate the interest till the 31st March 1911.

(1) The minimum balance in December 1910 after the 4th was Rs. 5-10-0. Since interest is calculated only on complete rupees he gets interest on Rs 5.

(2) In January, the minimum balance at the credit of the account is Rs. 5-2-0. And he gets interest on Rs. 5.

(3) In February since the deposit of Rs. 10 is made after the 4th, interest is allowed only on Rs 5.

(4) In March the minimum balance between the close of the 4th day and the end of the month is (Rs. 15-2-0)—(Rs. 5-0-0), *i.e.*, Rs. 10-2-0. ∴ the complete number of rupees is Rs. 10; and interest is allowed on Rs. 10 only.

The interest on Rs. 100 for 12 months = Rs. 3 or 48 as.

∴	„	„ 100	1 ... $= \frac{48}{12}$ or 4 as. = 48 p.	
∴	„	„ 5	1 m. $= \frac{48}{20}$ or 2 p.	(to the nearest pie.)
		... 10	1 ... $= \frac{48}{10}$ or 5 p.	„ „

The int. for December, January and February (in each of which int. is calculated only on Rs. 5) = 2 × 3 or 6 p. and that for March = 5 p. Total interest = 11 p.

§ 173. Graphical representation of interest.—Find the simple interest on £253 for 4 years at 6 °/₀.

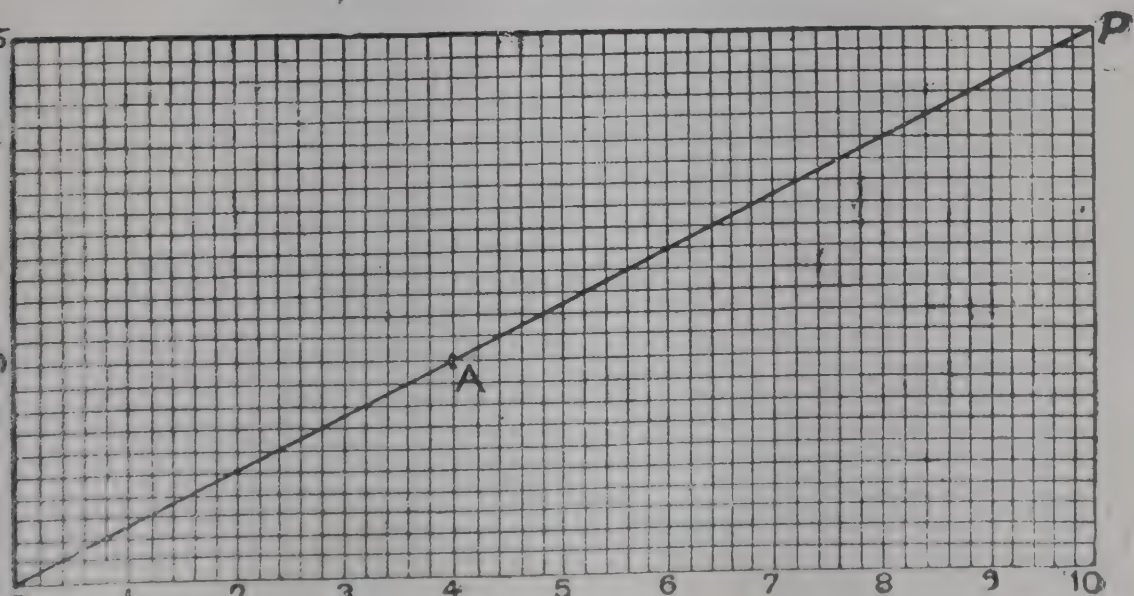


FIG. 170.

Since interest, as we have seen, is proportional to the principal, we have a straight line graph.

Let 1 division on the Y axis represent a principal of £10 and 5 divisions on the X axis represent £6. Now plot the point (4, 10) which corresponds to the interest on Rs. 100 for 4 years. Let A be the point. Join OA. Produce OA to meet the line through 25 (*i.e.*, Rs. 250) on the Y axis parallel to the X axis at a point P. The abscissa of P is 10. ∴ 10 times £6, *i.e.*, £60 is the interest.

Exercise XXI (b).

1. Find the interest due to depositors in the Post Office Savings Bank up to December 31st in the following cases :—

I.

Date.	Amount of withdrawal or deposit.
7- 4-09	Deposited Rs. 200.
17- 4-09	Withdrawn ,, 100.
12- 6-09	Do. ,, 20.
21- 6-09	Do. ,, 60.
21-10-09	Do. ,, 10.
30-12-09	Do. ,, 5.

II.

Date.	Amount of deposit or withdrawal.
3- 4-09	Deposited Rs. 5.
24- 7-09	Withdrawal ,, 3.
6- 9-09	Deposited ,, 25.
18- 9-09	Withdrawal ,, 6.
27-10-09	Do. ,, 20.
2-12-09	Deposited ,, 80.

2. A person has transactions with the Indian Bank (current deposit) as indicated by the following table: Terms of interest being those of the Post Office Savings Bank excepting that the

rate of interest is $4\frac{1}{2}\%$. Calculate the interest due up to the 31st December '09.

Date.	Amount of each deposit or withdrawal.	Amount deposited.	Amount withdrawn.	Balance at the credit of the depositor.
1909 Oct. 15	Deposited Rupees Seventy-five only.	Rs. 75-0-0		Rs. 75-0-0
Dec. 26	Withdrawn Rupees Twenty only.		Rs 20-0-0	Rs. 55-0-0

3. What sum lent out at 6 per cent. will produce the same interest in 5 years as Rs. 1120 in 6 months at 10 per cent.?

4. What sum lent out at $6\frac{1}{4}$ per cent. will produce the same interest in 4 years as Rs. 600 in 6 years at 5 per cent.?

5. What sum lent out at $3\frac{3}{4}$ per cent. for $6\frac{2}{3}$ years will amount to the same sum as £480 in 5 years at 7 per cent.?

6. A certain sum amounts to Rs. 560 in 5 years at $3\frac{1}{3}$ per cent. What will it amount to in 4 years at 5 per cent.?

7. What sum will amount to Rs. 600 in 4 years at the same rate of interest at which Rs. 450 amounts to Rs. 562 8 as. in 5 years?

8. A certain sum of money amounts to Rs. 640 in 4 years at 5 per cent. At what rate per cent. will the same sum amount to Rs. 650 in 5 years?

9. A certain sum of money amounts to Rs. 590 in 3 years at 5 per cent. In what time will the same sum amount to Rs. 587 8 as. at $3\frac{1}{2}$ per cent.?

10. A certain sum of money amounts to Rs. 833 5 as. 4 p. in 5 years at 5 per cent. At what rate per cent. will double the same sum produce Rs. 320 interest in 4 years?

11. A sum of money amounts in 8 years to £430 2*s*. at $4\frac{1}{2}\%$. What will it amount to in 15 years?

12. A sum of money amounts in 4 years to Rs. 3175 at $6\frac{1}{4}$ per cent. What will it amount to in $4\frac{1}{2}$ years at 6 per cent.?

13. A sum of Rs. 100 is annually deposited in a bank for 3 years. What amount will be standing to the credit of the depositor at the end of the third year, interest being allowed @ $2\frac{1}{2}$ per cent. per annum ?

14. In the following advertisement find at what rate per cent. per annum a person is charged interest if he chooses the instalment system instead of cash payment. Encyclopædia Brittannica cash price Rs. 456 12 as. or 4 monthly payments of Rs. 115 each.

15. A man owes Rs. 100. On April 1st he pays away the accumulated interest on the loan and Rs. 50, then Rs. 25 on May 1st, and the rest on Sep. 1st. If interest is charged at the rate of 9% on the money due, how much has he still to pay on Sep. 1st? Reckon each month as $\frac{1}{12}$th of a year.

16. A labourer pledges his vessel and borrows Rs. 3 paying interest at the rate of half-an-anna per rupee per month. To what rate per cent. per annum is this equivalent? Find also what sum he pays to redeem the vessel at the end of 2 years 8 months.

17. Find the interest on £x for 1 month at 5 per cent. per annum. Use your result to find the interest on £25 10*s*. for one month at the same rate to the nearest penny.

18. Find the amount of Rs. p in 4 years at $3\frac{1}{8}$ per cent., and from your result find the amount of Rs. 192 in the same period at the same rate.

CHAPTER XXII.

AREAS OF TRIANGLES, PARALLELOGRAMS. THEOREM OF PYTHAGORAS.

§ 174. Area of a right-angled triangle. It has been established that the area of a rectangle whose length is 4 inches and breadth 3 inches is 12 square inches, and in general, the area of a rectangle whose adjacent sides are a and b units is ab corresponding square units of area.

Let ABCD be a rectangle. Join BD. Now the diagonal BD divides the rectangle into two right-angled triangles which are congruent. Therefore the $\triangle$ DBC $= \frac{1}{2}$ the rectangle ABCD, *i.e.*, $\frac{1}{2}$ the length BC $\times$ the breadth CD $= \frac{1}{2}$ BC·CD.

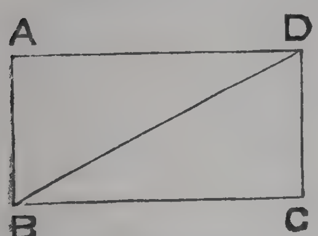


FIG. 171.

Thus **the area of a right-angled triangle is half the product of the sides containing the right angle.**

The result can be verified by drawing the figure on squared paper and counting squares.

Exercise—Practical.

1. Construct on squared paper a right-angled triangle having its sides about the right angle equal to 3 inches and 4 inches. Calculate the area of the triangle. Verify your answer by counting squares.

2. Construct a triangle having its sides equal to 6, 8, 10 cm Measure the angles and calculate the area of the triangle.

3. The area of a right-angled triangle is 30 sq. inches. One of the sides containing the right angle is 6″; what is the other side? Draw the triangle on squared paper and verify the area.

4. Construct a triangle having the sides equal to 13 mm., 12 mm. and 5 mm. On the longest side draw a perpendicular from the opposite vertex. Find the area of the whole triangle by adding the areas of the two small triangles and verify your answer by calculating the area of the whole triangle.

5. Find the area of each of your set squares.

§ **175. To find the area of a parallelogram.** ABCD is a parallelogram. From A and D draw AE and

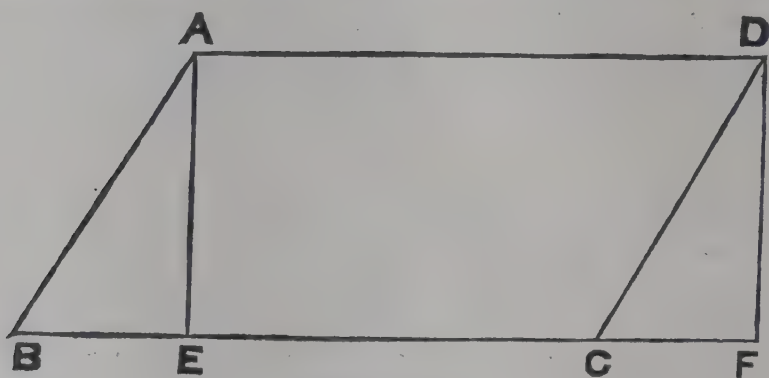


FIG. 172.

DF ⊥ to BC. The two triangles ABE and DCF are congruent; because the hypotenuses AB and CD are equal (being the opposite sides of a parallelogram) and AE and DF are equal being the perpendicular distances between two parallel lines).

∴ the area of the △ ABE = the area of the △ CDF.

From the parallelogram ABCD cut off the △ ABE and put it in the position of the △ CDF. The parallelogram ABCD can thus be fitted into the rectangle AEFD.

∴ ▭^{m} ABCD = rectangle AEFD = AD·AE = base × the altitude (or the perpendicular distance between the parallel sides) of the ▭^{m}.

Hence learn that (1) **a parallelogram and a rectangle on the same base and between the same parallels are equal,** (2) **the area of a ▭^{m} = the base × the altitude.**

In this figure there are two parallelograms, *viz.*, ABCD

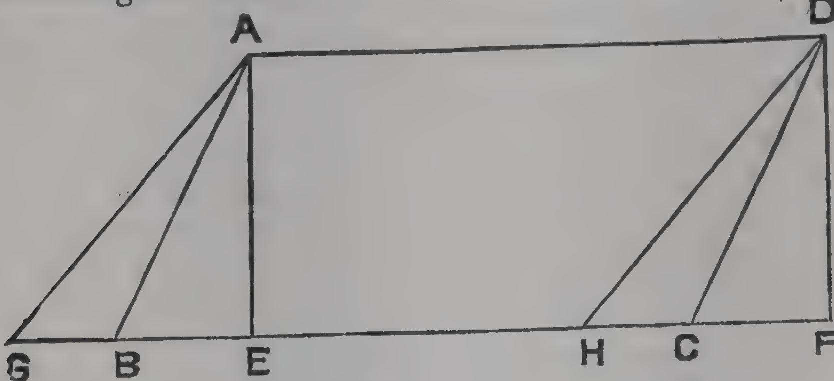


FIG. 173.

and AGHD standing on the same base AD and between the same parallels AD and BC. Now the ▭^{m} ABCD = the rectangle AEFD (by the previous result). The ▭^{m} AGHD = the same rectangle AEFD (by the previous result). ∴ the ▭^{m} ABCD = the ▭^{m} AGHD.

Or thus: △^{s} AGB, DHC can be shown to be congruent; cut off the triangle AGB from the parallelogram AGHD and put it in the position DHC. The parallelogram AGHD can thus be fitted into the parallelogram ABCD.

Hence learn that **parallelograms on the same base and between the same parallels** (*i.e.*, **of the same altitude) are equal.**

Similarly it can be shown that **parallelograms on equal bases and between the same parallels are equal.**

Exercise XXII (a).

1. Construct a parallelogram having its sides equal to 1·6″ and 2·4″ and the included angle equal to 70°. Find the area of the parallelogram in two ways and find the average of the two results.

2. Construct a parallelogram having given two sides to be 2·8″ and 1·6″, and one of the diagonals 2·8″. Find the area of the parallelogram by reducing it to an equivalent rectangle.

3. Construct a parallelogram having given a side to be 2 inches and the two diagonals to be 2 and 2·5 inches. Find the area in two ways and give the average of the two results.

4. The area of a parallelogram is 3·25 sq. in. The base is 1·6". Find the length of the other side given that the angle between the two sides is 40°; draw the parallelogram on squared paper and check the area by counting squares.

5. Two parallelograms have the same area and the same base of 2·5". The adjacent side of the first parallelogram is 1·8" and makes an angle of 48° with the base. If the adjacent side of the second parallelogram is 2", find the angle between the two sides of the second parallelogram.

6. If in the previous question the angle between the adjacent sides of the second parallelogram is 72°, find the length of the other side.

7. ABCD is a parallelogram. In AB take any point P. From P draw PE ⊥ to DC. Show that the area of the ▭^{m} is equal to the area of a rectangle on an equal base and of the same altitude by cutting off the quadrilateral APED and fitting it to PBCE so as to form a rectangle.

8. Construct a rhombus equal to a parallelogram.

§ 176. Area of a triangle. Let ABC be a triangle. Draw AD ⊥ to BC. From B and C draw perpendiculars BE and CF to meet the line drawn through A parallel to BC.

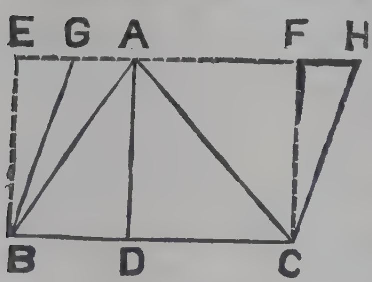


FIG. 174.

$\triangle$ ABC = $\triangle$ ABD + $\triangle$ ADC = $\frac{1}{2}$ the rectangle ADBE + $\frac{1}{2}$ the rectangle ADCF = $\frac{1}{2}$ the sum of the rectangles ADBE and ADCF, *i.e.*, $\frac{1}{2}$ the rectangle BCEF.

Or thus: $\triangle$ ABC = $\triangle$ ABD + $\triangle$ ADC

= $\frac{1}{2}$ AD. BD + $\frac{1}{2}$ AD. DC (by a previous result)

= $\frac{1}{2}$ AD (BD + DC) = $\frac{1}{2}$ AD. BC.

Hence learn that (1) **the triangle is half the rectangle**

on the same base and of the same altitude; (2) **the area of a triangle = ½ the base × the altitude.**

i.e., if $\triangle$ be the area of a triangle, b its base and h the altitude, $\triangle = \frac{1}{2} bh.$

Again the $\square^m$ GBCH and the rectangle EBCF stand on the same base BC and between the same parallels.

$\therefore$ $\square^m$ GBCH = the rectangle EBCF.

$\therefore$ the $\triangle$ ABC = $\frac{1}{2}$ the rectangle EBCF

= $\frac{1}{2}$ the $\square^m$ GBCH.

Hence learn that **if a triangle and a parallelogram stand on the same base and between the same parallels the triangle is half the parallelogram,** or in other words, **the parallelogram is double the triangle.**

Exercise XXII (b).

1. Construct a triangle of sides 4 cm., 3 cm., 3·5 cm. Find the area of the triangle in three different ways by drawing the three perpendiculars and take the average of the three results. Check your result by drawing the figure on squared paper and counting squares.

2. The area of a triangle is 30 sq. inches. The base of the triangle is 6·8 inches. Find the altitude.

3. The base of a triangle is 6·9 ft. and the height 3·4 ft. Find its area.

4. Find the area of a triangle whose sides are 18 ft., 19 ft. and 20 ft. respectively.

5. Construct an equilateral triangle on a side of 3 inches. Calculate the area of the triangle.

6. Find the area of a rhombus in terms of the diagonals.

7. Find the area of a square in terms of its diagonal.

8. In a triangular field ABC the angle A is 52°. the sides AC and AB are 48 yards and 54 yards. Find the side CB and the area of the field.

9. Two angles of a triangle are 40^0 and 60^0 and the side adjacent to these angles is 3 in. Find the area of the triangle.

10. Two triangles are equal in area; their bases are p in. and q in. respectively. If the altitude of the first is r in., find the altitude of the other.

11. A triangle, base 26 ft., altitude 20 ft. is $\frac{1}{4}$ the area of a rectangle on the same base. Find the length of a side of the rectangle adjacent to the base.

12. Find the area of an isosceles right-angled triangle whose longest side is 8 cm.

§ 177. Triangles equal in area. ABC and BDC are two triangles standing on the same base and AD the line joining their vertices is parallel to the base. Compare the areas of the two triangles ABC, DBC.

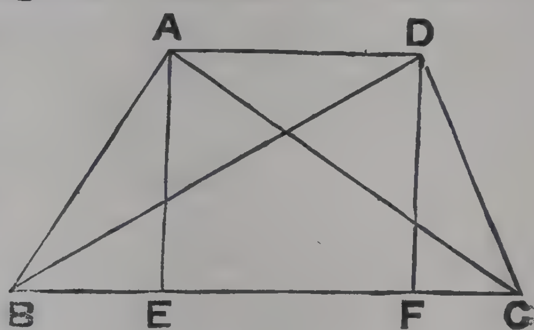


FIG. 175.

From A and D draw AE and DF $\perp$ to BC.

$\triangle$ ABC $= \frac{1}{2}$ the base $\times$ altitude

$= \frac{1}{2}$ BC.AE (1).

Similarly

$\triangle$ DBC $= \frac{1}{2}$ BC.DF (2).

But AE $=$ DF (being the opposite sides of a rectangle).

$\therefore$ $\triangle$ ABC $=$ $\triangle$ DBC.

Hence learn **that triangles on the same base and of the same altitude,** *i.e.*, **between the same parallels, are equal in area.**

Similarly, **triangles on equal bases and of equal altitudes are equal in area.**

It also follows that **the locus of the vertices of all equal triangles on the same base is a line parallel to the base.**

Exercise XXII (c).

1. ABC is a triangle. D is the middle point of the base BC. Show that the Δs ABD and ADC are equal in area.

2. In the same question trisect BC at D and E. Show that the whole triangle is divided into three equal triangles. Hence give a construction for cutting off a triangle equal to one-third of the whole.

3. Divide a triangle into 5 equal parts.

4. Construct a triangle having its sides equal to 5, 6, 8 cm. On the side 8 cm. construct an isosceles triangle equal in area to the triangle already got.

5. In the previous question on the base 8 cm. construct an equal triangle having one of the other sides = 9 cm. How many solutions will there be?

6. Construct a triangle on a given base 3·2″ long and of given altitude 2·4″. One of the sides makes an angle of 45° with the base. How many solutions are there?

7. Construct a triangle equal in area to a $\square^m$.

8. Construct a triangle equal in area to a square.

9. Draw a parallelogram equal in area to a given triangle.

10. Describe a rectangle equal to a given triangle.

11. Describe an isosceles triangle equal in area to a given parallelogram.

12. Describe an isosceles Δ equal in area to a given square.

§ 178. Right-angled triangle. Construct a triangle ABC having its sides BA, AC and BC equal to 3, 4 and 5-tenths of an inch. Measure the angle opposite to 5. You will find that it is a right angle. Describe squares on the sides and divide them up into small squares a side of each being $\frac{1}{10}$ in. The square on BC contains 5 × 5 or 25 small squares, the square on AC, 16 small squares and the square on AB, 9 small squares.

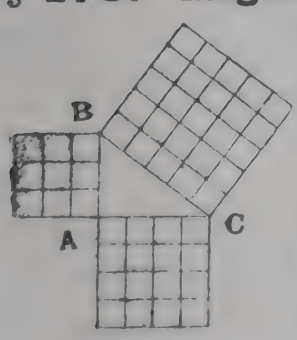


FIG. 176.

Since $25 = 16 + 9$ we see that the sq. on BC = sq. on AB + sq. on AC.

Draw another right-angled triangle having the sides containing the right angle equal to 12 cm. and 5 cm. Then by measurement the side opposite the right angle will be found to be 13 cm.

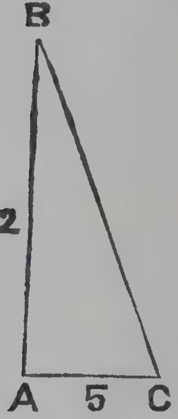


FIG. 177.

Now $13^2 = 169$; $12^2 = 144$ and $5^2 = 25$. $\therefore 13^2 = 12^2 + 5^2$. Construct on squared paper half a dozen right-angled triangles and see if the square on the hypotenuse is in each case equal to the sum of the squares on the other two sides.

Hence learn that **the square on the hypotenuse is equal to the sum of the squares on the sides containing the right angle.**

This proposition may be proved thus :—

I. AGFB and KGHE are two squares having their sides AG and KG in the same straight line. From AG cut off AC = KG and produce GH to D making FD = KG. Join DE, DB, CE and BC. Now the $\triangle$s ABC, CKE, HED, DFB are equal. ($\because$ AB = CK = HD = BF and AC = KE = EH = FD and the contained angles are right angles). Cut off the triangle ABC (marked 1 in the figure) and put in the position of the $\triangle$ HED (marked 3). Similarly cut off the $\triangle$ KCE (marked 2 in the figure) and put it in the position of the $\triangle$ DFB (marked 4). Thus the squares AGFB and KGHE can be fitted into the square BCED, *i.e.*, the square on BC = the square on AB + the square on KG, *i.e.*, AC.

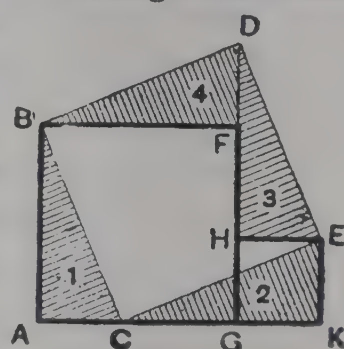


FIG. 178.

II. Choose the bigger of the two squares on the sides containing the right angle. Draw two straight lines through its centre (the point of intersection of the diagonals) one of them perpendicular and the other parallel to the hypotenuse. Cut up the square into 4 parts along those lines and arrange them within the square on the hypotenuse in the places marked, 1, 2, 3, 4. By so doing a square space in the centre will be formed which will be exactly equal to the other square not cut up.

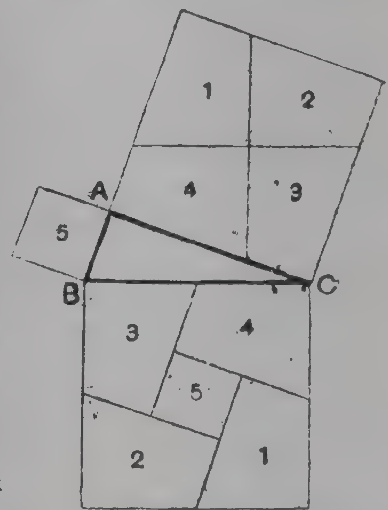


FIG. 179.

These two experiments prove that the square on the hypotenuse is equal to the sum of the squares on the sides containing the right angle.

*III. ABC is a triangle right-angled at B. On AC, AB, BC describe the squares AEDC, ABHK, BCFG. Join KC and EB. Through B draw BL parallel to AE.

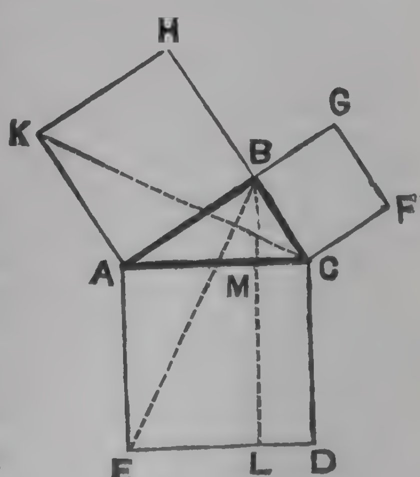


FIG. 180.

The two triangles BAE and KAC are congruent $\because$ AB$=$AK, AE$=$AC and the angle BAE$= \angle$KAC (for each is $90^\circ + \angle$BAC).

The square AH and the $\triangle$ KAC are on the same base KA and beetween the same parallels.

$\therefore$ square AH $= 2 \triangle$ KAC.

Similarly, the rectangle AL $= 2 \triangle$ BAE.

But the $\triangle$ KAC $= \triangle$ BAE (proved).

$\therefore$ square AH = rectangle AL.

Similarly, square BF = rectangle CL.

$\therefore$ AH + BF = AL + CL = AD,

i.e., the square on the hypotenuse is equal to the sum of the squares on the sides containing the right angle.

Exercise XXII (d).

1. If ABC be a $\triangle$ right-angled at C and if a, b, c be its sides, what is the relation between a, b and c?

2. Two persons start from a place O and travel in the northern and eastern directions respectively at the rate of 6 miles and 8 miles an hour. How far will they be apart at the end of an hour and a half? (Find the distance by calculation. Verify your answer by measurement).

3. Find, by calculation, the diagonal of a rectangle whose sides are 9 ft. and 12 ft. Check your result by measurement.

4. Construct a square equal to the squares on 2·3" and 2·4". Find the length of a side of the square constructed.

5. Calculate the altitude of an equilateral $\triangle$ described on a base 4" long.

6. A ladder placed with its foot 6 ft. from the base of a wall reaches a point on the wall 8 ft. above the ground. Calculate the length of the ladder. Verify by measurement.

7. The side of an equilateral triangle is 6 inches. Find the length of the perpendicular drawn from the vertex to the base.

8. Two ships are observed from a signal station to bear N.-E. and N.-W., and their distances from the station are respectively 12 miles and 5 miles; how far are they apart?

9. A vessel on leaving port sailed due North for 6 hours at the rate of 5 miles an hour and then due East for 5 hours at the rate of 8 miles an hour. If she took 10 hours to return to port in a direct line, at what rate did she sail?

10. A vessel making for a port 25 miles distant reaches it in 5 hours by sailing for half this time due East at the rate of 8 miles

an hour and for the rest due North ; at what speed must she have sailed northward ?

§ 179. Horizontal and vertical lines. Hold a small stone in your hand and allow it to drop. It falls in a straight line being attracted towards the centre of the earth. Its path is said to be **vertical.** If one end of a piece of thread is tied to a weight and allowed to hang freely, the direction in which the thread stands when it is at rest is *vertical.* This is exactly how a mason tests with a *ptumb line* whether a wall is upright or not. (See Fig. 181).

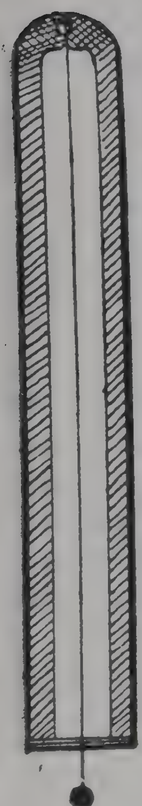


FIG. 181.

Draw a straight line on the drawing board and make a pencil stand erect on it. The pencil is in a vertical position and makes a right angle with the line on the board. This line on the board which is at right angles to the vertical line is called a **horizontal line.**

We may draw a number of straight lines on the board passing through the foot of the pencil ; all these lines are at right angles to the pencil and hence horizontal. Thus any *number of horizontal lines* can be drawn through a point but *only one vertical* can be drawn passing through the point. It is plain that all such horizontal lines lie on a plane called **the horizontal plane.** The surface of a sheet of water in a vessel or of a calm pond is a very good example of a horizontal surface.

Mark a point P on a pole in a level with your eyes. Go back a few yards from the pole and hold your pencil with one end close to your right eye and the other end pointing to the mark P. Now, without changing the position of the

eye and that of the pencil end near the eye, direct the pencil to the top of the pole. The angle through which the pencil has been raised or *elevated* is called the **angle of elevation** of the top of the pole. If instead of pointing the pencil to the top suppose you direct it to the foot of the pole, then the angle through which the pencil has been lowered or *depressed* is called **the angle of depression** of the foot.

Hence learn that **the angle contained between a horizontal line and the line joining the object and the point of observation is called the angle of elevation when the object is above the point of observation; and the angle of depression when the object is below.**

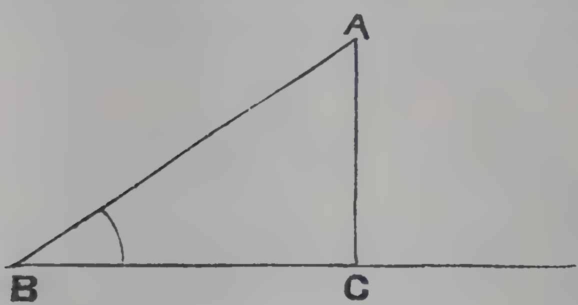


FIG. 182.

Thus if B be the point of observation and A the given object (Fig. 182) the angle made by the line joining B to A with the horizontal BC is called the angle of elevation of A.

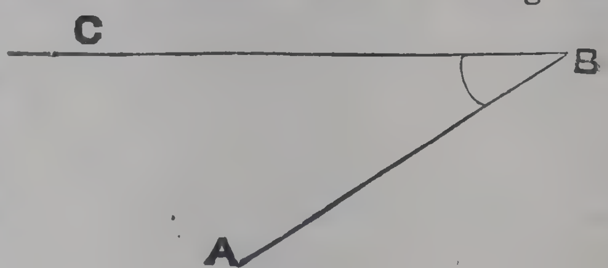


FIG. 183.

In a similar manner if B be the point of observation above A the object (Fig. 183) the angle between the horizontal line BC and the line BA is called the angle of depression of A.

Ex. 1. At a distance of 60 ft. from the foot of a tower the angle of elevation of the top of the tower is found to be 60°. Find the height of the tower.

Let T be the foot of the tower (Fig. 184). Through T draw a horizontal line TA so that TA = 60 ft. (use some convenient scale, say 1 cm. to 20 ft.) At A make an angle of 60° meeting the vertical through T in R. Measure TR in centimetres and convert the measure at the rate of 20 ft. to one cm. (the scale used). The result is the height of the tower in feet.

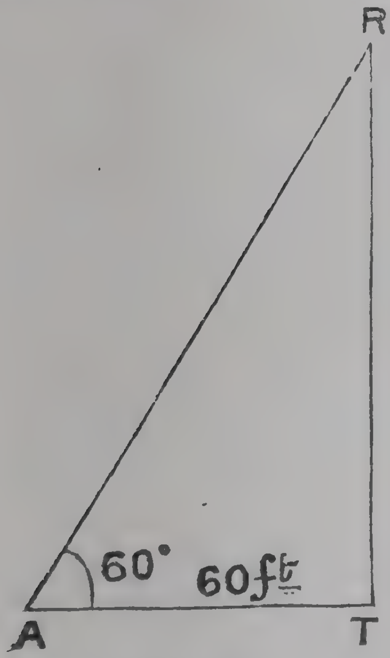


FIG. 184.

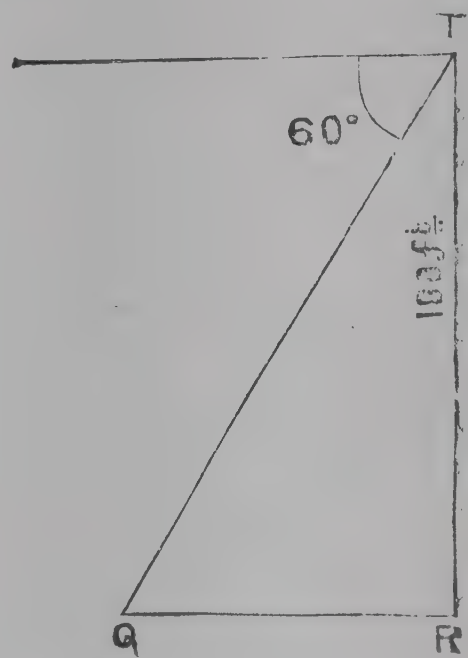


FIG. 185.

Ex. 2. From the top of a tower the height of which is 100 ft. the angle of depression of an object on a straight level road on a line with the base of the tower is found to be 60°. Find the distance of the object from the tower.

Draw a vertical line TR = 5 cm. (Fig. 185) scale 20 ft. to 1 cm.) Through T draw a horizontal, and draw TQ making an angle of 60° with the horizontal meeting the horizontal through the foot of the tower at Q. Measure QR and calculate the required distance as in Ex. 1.

Exercise XXII (e).

1. The angles of elevation of a tower as observed from two points A and B and 500 ft. apart, in the same horizontal line with the base of the tower, are 60° and 25° respectively. Find the height of the tower.

2. The angle of elevation of the top of a steeple is 25°. From a point 40 yds. nearer, the angle of elevation is 45°. What is the height of the steeple ?

3. A vertical flag-staff 40 ft. high stands on a horizontal plane. Find the angle of elevation of the top of the flag-staff from a point on the horizontal plane 25 ft. from the foot of the flag-staff.

4. A torpedo-boat passes at a distance of 200 yds. from a fort the guns of which are 180 ft. above sea-level ; to what angle should the guns be depressed so that they may point straight at the torpedo-boat ?

5. An observer stands at a distance of 100 yds. from the foot of a hill and finds the angle of elevation of the top of a temple on the hill to be 60°. If the height of the observer be 6 ft. and the height of the temple 80 ft., find the height of the hill.

6. Two poles are 30 ft. and 40 ft. high, and 50 ft. apart. A boy stands at a suitable distance from the poles and sees their tops in a line with his eye. Find the angle of elevation of the top of either pole and the distance of the bottom of each pole from the boy.

7. A flag-staff stands on the top of a tower. At a distance of 50 ft. from the base of the tower the angles of elevation of the top of the tower and that of the flag-staff are found to be 35° and 50°. Find the length of the flag-staff and the height of the tower.

8. From the top of a mast 60 ft. high the angle of depression of a buoy is 20°. From the deck it is 6°. Find the distance of the buoy from the ship.

9. A man walking along a road observes the angle of elevation of the top of a building situated on the road-side and finds it to be 30°. He walks 10 yds. towards it and then finds the angle to be 35°. What is the height of the building ?

10. At a window 20 ft. from the ground a tree subtends an angle of 45°. If the angle of depression of the foot be 12°, find its height.

11. At two points on opposite sides of a palmyra tree and in a line with it the angles of elevation of its top are 35° and 40°. If the distance between the points be 80 ft., find the height of the tree.

12. An observer in a balloon, 1500 yds. high observes the angle of depression of a temple to be 42°. After ascending vertically for 15 minutes he observes the angle of depression to be 50°. Find the rate of ascent in miles per hour.

13. An observer finds that the line joining P and Q subtends a right angle at a point R. PR=150 yds., QR=200 yds. Calculate the length of PQ and verify by measurement.

14. If, in the previous question, the observer walks from R towards P 100 yds., find the angle subtended at this point by PQ.

15. A man on the top of a hill sees a level road in the valley running straight away from him. He notices two consecutive mile stones on the road and finds their angles of depression to be 32° and 15°, respectively. Find the height of the hill.

16. Find the area in squares of your squared paper of each of the figures formed by joining in succession the points given in each set by dividing up the figures into rectangles and right-angled triangles.

(i) (3, 1), (11, 1), (9, 7), (8, 4).

(ii) (6, 0), (4, 5), (8, 9), (11, 12).

(iii) (0, 0), (2, 3), (6, 8), (10, 11).

17. Draw a rectangle on base 10 cm. and of altitude 8 cm. On the same base construct an equivalent parallelogram of angle 18° and measure its longer diagonal.

18. Transform a parallelogram of sides 8·5 and 12·6 and angle 15° into a rhombus of equal area with sides equal to 8·5 cm. Measure the angle of the rhombus.

19. Find the area of a ▭^{m} having its diagonals equal to 3·6″ and 2·4″ and the angle between the diagonals 55°.

20. Find the area of a rhombus of side 2 inches and angle 60°.

21. The vertices of a triangle are the points (3, 0), (— 2, — 4), (— 5, — 8); find its area by measuring the sides and altitudes. Verify your result by counting squares.

22. Transform a given triangle into a right-angled triangle of equal area.

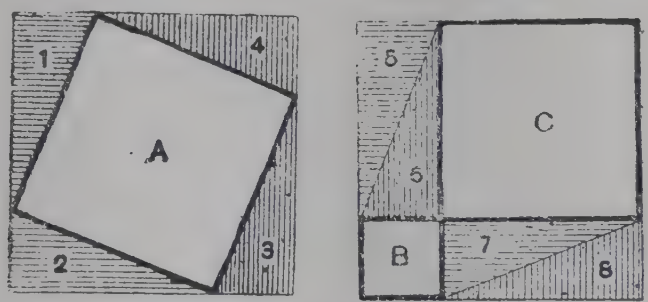


FIG. 186.

23. Another proof of the Pythagoras' Theorem (see Art. 178) is shown in the accompanying figure (186). Prove it with the aid of this figure.

24. A flag-staff 36 ft. high is held up by several each 45 ft. long ropes. Each rope is fastened at one end to the top of the flag-staff and at the other end to a peg in the ground. Find the distance between the peg and the foot of the flag-staff.

25. Calculate the distance from one corner to the opposite corner of (1) your slate, (2) the board in your class-room, and verify by measurement.

26. Two men are conversing across a street 20 ft. wide from the windows of their respective rooms. Their heads are 12 ft. and 27 ft. above the level of the pavement. Find the distance between the men.

27. Show that the triangle whose sides are $2n$, $n^2 - 1$ and $n^2 + 1$ units must be right-angled. Give 10 different values to n and obtain 10 different sets of three lengths that can be the sides of a right-angled triangle.

28. What is the remaining side of a right-angled triangle which has its hypotenuse $= x$ in. and one side $= y$ in.?

29. Calculate the square of the distance between the following pairs of points: (*a*) (3, 1), (4, 8); (*b*) (0, 0), (3, 1); (*c*) (x_1, y_1) and (x_2, y_2).

30. Calculate the squares of the lengths of the sides of the triangle whose vertices are (4, $-$ 4) (0, $-$ 6) and ($-$ 4, 1).

CHAPTER XXIII.

MISCELLANEOUS.

Proportional Parts and Partnership.

§ 180. Here we deal with quantities which have to be divided into a number of parts in a particular way, usually in a given ratio or proportion.

Example 1. Divide the number 1045 into 3 parts in the ratio of 5 : 4 : 2.

The ratio $5x : 4x : 2x$ is the same as the ratio $5 : 4 : 2$ We may suppose that the 3 parts are $5x$, $4x$ and $2x$.

Then, by the question $5x + 4x + 2x$ or $11x = 1045$. $\therefore x = 95$.

$\therefore$ the three parts are 5×95, 4×95, 2×95, *i.e.*, 475, 380, 190. (Check : $475 + 380 + 190 = 1045$).

Example 2. Divide Rs. 1240 in the ratio of $\frac{1}{2} : \frac{1}{3} : \frac{1}{5}$.

The ratio $\frac{1}{2} : \frac{1}{3} : \frac{1}{5} = \frac{1}{2} \times 30 : \frac{1}{3} \times 30 : \frac{1}{5} \times 30 = 15 : 10 : 6$.

Hence we may take Rs. $15x$, Rs. $10x$ and Rs. $6x$ to be the required parts.

Then, by the question $15x + 10x + 6x = 1240$, *i.e.*, $31x = 1240$. $\therefore x = 40$.

$\therefore$ the parts are Rs. 15×40, Rs. 10×40, Rs. 6×40, *i.e.*, Rs. 600, Rs. 400, Rs. 240.

(Check : $600 + 400 + 240 = 1240$).

Example 3. Three partners respectively contribute Rs. 4000, Rs. 3000, Rs. 7500 towards the capital in a business, and the profits amount to Rs. 580. How should the profits be divided between them ?

The profits shared by them must be proportional to their investments ; $\therefore$, we have to divide Rs. 580 in the ratio of

4000 : 3000 : 7500, *i.e.*, 40 : 30 : 75, *i.e.*, 8 : 6 : 15. We may take Rs. $8x$, Rs. $6x$ and Rs. $15x$ to be the parts.

Then $8x + 6x + 15x = 580$.

$\therefore x = \frac{580}{29} = 20$.

$\therefore$ Rs. 8×20, Rs. 6×20, Rs. 15×20 are the parts required, *i.e.*, Rs. 160, Rs. 180, Rs. 300. (Check : $160 + 120 + 300 = 580$.)

Exercise XXIII (a).

1. Divide 689 into 3 parts in the ratio of 3 : 4 : 6.

2. Divide Rs. 850 in the ratio of $\frac{1}{3} : \frac{1}{6} : \frac{1}{8}$.

3. A worked for 6 days, B for 8 days and C for 10 days and their total wages amounted to Rs. 3-12-0. Find how much each man received.

4. Three men contribute Rs. 6800, Rs. 14400, Rs. 3000 respectively to a business and make a profit of Rs. 1573. Find how this profit should be divided amongst them.

5. A sum of money was divided among A, B and C in the proportion of 8, 4 and 7. Rs. 56-1-4 was the amount of A's share. What was the sum of money ?

6. A man divided his property worth Rs. 16800 into two parts in the ratio of 9 : 5. He left the first part in equal shares to 6 daughters and the other part in equal shares to 8 sons. How much did each daughter and each son receive ?

7. Divide Rs. 19840 among 24 men, 36 women and 72 children giving each man twice as much as each woman and each woman twice as much as each child.

8. On a certain railway the cost of a first class ticket is double that of a second class ticket and that of a second class ticket is treble that of a third class ticket. If 3 first class tickets, 2 second class tickets and 1 third class ticket for a journey of 200 miles cost Rs. 52-1-4, find the price of each ticket and the second class rate per mile.

9. Divide Rs. 7000 among A, B and C, such that A's : B's :: 5 : 3 and B's : C's :: 1 : 2.

10. 3 boys A, B and C gain 48, 96, and 105 marks respectively in an examination, and the examiner alters the marks proportionally so that C gets 100. Find to the nearest integer the altered marks of A and B.

11. A person with a monthly income of Rs. 328 spends as much in 8 months as he earns in 6 months. After 6 years he divides his savings among his three children in such a manner that the eldest has twice as much as the second and thrice as much as the youngest. How much does eaeh receive ?

12. A certain number of men, twice as many women and three times as many boys together earn Rs. 250. Find the number of men, women and boys, supposing that a man earns Rs. 4, a woman Rs. 2 and a boy Re. 1-8.

Time and Work.

§ 181. *Example* 1. A can do a piece of work in 4 days, B in 6 days, and C in 12 days. In what time can the three do it together ?

In 1 day A can do $\frac{1}{4}$ of the work

............B............$\frac{1}{6}$..................

............C............$\frac{1}{12}$..................

$\therefore$ in 1 day A, B and C can do $\frac{1}{4}+\frac{1}{6}+\frac{1}{12}$ or $\frac{1}{2}$ of the work, *i.e.*, A, B and C can do $\frac{1}{2}$ the work in 1 day.

$\therefore$ ....the whole work in 2 days.

[*Check.* In 2 days A does $\frac{2}{4}$, *i e.,* $\frac{6}{12}$

..............B........ $\frac{2}{6}$, *i.e.,* $\frac{4}{12}$

..............C...... $\frac{2}{12}$ and $\frac{6}{12}+\frac{4}{12}+\frac{2}{12}=1$.]

Example 2. A cistern is filled by the pipes A and B in 3 and 4 hours respectively but can be emptied by a tap in 6 hours when full. If all the three are opened and the cistern at first is empty, when will the cistern be full ?

In 1 hr. A and B fill $\frac{1}{3}+\frac{1}{4}$ cis. or $\frac{7}{12}$ of the cistern.

In 1 hr. C empties $\frac{1}{6}$ cis.

$\therefore$ In 1 hr. all the three fill $\frac{7}{12} - \frac{1}{6}$ or $\frac{7-2}{12}$ or $\frac{5}{12}$ cis.

i.e., $\frac{5}{12}$ cis. is filled by all the three pipes in 1 hr.

$\therefore$ 1 cis...$\frac{12}{5} = 2\frac{2}{5}$ hrs.

[*Check*: $\frac{2\frac{2}{5}}{3} + \frac{2\frac{2}{5}}{4} - \frac{2\frac{2}{5}}{6} = \frac{12}{5}\left(\frac{1}{3} + \frac{1}{4} - \frac{1}{6}\right) = \frac{12}{5}\left(\frac{4}{12} + \frac{3}{12}\right.$ $\left.- \frac{2}{12}\right) = \frac{12}{5} \times \frac{5}{12} = 1.$]

Graphical Solution.

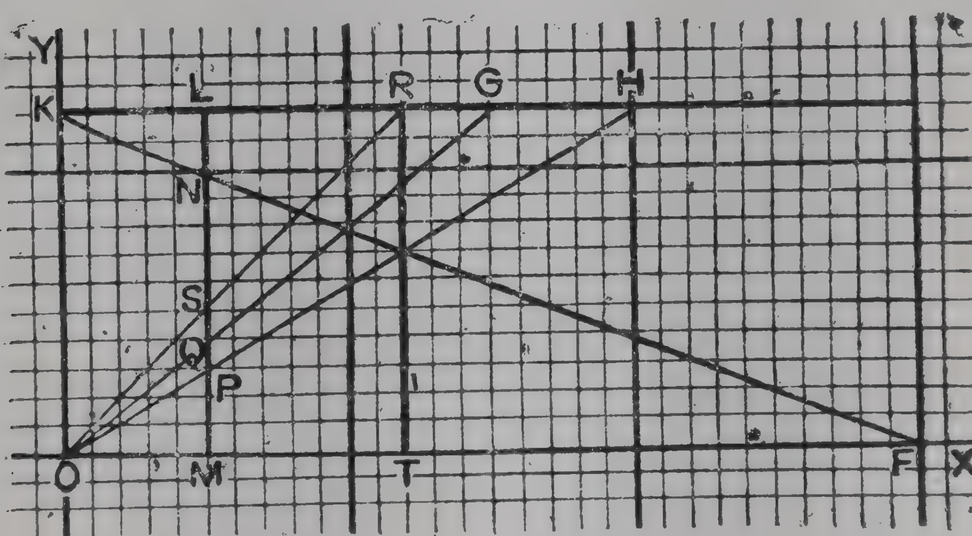


FIG. 187.

Let 12 divisions on the y axis represent the whole cistern. Let 5 divisions on the x axis represent 1 hour. Plot the point G (3, 1). Join it to O. Then OG (Fig. 187) gives the graph of the pipe A. Similarly plot H (4, 1). Join it to O. Then OH (Fig. 187) gives the graph of pipe B. Similarly plot K (0, 1). Join it to the point F (6, 0.) Then KF (Fig. 187) gives the graph of pipe C (since C empties the cistern in 6 hrs. if at the beginning it is full). Note the downward course of this graph and contrast it with the rising slope of the graphs of the 2 pipes A and B.

In 1 hour A supplies water equal to the ordinate MQ

$\therefore$ ....B...MP

$\therefore$ in 1 hour A and B together supply water equal to MP + MQ, and C empties in 1 hour water equal to LN. Hence with your dividers find an ordinate MS = PM + QM − LN. Join S to O. Produce OS to meet the line parallel to the x axis through 1 on the y axis. The point of intersection has for its abscissa $2\frac{2}{5}$ hours which denotes that the whole cistern can be filled in $2\frac{2}{5}$ hours by all the three pipes working simultaneously.

Example 3. A can do in 2 days as much as B in 5 days. They both finish a piece of work in 10 days. Find in how many days each working alone can do it.

Work done by A in 2 days = work by B in 5 days.

$\therefore$,, 10 days = ,, $5 \times 5 = 25$ days.

$\therefore$ Work done by A and B in 10 days, *i.e.*, the whole work = the work done by B in 25+10, *i.e.*, 35 days,

i.e., B can do the work in 35 days. $\therefore$ A can do it in 14 days.

Exercise XXIII (b).

1. A and B can do a piece of work in 20 days and 30 days respectively. A works alone for the first 5 days when he is joined by B. Find when the work will be completed. Illustrate your answer graphically.

2. If, in the previous question, at the end of 5 days A stopped and B alone did the work, in how many days would B have completed the work. Illustrate graphically.

3. A can do a piece of work in 8 days, B in 12 days and C in 16 days. In what time will they finish the work together? Work out graphically.

4. A, B and C can do a piece of work in 8 days. A and B in 12 days, B and C in 14 days. Find in what time each of them will do the work.

5. A and B can do a piece of work in 6 days, B and C in 8 days and A and C in 10 days. In what time will (i) they all do the work together? (ii) each do it separately?

6. A in 4 days can do as much as C in 5 days and B in 8 days as much as C in 7 days. What time will C require to finish a piece of work which they all working together can do in 25 days?

7. A is thrice as good a workman as B. If they both finish a piece of work in 8 days, in what time will they do it separately?

8. A and B undertake to do a piece of work for Rs. 10; A could do it alone in 14 days; B could do it alone in 16 days. How should the money be divided?

9. 8 men or 16 women or 12 boys can do a piece of work in 15 days. In how many days will 4 men, 5 women and 6 boys do it?

10. A person employs 129 men to make a railway 126 miles long in 12 months. But after 3 months he finds that he has finished only 21 miles. How many more men should be employed to finish the work within the time required?

11. A's rate of working is to B's as 3 : 4; B's to C's as $\frac{1}{4} : \frac{1}{5}$; C's to D's as $\frac{1}{5} : \frac{1}{6}$, how long would they altogether take to do a piece of work which A could do in 250 hours?

12. A besieged garrison has a supply of water for 30 days. Owing to a leak, however, in the bottom of the reservoir 20 gallons waste away every day. The supply lasts for 25 days. Find the quantity of water in the reservoir.

13. Two taps A and B fill a cistern in 6 and 9 hours respectively. In what time will they fill it together? Illustrate your answer graphically.

14. A cistern is filled by two taps in 10 and 15 hours respectively and is emptied by a tap in 8 hours. If all the three taps are open, in what time will the cistern be filled?

15. A cistern is filled by two pipes in 10 and 12 minutes respectively and emptied by a tap in 15 minutes, what part of it will be filled in 5 minutes when they are all left open together?

16. A cistern having a leak in the bottom is filled in 10 hours. Had there been no leak it could have been filled in 8 hours. If the cistern is full, find in what time the leak can empty it.

17. A bath is filled by a pipe in 15 minutes and is emptied by a waste pipe in 12 minutes. If the waste pipe also is opened after the bath has been half filled by the first pipe, find the time in which it will be emptied.

18. Three boys begin to fill a cistern. One of them brings a pint at the end of every 4 minutes, another a quart every 6 minutes, and the third a gallon every 8 minutes. If the cistern holds 110 gallons, in what time will it be filled?

Time and Distance.

§ 182. *Example* 1. A and B start from a place P and walk in the same direction at 3 miles and 4 miles an hour. How many miles will they be apart at the end of 5 hours and after what time will they be 8 miles apart?

1. If they walk for 1 hour they are 1 mile apart.

∴ ....5 hours............5 miles apart.

2. To be 1 mile apart they must walk for 1 hour.

.........8 miles...................................8 hours.

[*Check*: **1.** $5 \times 4 - 5 \times 3 = 5$.

2. $8 \times 4 - 8 \times 3 = 8$.]

Example 2. If, in the previous example, they walk in opposite directions how many miles will they be apart at the end of 5 hours and after what time will they be 28 miles apart?

1. At the end of 1 hour they are 7 miles apart

..................5 hours............35 miles apart.

2. They are 7 miles apart in 1 hour.

∴ ....28..................4 hours.

[*Check*. $5 \times 4 + 5 \times 3 = 35$.

$4 \times 4 + 4 \times 3 = 28$.]

Example 3. A goods train starts from Madras at 1 A.M. and reaches a place 120 miles from Madras at 10 A.M. Find its average speed; and assuming that it goes at the same rate, find how many miles will it be distant from Madras at 9-30 A.M.? When will it be at the 100th mile?

In 9 hours (the interval between 1 A.M. and 10 A.M.) it travels 120 miles

$\therefore$ in 1 hour........................$\frac{120}{9}$, *i.e.*, $\frac{40}{3}$ or $13\frac{1}{3}$ miles.

In $8\frac{1}{2}$ hours (the interval between 1 A.M. and 9-30 A.M., it must be $\frac{40}{3} \times \frac{17}{2}$ or $\frac{340}{3}$ or $113\frac{1}{3}$ miles distant from Madras.

To travel 100 miles it takes $\frac{100}{13\frac{1}{3}}$ or $7\frac{1}{2}$ hrs. $\therefore$ it will be at the 100th mile at 8-30 A.M.

Example 4. I leave a place P at 2-55 P.M. and walking at 3 miles an hour reach a place Q 15 miles distant from P. I find I am half an hour late. At what rate should I walk to reach Q in time?

It takes 5 hours for me to walk 15 miles at 3 miles an hour; so I reach the place at 2-55 + 5 or 7-55 P.M. I am late by 30 minutes. I must be there at 7-25 P.M. To reach Q at 7-25, starting at 2-55 P.M., I must walk 15 miles in 4 h. 30 m.

$\therefore$ My rate of walking $= 15 \div 4\frac{1}{2}$ miles per hour.

$= \frac{15 \times 2}{9}$ or $3\frac{1}{3}$ miles an hour.

Exercise XXIII (c).

1. A and B start from the same place at the same time and travel in the same direction walking at the rate of 10 and 12 miles an hour respectively. In how many hours will they be 20 miles apart? Find how many miles apart they will be at the end of $5\frac{1}{2}$ hours. Illustrate your answer graphically.

2. A and B set out from X to Y a distance of $31\frac{1}{2}$ miles walking at $3\frac{1}{2}$ and $4\frac{1}{2}$ miles an hour respectively. If both leave X at the same time, what will be the difference in time between A's and B's arrival at Y?

3. In example (2) how many hours later than A should B start in order that they might reach Y at the same time?

4. A steamer sailing at the rate of 20 miles an hour discovers a boat 24 miles away making way at 12 miles an hour. How many miles must the steamer sail before the boat is overtaken?

5. Two clocks are set right at the same time. One gains $1\frac{1}{2}$ minutes a day and the other $\frac{3}{4}$ minute a day. When will the times indicated by them differ by 9 min.?

6. A man starts for a walk at 4 miles an hour. Five hours later, a cyclist starts after him at 10 miles an hour. How long after will the cyclist overtake the man? How far will they then be from the starting point?

7. A man goes from A to B at 3 miles an hour and returns at 4 miles an hour. What is his average rate for the whole journey?

8. From A to B is 40 miles. An express train travelling at 30 miles an hour leaves A 50 minutes after a slow train, and on reaching B, finds that the slow train is signalled as having left the previous station 5 miles away. Find the rate of the slower train assuming that it travels uniformly.

9. How long will an express 120 yds. long running at the rate of 40 miles an hour take to pass a man standing on the platform of a station?

10. How long will a train 80 yds. long and running at 20 miles an hour take to pass completely a station platform 300 yds. long?

11. A man can row 5 miles per hour in still water. He rows 5 miles down stream in 45 minutes. How fast is the stream flowing?

12. A man can row 5 miles per hour in still water. He rows 5 miles up stream in $1\frac{1}{4}$ hours. At what rate is the stream flowing?

13. If a man rows at 8 miles an hour down stream and at 4 miles an hour up stream, at what rate does he row in still water, and at what rate does the stream flow?

14. A boy leaves Madras at 12-30 P.M., travels by train to Pallavaram 14 miles distant at 15 miles an hour and cycles to Chingleput (28 miles) at 10 miles an hour from Pallavaram. When does he reach Chingleput?

15. At what time must a motor-car leave Villupuram so as to reach Chingleput (70 miles) at the same time as the boy of Ex. 14 travelling 20 miles an hour for the first 25 miles and the rest at 30 miles per hour?

16. A starts from P at 6 A.M. and walks to Q ($136\frac{1}{2}$ miles) at $3\frac{1}{2}$ miles an hour. B starts an hour later from P and reaches Q at the same time as A. What was B's speed?

Profit and Loss.

§ **183.** *Example* 1. A horse which costs Rs. 200 is sold for Rs. 250. Find the gain per cent. If he were sold for Rs. 180, what would be the loss per cent ?

The gain = Rs 250 — 200 = Rs. 50.

i.e., the gain on Rs. 200 is Rs. 50.

∴ ....100 ....25.

∴ the gain is 25°/₀.

If sold for Rs. 180 there would be a loss of Rs. 20. The loss on Rs. 200 would be Rs. 20.

∴ the loss on Rs. 100 would be Rs. 10, *i.e.*, the loss would be 10°/₀.

Example 2. By selling a watch for Rs. 40 a shop-keeper loses 25°/₀. What would be the gain per cent., if it were sold for Rs. 60 ?

If Rs. 75 is the S. P., the C. P. is Rs. 100.

∴ if Rs. 40...................... $\frac{\overset{4}{\cancel{100}}}{\underset{3}{\cancel{75}}} \times 40$ or $\frac{160}{3}$ Rs.

The gain, if the watch is sold for Rs. 60 = Rs. 60 — Rs. $53\frac{1}{3}$ or $6\frac{2}{3}$ or $\frac{20}{3}$ Rs.

On $\frac{160}{3}$ the gain is $\frac{20}{3}$

,, 1.............. $\frac{20}{3} \times \frac{3}{160} = \frac{1}{8}$

,, 100.............. $\frac{1}{8} \times 100$ or $12\frac{1}{2}$. The gain is $12\frac{1}{2}$ °/₀.

Example 3. A person bought 192 mangoes for Rs. 8 and sold them in such a way that he gained the cost price of 24 mangoes. What was each mango sold for ?

C. P. of 192 mangoes = Rs. 8.

∴ C. P. of 24 mangoes = Re. 1

∴ he sold 192 mangoes for Rs. 9

∴ he sold 1 mango for $\frac{9 \times 16 \times 12}{192}$ pies,

i.e., he sold the mangoes at 9 pies each.

Example 4. If, in the previous question, he sold the mangoes so as to gain the selling price of 48 mangoes, how did he sell the mangoes ?

C. P. of 192 mangoes = Rs. 8.

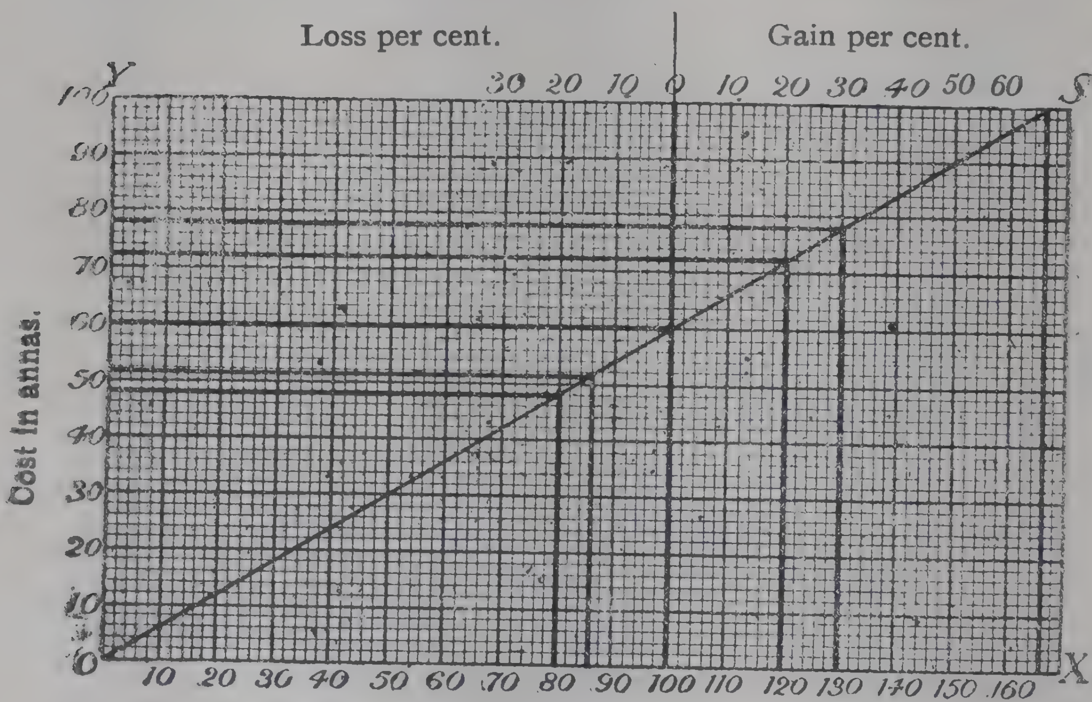
Gain = S. P. of 48 mangoes.

$\therefore$ S. P. of 192 mangoes = C. P. of 192 mangoes + the gain = Rs. 8 + S. P. of 48 mangoes.

$\therefore$ S. P. of 144 mangoes or 12 dozen = Rs. 8 $\times$ 16 or 128 as.

$\therefore$............1 dozen mangoes = $\frac{128}{12}$ as. = 10 a. 8 p.

Example 5. The cost price of an article was Rs 3-12. How much per cent. was gained by selling it at Rs. 4-14-0 ? Rs. 4-8-0 ? What would have been the loss per cent. had it been sold for Rs. 3 ? Rs. 3-3-6 ?



Percentages.

FIG. 188.

On your squared paper set off cost in annas along the y-axis and percentages on the x-axis, (*vide* Fig.188). Since the cost price, *i.e.*, 100 % of it, is Rs. 3-12 or 60 as., find the point where the line

representing 60 as. meets the line corresponding to 100 % and join it to O. Then the co-ordinates of any point on this line will give the percentage and the corresponding price in annas.

Observe that Rs. 4-14 corresponds to a percentage of 130.

$\therefore$ by selling at Rs. 4-14, the gain is 30 %. Similarly by selling at Rs. 4-8-0 the gain is 20 %.

Also observe that by selling at Rs. 3, only 80 % would be realized, *i.e.*, there is a loss of 20 %.

Similarly if the selling price were Rs. 3-3-6, the loss could be seen to be nearly 14 %.

Work out the question by calculation and compare the results. It will be seen from this *that great care is necessary in working by graphs and that the answers would be only approximate.*

Exercise XXIII (d).

1. A person buys 500 copies of a book for Rs. 100. He sells 300 copies at Rs. 2-3-0 per dozen and the rest at 3 as. a copy. How much does he gain or lose, and how much per cent. ?

2. Goods which cost Rs. 40 are sold for Rs. 50. Find the gain per cent. If the cost price is p pounds and the selling price is q pounds, find the gain per cent.

3. An article is sold for Rs. 6-12-6 at a profit of 20 per cent. For what must it be sold so as to gain 12 per cent. ?

4. A dealer buys 40 horses at £25 each ; he sells 12 at £32 each and the rest at £35-10*s*. each ; find his total gain and the gain per cent.

5. A basket woman bought a certain number of fruits at 4 for 3 annas and sold them at 5 for 4 annas thus making a profit of Re. 1-4-0. How many fruits did she buy ?

6. If I get a book of 50 tramway anna-tickets for Rs. 3, instead of buying them separately at an anna each, how much per cent. do I gain by taking a book ?

7. A packet of 100 fancy cards costs 2 annas ; each card requires a quarter-anna stamp to be affixed when posted ; but an East India post card can be purchased for a quarter-anna. Find the loss per cent. in buying fancy cards and affixing postage stamps.

8. In the year 1910, a cart of paddy containing 80 marakkals was sold at the rate of Rs. 40. In the year 1911, five marakkals were sold for Rs. 3. What is the increase or decrease per cent. in the price?

9. A manufacturer of silver jewels makes up the selling price (say, a rupee of it) as follows :—

Cost of silver	... ...	As. 10
,, manufacture	... ...	,, 4
Expenses of selling	... ...	,, 1
Profit ...	... ...	,, 1

Express each item as a percentage of the selling price.

10. Photograph mounts which are sold at Re. 1-8 as per dozen cost Rs. 11-4-0 per 100. Find the gain per cent. in buying by hundreds.

11. A tradesman's sale prices are 20 per cent. above cost price. If an article is sold for Rs. 15-8 as, find the cost price.

12. A buys an article for Rs. 20 and sells it to B at a gain of 15 per cent. B sells it to C at a gain of 20 per cent. What does C give for the article?

13. A tradesman buys articles at Rs. 8 per gross; he divides them into 3 equal lots which he respectively sells at Re. 1 a score, 12 as. a dozen and 1 a. 4 p. each. Find the profit per cent.

14. A person buys a French watch bearing a duty of 25 per cent. and sells it at a loss of 5 per cent. If x is the cost price of the watch, find its selling price.

15. A smuggler buys 6 cwt. of tobacco at 1*s*. 3*d*. per lb.; he meets with a revenue officer who seizes $\frac{1}{6}$ of it; at what rate per lb. must he sell the remainder so as (1) neither to gain nor to lose? (2) to gain 3 guineas? (3) to gain 5 per cent.?

INCOME-TAX.

§ 184. *Example* 1. The income-tax being 4 pies in the rupee, what is the net income on an income of Rs. 160?

On Re. 1 the tax is 4 p.

$$\therefore \quad \text{Rs. } 160 \ldots\ldots\ldots \text{Rs. } \frac{\overset{10}{\cancel{160}} \times \cancel{4}}{\cancel{12} \times \underset{3}{\cancel{16}}} = \text{Rs. } \frac{10}{3} = \text{Rs. } 3\text{-}5\text{-}4.$$

$\therefore$ the net income is Rs. 160 — Rs. 3-5-4 = Rs. 156-10-8.

Example 2. A person after paying an income-tax of 4*d*. in the £ had £147-10*s*. left. Find his gross income.

If the net income is £1—4*d*. or £$\frac{59}{60}$, the gross income is £1,

$$\ldots\ldots\ldots £\frac{295}{2}\ldots\ldots\ldots \frac{\overset{30}{\cancel{60}}}{\cancel{59}} \times \frac{\overset{5}{\cancel{295}}}{\cancel{1}} \text{ or } £150.$$

Or thus : let £x be the gross income :

Then £$\frac{x \times 4}{240}$ is his income-tax.

$\therefore$ the net income is £$\left(x - \frac{x \times 4}{240}\right)$ which is given to be £$147\frac{1}{2}$.

$$i.e., \; x - \frac{x}{60} = \frac{295}{2},$$

$$i.e., \quad \frac{59x}{60} = \frac{295}{2}.$$

$$\therefore \quad x = \frac{295}{2} \times \frac{60}{59} = 150.$$

$\therefore$ the gross income is £150.

Exercise XXIII (e).

1. What is the net income on a gross income of £3000 when the income-tax is 6*d*. in the pound ?

2. Incomes above Rs. 1000 a year are subject to an income-tax of 4 pies in the rupee. Find the net income of a person whose monthly salary is Rs. 125.

3. In the previous example, what income should a person have so that after paying the income-tax his net income might be exactly Rs. 150 (no income tax is calculated on fractions of a rupee) ?

4. If the net income of an estate after paying all taxes be Rs. 775 and gross income be Rs. 800, how much in the rupee do the taxes amount to ?

5. What is the income of a person who, when the income-tax is 7*d*. in the pound pays £8-15*s*. more than when it was 5*d*. in the pound ?

6. One-tenth of a man's income is exempt from income-tax. If the tax he pays, at 5 pies in the rupee, amounts to Rs. 5-2-6, what is his entire income ?

7. When the income-tax was reduced from $6d.$ to $4d.$ in the pound, a person found that he had £14 1$s.$ 3$d.$ less income-tax to pay. What was his income?

8. A person after paying out of his income a tax of 8 pies in the rupee and ·75 of the remainder for his expenses, has Rs. 65-0-9 remaining. What is his gross income?

BANKRUPTCY.

§ 185. A bankrupt is one who is unable to pay his debts, *i.e.*, whose property is less than the amount of his debts.

The possessions of the bankrupt are called his *assets* and his debts are called his *liabilities*.

Book debts are those entered in the book of accounts.

Good debts are those that are recovered in full whereas *bad* debts are those a portion only of which is recoverable.

Example 1. A bankrupt whose assets amount to Rs. 3000, pays his creditors 5 as. 4 p. in the rupee. What is the amount of his debts?

5 as. 4 pies or $\frac{1}{3}$ rupee is paid on Re. 1 of the debts.

$\therefore$ 1........................Rs. 3.

$\therefore$ Rs. 3000........................Rs. 9000.

Example 2. A bankrupt's assets are Rs. 650 but he owes his creditors Rs. 1000. How much in the rupee can he pay?

On Rs. 1000 he pays Rs. 650.

On Re. 1 ...... Rs. $\frac{65\cancel{0}}{100\cancel{0}}$ = Rs. $\frac{13}{20}$ = 10 a. 4·8 p.

Exercise—XXIII (f).

1. A bankrupt pays 10 as. 8 pies in the rupee and his assets amount to Rs. 960. What is the amount of his debts?

2. If a bankrupt's debts amount to £1,600 and his assets to £960, how much in the pound can he pay?

3. A bankrupt owes Rs. 20,000 to two creditors, namely,

Rs. 12,000 to one and Rs. 8,000 to the other; his assets are Rs. 6000, what does each creditor lose?

4. A bankrupt's dividend being at the rate of 12*s.* in the pound, how much is lost by a creditor who has sold him 8 cwt. 2 qr. 8 lb. of goods at £1-2-6 a cwt?

5. The property of a bankrupt amounts to Rs. 3,000. It is estimated that his creditors will receive a dividend of 5 annas in the rupee; but another liability of the value of Rs. 2000 turns up. By how much is the dividend in the rupee reduced?

6. A creditor received on a debt of Rs. 500 a dividend of 10 annas in the rupee and a further dividend of 2 as. upon the remaining debt. What did he receive altogether and what fraction was it of the entire debt?

7. A bankrupt has debts equal in amount to his liabilities. On half his debts he can recover only 8 as. in the rupee. How much can he pay in the rupee?

8. A speculator sells an article at a profit of 75 per cent. but his purchaser fails and pays only 4 as. in the rupee. How much per cent. does the speculator gain or lose by the venture?

REVISION PAPERS—III SERIES.

1.

1. (*a*) Express 35 pence as the decimal of a pound. (*b*) Find the value of 6 per cent. of £9. (Neglect fractions of a penny.)

2. A and B own a field in shares proportioned as 11 : 9. If A's share is $\frac{5}{8}$ of an acre, what is the size of the field in square yards?

3. A sum of money put out at simple interest amounts to £684 when the rate is $3\frac{1}{2}$ per cent. and the time 4 years. What would be the amount if the rate were 3 per cent. and the time 2 years?

4. Plot the graph of the equation $y = 6x + 1$, and from the graph find the value of x when $y = 25$.

5. ABC is an equilateral triangle, each side being 12 inches. Find (*a*) its altitude: (*b*) its area (by formula).

6. If the weight of a square sheet of metal, length of the side being 15 inches, be 12 lbs., what would be the weight of an equilateral triangle of side 9 inches cut from the same sheet?

7. A signals 144 words in 23 minutes, B 95 words in 15 minutes and C 432 words in 21 minutes. Represent the times taken by horizontal distances and the number of words signalled by vertical distances. Mark the three points representing the above results. And find who is the fastest signaller and who the slowest.

8. The sides of a rectangle are 156 yds. and 117 yds respectively. Calculate the length of the diagonal. Verify by measurement. State the scale you choose.

2

1. (*a*) Express 4 tons 11 cwt. 2 qr. as the decimal of 6 tons.
   (*b*) Find the value of 4 ac. 45 cents. at Rs. 150 per acre.

2. An embankment 16 feet high and 80 ft. long is built by 40 men of equal strength in 70 days; another embankment (of the same thickness) 15 feet high and 75 ft. long is built by 50 men (also of equal strength) in 60 days. What is the ratio of the strengths of the two classes of men?

3. From the formula $\frac{P \times T \times R}{100} + P = A$, show that

$$R = \left(\frac{A}{P} - 1\right) \times \frac{100}{T}.$$

£25 amounts in $3\frac{1}{2}$ years to £28·18s.·9d.; what is the rate per cent.? Verify your answer by using the above formula.

4. A clock gains 3 minutes every day. How should its hands be placed at noon that it might show correct time at 8 P.M.?

5. If a, b, c are the lengths of the sides of a right-angled triangle having the angle C a right angle, what is the relation connecting a, b and c? If $c = 82$, $b = 18$, find the value of a.

6. Show that a velocity of 15 miles per hour = 22 ft. per second; and from this realation convert a velocity of 40 miles per hour into feet per second. Draw a graph for converting a velocity in miles per hour into a velocity in feet per second.

7. Three roads, each 600 yds. long form an equilateral triangle. Find the area of the enclosed triangle and its value at £75 an acre.

8. Divide Rs. 540 among A, B and C, so that A may receive Rs. 10 more than a third of what B receives, and C may have as much as A and B together.

3

1. What is the difference in value between a thousand pagodas and a million pies (a pagoda = $3\frac{1}{2}$ Rs.)?

2. A rate of 6 pies in the rupee is levied on a district whose rateable value is Rs. 6,500. What sum does this rate produce?

3. If 20 gallons of wine worth 12s. per gallon are mixed with 64 gal. at 18s. 6d. per gallon, what is the cost per gallon of the mixture?

4. A train leaves Madras at 1 P.M. and reaches Arkonam (42 miles) at 2-15 P.M. Draw a graph representing the motion of the train assuming it to travel uniformly. Find from your graph at what times it passes through (1) Avadi (13 miles); (2) Tiruvallur (27 miles); (3) Thiruvalengadu (30 miles).

5. If a rupee is lent bearing interest at the rate of quarter anna per month, at what rate per cent. is interest charged? Find in what time money will double itself at this rate?

6. Find the value of $(c-a)^2 (3c-b)$ when $c = 12$, $b = 8$ and $a = 5$.

7. Plot the points P (18, -9), Q(3, -24), R (17, 3). Supposing the triangle PQR represents the plan of a field drawn to scale of 1 to 100, find its area in acres.

8. The populations of two towns are 150,000 and 284,000, their birth-rates per 1000 are 32 and 35. Find the birth-rate for the two towns taken together.

4.

1. Express the difference between $3\frac{1}{7}$ and $\frac{355}{113}$ as a vulgar fraction.

2. The paddy produced from 12 cawnies is worth Rs. 650 at Rs. 40 per bandy load of paddy. How many cawnies at the same rate will produce paddy worth Rs. 780 at Rs. 36 per bandy ?

3. In the equation $xy = 144$, find the values of y corresponding to the values; 2, 3, 4, 6, 8, 10, 11, 12, 13, 14, 16, 18, 24, 36, 72 of x. Arrange these in tabular form as co-ordinates ; plot the points on squared paper, and freely draw a curve through these points.

4. With centre O and radius 10 units describe on squared paper a circle ABC. Suppose a point P starts from A (10,0) and travels along the circumference of this circle in a direction opposite to that of the hands of a clock. Write down the co-ordinates of P when the angle AOP is 30°, 60°, 108°.

5. If 36 bullocks can plough 72 acres of land in 25 days, how many days will 39 bullocks take to plough 104 acres at the same rate ?

3 The following are the rates of commission charged on money orders :—

Rs. 5	...	...	...	...	1 a.
,, 10	...	...	...	...	2 as.
,, 15	...	...	...	...	3 as.
,, 25	...	...	...	...	4 as.

In how many ways can you remit Rs. 30 ? Which is the cheapest method ?

7. A sum put to interest amounts in 6 years to Rs. 720 and in 7 years to Rs. 810. Find the principal and the rate per cent.

8. Draw the plan of a triangular field whose sides are 100, 120 and 40 yds. respectively. On the side 40 construct an isosceles triangle equal in area to the original triangle. Find the length of each of the equal sides.

5.

1. The rainfall on a flat roof 30 ft. long and 16 ft. wide is collected in a covered rectangular cistern 8 ft. long, 5 ft. wide, and 4 ft. deep ; what depth of water will be found in the cistern after a fall of 3 inches of rain ?

2. A man travelled by train 64 miles in one day, half as many more on the second day, as many miles on the third day as he had done in the two previous days taken together. What percentage of the whole journey did he travel on the second day ?

3. Find the simple interest on Rs. 1,220 for $3\frac{3}{4}$ yrs. at $5\frac{1}{2}$ per cent. per annum.

4. If a family of 9 persons consume Rs. $4\frac{1}{2}$ worth of sugar in 2 weeks when sugar is sold at Rs. 3 a maund, what will it cost a family of 12 persons for 4 weeks when sugar is sold at 5 annas a viss ?

5 If $z = xy$ and $z_1 = x_1 y_1$, find the ratio $\frac{z}{z_1}$ when $x = 6$, $y = 10$, $x_1 = 12$ and $y_1 = 15$.

6. A ladder 33 feet 9 inches long with 44 rungs dividing the whole length into 45 equal divisions, is leaning against a wall, the foot of the ladder being 15 feet from the wall. Find the distance from the wall of the 9th, 16th and 20th rungs from the top.

7. A train leaves Madras at 6-45 P.M. and reaches Madura (350 miles) next day at 12-15 P.M. Find graphically (1) the speed of the train in miles per hour : (2) the time at which it would reach Tanjore (204 miles from Madras) supposing that it travelled uniformly.

FIG. 189.

8. The diagram of Fig. (189) represents the section of a stack

of coal on the scale 1 cm. to the foot. Find roughly the number of cubic feet of coal per yard run.

6.

1. The area of a river basin is 4,000 sq. miles, and the annual rainfall is 32 inches. The river discharges 200,000 c. ft. per minute. What percentage of the rain that falls is discharged by the river ?

2. In a certain year the national expenditure was £183,592,264. Of this the army cost £91,710,000, the navy £29,520,000, and the civil service £23,500,000. Express each of these as a percentage of the whole to the nearest integer.

3. A man sells a horse for Rs. 450 and loses 5 per cent. on what the horse cost him. What was the original cost ?

4. Find the simple interest on Rs. 1,800 from 3rd February 1908 to 6th June 1908 (both days inclusive) at 4 per cent. per annum.

5. A bankrupt owes £6,000 and has good debts to the amount of £1,800 and bad debts to the amount of £1,200 for which he receives on the average 10*s.* in the pound. How much can he pay in the pound ?

6. Given that $y + 6x = 15$, find the corresponding values of y when $x = 0, 1, 2, 3, 4, 5, 6, 7, 8, 9$. Plot these points upon squared paper and draw a line through these points.

7. Find the area of a triangle of angles 60° and 90° of which the shortest side is 6 inches.

8. Brass is composed of seven parts of copper and three of spelter ; copper costs £1-18*s.* a cwt. and spelter £1 a cwt. What will be the cost of the materials for 6 tons of brass ?

7.

1. A wall is to be built 480 ft. long, 12 in. thick and 8 ft. high. Supposing a brick is 8 in. long, 4 in. broad and 2 in. thick, how many bricks would be required to build the wall. And what would be the cost of building at 1 a. 10 p. per cubic foot ?

2. A and B can do a piece of work in 20 and 24 days respectively ; they work together at it for 4 days, when B leaves ; but A continues and after 4 more days is joined by C and they finish it together in 5 days ; in what time could C do the piece of work by himself ?

3. An estate in the joint occupation of A, B and C is sold for three lakhs of rupees. Their shares in it are as follows :—A's share : B's share = 2 : 3. B's share : C's share = 4 : 5. Find the share of each in the sale proceeds.

4. Given that $y = \frac{1}{2}x + 12$, find the values of y when $x = 0$, 12 and 24 respectively. Plot the three points denoted by the corresponding values of x and y upon squared paper and draw a straight line through these points.

5. Find the value of $\frac{a^3 + b^3}{a^2 + b^2}$ when $a = -2$, $b = \frac{1}{3}$.

6. A wire is stretched from the top of a flagstaff to a point on the ground 20 ft. away from the foot of the flagstaff. The wire is 101 ft. long. What is the height of the flagstaff ?

7. A postman has 10 miles to walk in 3 hours ; he walks the first 4 miles at a speed of $3\frac{3}{4}$ miles an hour, the next 2 miles at a speed of $3\frac{1}{3}$ miles an hour, the next mile at a speed of 3 miles an hour ; in what time should he walk the last three miles ?

8. Divide 7 tons 16 cwts. into two parts in a ratio equal to that of £1-10*s*. : £2-10*s*.

8.

1. Find the greatest common measure and the least common multiple of the numbers 30,992 and 25,330 and show by general reasoning that these 4 numbers (*viz*., the G. C. M. of the two numbers, the numbers themselves and their L. C. M.) form a proportion.

2. A tradesman bought 5 cwt. of tea for Rs. 385 ; at what price per lb. should he retail it in order to make aprofit of 25 per cent. ?

3. Find the acreage of a field ABC having 3 straight boundary lines. AB = 2·4 chains. BC = 3·8 chains. $\angle$ BAC is a right angle. (1 chain = 22 yds.)

4. A certain sum amounts to Rs. 1,430 at the end of one year, and to Rs. 1,560 at the end of two years. What is the principal ?

5. Two poles 20 ft. and 12 ft. high are fixed upright 15 ft, apart. What is the distance between the tops of the 2 poles ?

6. PQRS is a quadrilateral with the sides PQ, RS parallel and $\angle$ PSR = 60°, = $\angle$ SRQ = 90°, base SR = 2·5″, height QR = 2″. Show how to convert this by paper cutting into a rectangle (shade the part to be cut off.) Draw the rectangle and write down its dimensions and area.

7. Find correct to 2 decimal places, the value of $\frac{a^3 + b^3}{a^3 - b^3}$ when $a = + 8$ and $b = - 6$.

8. A man walks five miles along a straight road at the rate of 4 miles an hour. Draw the graph of his path.

9.

1. The three sides of a triangle are respectively 3 yds. 2 ft., 4 yds. 0 ft. and 4 yds. 1 ft. What is the length of its perimeter (or measure round the triangle)? What is the area of a square having the same perimeter as the triangle?

2. A marwadi lends Rs. 3-8-0 to a woman on her mortgaging a vessel worth Rs. 9-8-0 and charges interest at the rate of one anna in the rupee per month. In what time will the vessel cease to be hers in case she makes no payment whatever? Find also the rate of interest charged.

3. Construct a parallelogram having its sides 4·5 cm. and 3·8 cm. and having an included angle of 58°. Calculate the area in 2 ways and find the average of the two results.

4. If 625 metres of silk cost 4375 francs, what is the equivalent price per yard in English money (1 metre = 40 in.) and £1 = 25 francs?

5. In the above question construct a graph and from your graph find the price of 25 yds.

6. A farmer has gram enough for 42 bullocks for 18 days. If he buys 12 more bullocks, in how many days will the corn be exhausted.

7. Plot the points, (32, 11) and (17, 3) and calculate the length of the line joining the two points from a right-angled triangle and verify your result by measurement.

8. A man leaves by will, the sum of Rs. 2800 to be divided among his family. Each of his two sons is to have 3 times as much as each of his 3 daughters. Find the share of each.

10.

1. Construct a graph to show the cost of eggs at 32 for a rupee, and from it find the price of eggs.

2. What difference in amount of simple interest shall I obtain by lending Rs. 5,200 for 2 years at $4\frac{1}{2}$ and $5\frac{1}{2}$ per cent. per annum?

3. A train moving at the rate of 42 miles per hour covers the distance between two towns in 160 minutes. How long will it take a train moving at the rate of $35\frac{1}{3}$ miles per hour to travel the same distance?

4. (*a*) The sides of a rectangular piece of paper measure p inches and q inches. Find an expression for the square of the length of the diagonal.

(*b*) A cross-section of a verandah having a sloping roof is shown in the following figure (Fig. 190). Find the breadth of the roof.

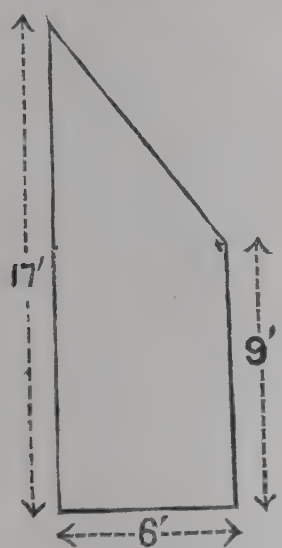


FIG. 190.

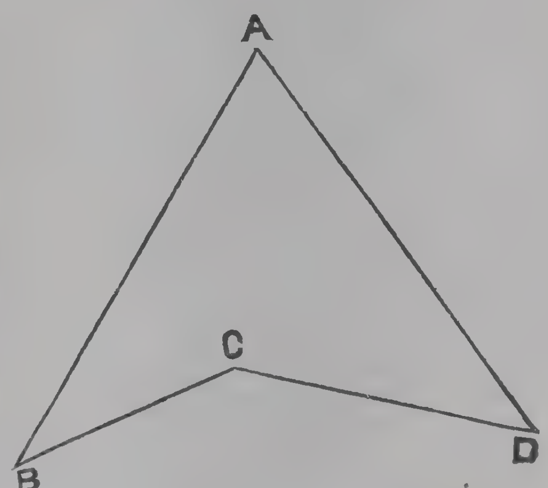


FIG. 191.

5. Find in acres and decimals of an acre the area of a field which is represented by the figure ABCD (Fig. 191) on a scale of 4 ft. to the mile.

6. A can do a piece of work in $5\frac{1}{2}$ days, B in 10 days and C in $27\frac{1}{2}$ days; how soon can the work be done if A is assisted by B and C on alternate days?

7. A man's income is £280 and he has to pay an income-tax of 6*d*. in the pound; what amount of tax does he pay and what income has he left?

8. Give an expression for the number of bricks measuring 8 inches by 4 inches by 2 inches required for building a wall a yds. long, b ft. high, c inches thick. Give the result to the nearest thousand when $a = 15$, $b = 8$ and $c = 9$.

MISCELLANEOUS EXAMPLES.

1. Define a prime number. When are numbers said to be prime to one another? Write down all the prime numbers below 80. Find the prime factors of 999,999.

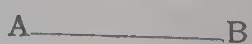
2. Use the identity $\frac{16}{63} = \frac{1}{7} + \frac{1}{9}$ to find the value of $\frac{16}{63}$ of Rs. 5-8-4.

3. Write down in factors (1) the L.C.M., (ii) the H.C.F. of $3^3 \times 4^2 \times 19$ and $3^4 \times 4^3 \times 41$.

4. The Principal of a College is to be given an address and presented with a souvenir on the eve of his retirement. The expenses amount to Rs. 300 to be borne by the 5 Professors, 4 Assistant Professors, 3 Lecturers and 2 Tutors in proportion to their salaries. The salaries of a Professor, an Assistant Professor, a Lecturer and a Tutor are Rs. 400, 250, 150 and 75 per month respectively. Express this contribution as so much in the rupee.

5. A gentleman with his family travels from Madras to Conjeevaram. He buys p tickets and 4 children's tickets for Rs. 4-13 as. If the price of each ticket is 11 as. and each child's ticket is half of each ticket for an adult, find the value of p.

6. Measure the line AB in centimetres. Calculate the length of a line 3·5 times as long. Verify your answer by drawing a line and measuring it.



7. Express the angle between the minute-hand and the hour-hand of a watch at 12-30 P.M. in degrees, minutes and seconds.

8. Convert a price of Rs. 4 per maund into so many annas per viss. Draw a graph to read off the prices in annas per viss corresponding to the following prices in rupees per maund :— Rs. $4\frac{1}{2}$, 5, $5\frac{1}{2}$, 6, 10 per maund.

Use your graph to find the value in rupees per maund equivalent to a price of 10 as. per viss.

9. C is a point in AB, prove that if AC = $\frac{5}{4}$ CB, AC = $\frac{5}{9}$ AB.

10. The expenses of a family amount to Rs. 40 when rice sells at 5 measures a rupee and to Rs. 42 when rice sells at 4 measures per rupee, other expenses remaining unaltered. Find the quantity of rice consumed. (Use x to denote the amount of measures of rice and y to denote other expenses in rupees and frame equations).

11. The percentage composition of a kind of mortar is as follows :—

31·5 lime.
68·5 river sand.

How much sand is there in 5 ton. 4 cwt. of the mortar?

12. If the above mortar is prepared in a circular groove made in the ground and a bullock yoked to a revolving machine goes round pounding the mortar in the groove at the rate of 25 lbs. in every 6 rounds, each round being 37 ft., find how many miles the bullock would have described in preparing 5 tons 4 cwt. of the mortar?

13. A square room is 16 ft. long. The space formed by joining the middle points of the sides taken in order is to be carpeted at 8 as. per square foot ; and the rest to be matted at 4 as. per sq. foot. Find the total cost.

14. A street is 24 ft. broad and has parallel rows of houses on both sides of it. The main water-pipe is laid in the middle and runs throughout the entire length of the road. The Municipal authorities want to widen the street and encroach upon the houses on both the sides to the extent of 4 ft. and 6 ft. respectively. How should the main be shifted to occupy the central position of the road? Draw a plan indicating the old and the new position of the rain.

15. A farmer owns 4 adjacent plots of land each 48 ft. by 32 ft. which is supplied with water drawn from a well by a pair of bullocks in a bucket whose capacity is 3 gallons. If each time the bullocks advance 5 ft. water is drawn up, and each time the bullocks recede, the bucket goes down, find what distance on the whole, the bullocks should walk in this manner if water should stand to a depth of 3 inches in those plots, assuming that no water sinks below the surface. [1 gallon = 277·274 cub. in.]

16. Illustrate the formula $(x+a)(x-b) = x^2+x(a-b)-ab$

by a geometrical diagram; and using this formula find the product of 1008 and 993.

17. A gentleman buys a *rickshaw* for Rs. 40 and employs a cooly with whom he stipulates that a sum of 4 annas should be paid each day for the use of the vehicle. The cooly on an average earns Re. 1-4-0 a day. What rate of interest does the merchant get on his investment if he has to spend 1 anna a day for lighting charges and 12 annas per month for municipal taxes? How would his income be affected if the cooly were paid a monthly salary of Rs. 15?

18. If there are 199 boys in the I, II and III Forms paying Rs. 24 each and 120 boys in the IV, V and VI Forms paying Rs. 38 each per year, what is the total fee income in the Middle School and High School departments?

19. In a High School chemical laboratory provision is made for the supply of 2 gallons of water to each student. 48 students can work at a time in the laboratory. If 3 such batches meet every day, to what depth should water be stored daily in a tank 8 ft. by 4 ft. placed at a height for the distribution of water (1 gallon = 277·274 cub. in.) Give your answer correct to a tenth of an inch.

20. If $n\dfrac{(n+1)}{2}$ is the sum of numbers from 1 to n, find the sum of numbers from 1 to 10. Verify your answer by actual addition.

21. The accompanying figure represents the two faces of an ordinary brick which meet along one edge.

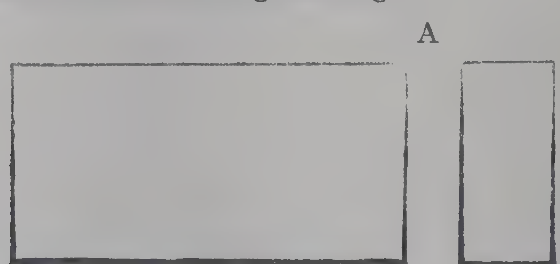


FIG. 192.

(the figure should however be enlarged 8 times to give a true representation). Draw a figure representing the other face which meets the two given faces at the corner A and write down the length, breadth and thickness of the brick.

22. If 0·917 of a sovereign be pure gold and the weight of a sovereign when turned out from the mint be 123·27 grains, what is the weight of pure gold that a sovereign contains?

23 The weight of 1 cub. ft. of iron is 485 lbs and the weight of 1 cubic foot of water is 62·5 lbs. How many times is iron heavier than water?

24. Given that one metre = 39·37 in., what multiplier must be used to convert metres into yards and to convert yards into metres to three places of decimals?

25. When acertain screw is turned round through one complete revolution its head goes forward by 0·145 of an inch. How many turns must you give such a screw to drive it into a piece of wood to a depth of 1·716 inches?

26. The internal section of a medicine bottle is a rectangle $2\frac{1}{2}$ in. long and $1\frac{1}{2}$ in. broad. What is the distance in inches between the ounce marks on the bottle, given that a fluid ounce = 1·73275 cub. in.?

27. A litre of water weighs 1 kilogramme and a gallon of water weighs 10 lbs.; how many litre measures could be run out of a 12 gallon cask and what will be then left in the cask? Assume 1 kilogram = 2·205 lbs.

28. The edges of a carpet in a room a ft. long, b ft. broad are c in. distant from the nearest walls. Find the area of the carpet in terms of a, b and c Find the area of the carpet when $a=24\frac{1}{2}$ ft., $b = 17$ ft, and $c = 16$ in.

29. A gravel walk g feet wide runs round a grass plot l ft. long and w ft. wide If gravel is c annas per cubic yard, find the cost of a layer of gravel on the path d inches deep. Find the cost of the layer of the gravel when $g = 5$, $l = 70$, $w = 50$, $c = 4$ and $d = 4$.

30. An open tank measuring on the outside 6 yds. in length, 3yds. in width and 6 ft. in depth is made with sides and floor of brickwork $\frac{3}{4}$ ft. thick and is filled with water. Find the weight of the tank and its contents, it being given that one cubic foot of water weighs 1000 oz. and that brick is twice as heavy as water.

31. A cistern is 6′ long, $4\frac{1}{2}$′ broad and $2\frac{1}{4}$′ high and is full of water. By how much will the level of the water sink in the cistern

if 100 gallons run out of it, assuming that 1 gallon = $277\frac{1}{4}$ cubic inches? Answer correct to a tenth of an inch.

32. Find the smallest number which is a perfect square and contains 7296 as a factor.

33. Find the least number which is a perfect cube and which contains 2275 as a factor.

34. Write down a series of numbers less than 300 each of which when divided by 5 leaves a remainder 2; write down a series of numbers also less than 1[illegible] each of which when divided by 8 leaves a remainder 7. Note the numbers common to the two series and then write down a series of numbers such that when divided by 5 the remainder is 2 and when divided by 8 the remainder is 7.

35. If the cost of painting a cubical block of 10 ft. edge be Rs. 9, what will be the cost of painting a cubical block of 8 ft edge?

36. The receipts per mile per annum in a Railway are made up as follows:—

Passengers	... ..	Rs. 22655
Goods	... ...	Rs. 23625

If the total receipts be Rs 555480, what are the receipts from passengers and what from goods?

37. The perimeter of a given triangle is 72 inches, the triangle being of the same shape as another triangle whose sides are 12·22 inches, 12·19 inches, 9·34 inches. What are the lengths of the sides of the given triangle?

38. The difference of the base angles of a triangle is [illegible] and the complement of the vertical angle is 38°. Find the angles of the triangle.

39. Find the area of the quadrilateral ABCD making any necessary measurements.

* A   * D

* B   * C

40. Plot the graph of $y = 2x + 3$. What are the co-ordinates of the points of intersection of the graph with the axes?

41. A workman was hired for 5[illegible] days at 4 as. 8 ps. a day for every day he worked, with the condition that for every day he did not work he was to forfeit 1 a. 4 pies, and on the whole he

received Rs. 7-1-4. How many days did he work? (Use x, frame and solve the equation.)

42. Find to the nearest pie the cost of a draft on London for £2420-17-6 at 1s, $3\frac{1}{16}$d. per rupee.

43. A staircase consists 12 steps each step is 8 inches broad and 6 in. high. What is the length of a plank that will just reach from the bottom of the stairs to the top?

44. Draw the plan of a triangle whose sides are 40 yds., 44yds, and 46 yds. Construct an equilateral triangle having the same perimeter as this triangle and find its area.

45. The sum of the diagonal and the side of a square is 20 ft. Find the side geometrically.

46. ABC is a right-angled isosceles triangle. Show that the square on BC the hypotenuse is double the square on either of the sides.

47. ABC is a triangle right-angled at C. Given that $a=50$ ft., $b=12$ ft., find the length of c and of the perpendicular from C on AB.

48. A cow is tethered by a rope 10 ft. long to a stake in the centre of a rectangular field surrounded by walls 40 ft. by 30 ft. Draw a plan and show her position when she is at a distance of 8 ft. from a lengthwise wall with the rope tightly stretched. Indicate also her positions when she is at the least distance from (1) the lengthwise walls; (2) the breadthwise walls.

49. A box 3 ft. long, 2 ft wide and $1\frac{1}{2}$ ft. deep (internal measurements) weighs 10 lbs. If it be filled with water (which weighs 1000 oz. for each cubic foot) calculate the total weight of the box and water.

50. Three cyclists who are riding together have machines with wheels respectively 75, 81 and 87 inches in circumference. What is the least distance in yards that they must travel in order that their wheels shall be simultaneously in the same position as at starting?

51. A merchant imports 2 tons of tea at £4 8s. per cwt. He pays £9 6s. 8d. for freight and a duty of 3d. per lb. He sells the tea at £6 4s. 3d. per cwt. Find the gain or loss per cent.

52. Prove that the product of any two numbers which consist

of two figures and three figures respectively must be a number consisting of not less than 4 and not more than 5 figures.

53. If the manufacturer makes a profit of 20 per cent., the shopkeeper one of 15 per cent., what is the cost to the manaufacturer of an article which is sold in the shop for Rs. 40?

54. A man buys an article and sells it at a profit of $12\frac{1}{2}$ per cent. If he had sold it at 8 as. less he would have made a profit of $2\frac{1}{2}$ per cent. Find the cost price.

55. A milkman adulterates his milk by taking an ollock out of each measure of milk and replacing it by water and he sells his adulterated milk at the price per measure that the pure milk originally cost him. What is his gain per cent.?

56. A man borrows Rs. 100 to be repaid by instalments and agrees to pay 10 per cent. interest on what he owes at any time. He pays off Rs. 40 (including interest) at the end of each year until the debt is cleared. Find the amount of the last instalment.

57. A bar of iron 20 ft. long has a rectangular section shown below. Measure the thickness and breadth, and find the area of the section. Also find the weight of the bar, a cub. foot of iron weighing 489 lbs.

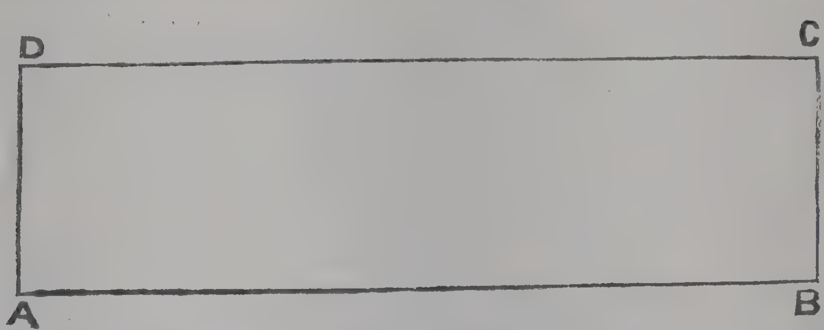


FIG. 193.

58. A, B and C go into partnership, A investing Rs. 3000, B Rs. 4000 and C Rs. 5000. If the profits of the first year amount to Rs. 1550, how must they be divided?

59. There is a square lawn ABCD containing 20 acres 2425 sq. yds. How much shorter is the diagonal AC than the sum of the two sides AB, BC?

60. Find the cost of fencing a rectangular lawn 65 ft. long and 30 ft. broad at 2 as. per yard.

61. A boat starts at A and takes the following courses :—

Moves 2 miles to the East, but at the same time 4 miles to the N; then 5 miles to the East, but at the same time 6·3 miles to the N. Draw a plan and find its final position.

62. One train A travels at 25 miles per hour, another train B travels at 20 miles per hour. If A completes ajourney in 4 hrs. 15 min., how long will B take to complete the same journey?

63. Two equal sums of money are put out at simple interest, and both obtain the same return: in one case the rate per cent. per annum was 8 and the time 6 months, in the second case the rate was 6 per cent. What was the time in the latter case?

✓ **64.** If 3080 soldiers have provisions for 98 days, how many men must be sent away to make the food last for 112 days?

65. If the hot water pipe can fill a bath in 20 min. the cold water pipe in 15 min. and if the waste pipe can empty it in 5 min., calculate how much water enters or leaves the bath in 2 minutes when (*a*) the hot water pipe alone is open; (*b*) the hot and cold water pipes are open; (*c*) the three pipes are open. Supposing the cistern is full, find also the time taken in emptying the bath if all the pipes are open. Illustrate your answer by a graph.

66. A cistern which can be filled by 1 tap in 24 hours is emptied by another. The latter alone would empty it in 9 hours. In what time would it be emptied if both the taps were set running together? Illustrate graphically.

67. Two clocks are set correct at 12 o'clock: one loses 6 sec. and the other gains 9 sec. in 12 hours; when will the faster be half an hour before the other, and what o'clock will it then show?

68. Two watches are together at 12 noon. One gains 35 sec. and the other loses 25 sec. per hour. When will they be together again at 12 noon?

69. A can do a piece of work in 10 days, B in 20 days and C in 40 days. How soon can the work be done if A is assisted by B and C on alternate days?

70. Hackney bullock carriage rules require that 2 as. should be paid for the first mile and 1 a. for every subsequent mile. Find what amount a person hiring a bullock cart on these terms will

have to pay for travelling x miles; and from this expression find the hire when (1) $x=5$, (2) $x=9$. Draw a graph and from the graph find for how many miles the cost of hire will be 10 as.

71. A certain number of rupees, an equal number of annas and an equal number of pies amount to Rs. 80-1-3. Find the number. (Frame an equation.)

72. The total fare for a journey of 504 miles partly by main line and partly by branch line was Rs. 17-11-6. The rate per mile being 6 pies on the main line and 8 pies on the branch line, what distance was travelled on the branch line? (Use x, frame and solve the equation.)

73. On a certain railway the second and third class fares are 7 pies and 3 pies per mile respectively. A man with Rs. 2-6-6 travels 100 miles by going part of the distance second class, and the remainder third class. How far did he travel second class? (Use x, frame and solve the equation.)

74. A merchant wishing to clear out his old stock sold one lot of goods at a reduction of $12\frac{1}{2}$ per cent., another at a reduction of 25 per cent., off the marked prices. He realised Rs. 682-8 as., and Rs. 617-8 as. and found that on the whole he had a loss of $2\frac{1}{2}$ per cent. on the price paid by him for the goods. What would he have lost or gained per cent. if he had sold all the goods at a reduction of 20 per cent. on the usual prices?

75. A merchant buys a quantity of tea at an average rate of 12 as. 6 p. per lb. He assorts the tea into two kinds which he sells at Re. 1-14 as. and 9 as. per lb. respectively. If, in the process of assortment, $2\frac{1}{2}$ per cent. of the tea is lost and of what remains 36 per cent. is of the dearer kind, find the merchant's gain on the transaction.

76. A merchant pays a lakh of rupees for a season's goods. He marks the goods 25 per cent. over the prime cost, and from what he sells at this rate realises Rs. 1,12,500. At the end of the season he sells the remaining goods at a reduced rate of 20 per cent. If the expenses of the business amount to 12 per cent. of the sale receipts, what is his rate of profit on the transaction of the season?

77. For printing a circular, a printer's estimate is Re. 1 for 50 copies or Re. 1 6 as. for 100 copies. Assuming that each of these estimates consists of (i) a charge for setting up the type independent

of the number of copies printed ; (ii) a charge for printing and paper proportional to the number of copies printed, find what his estimate for printing 500 copies would be.

78. Add together 108·7, 2369·4, 937·9, 845·3 and 64 so as to get the answer to the nearest integer. Find how much difference it would have made in your answer if you had written down each number to the nearest integer and then added.

79. I wish to form a rough estimate of the acreage of a rectangular field by stepping along two adjacent sides, If my steps average 25 inches and the two sides are 56 steps and 156 steps respectively, how many acres does the field contain? (Answer to the nearest hundredth of an acre).

80. A kilogram = 2·20464...lb. If it is taken to be $2\frac{29}{250}$ lbs., to how many figures is it correct?

81. A map drawn to a scale of a cm. to 12 Km. measures 12·5 cm. by 9·8 cm. Find in square miles the area that it represents, given that 1 km. =0·62 mile. The answer should be correct to 10 sq miles.

82. ABCD represents a rectangular field to scale (1 to 2000). Find its perimeter and diagonal, and its area in acres.

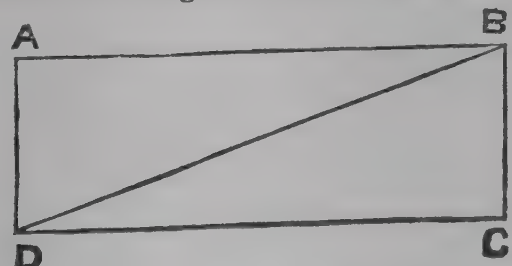


FIG. 194.

83. The average of x numbers is p ; of y other numbers is q. What is (1) the sum of all the numbers ; (2) the average of all the numbers ?

84. If the average marks in Arithmetic for a class of 39 be 20 and those for a class of 38 be 24, find the average for the two classes.

85. The average of x numbers is p : the average of y of these is q. What is the average of the remainder ?

86. The average monthly earnings, during the year 1910, of a merchant were Rs. 575-12-0 ; but the average during the first five

months was Rs. 525-8-0. Find his average monthly earnings during the remaining seven months.

87. By the following method determine which of the following express runs is the fastest ; (1) 28 miles in 56 minutes ; (2) 21 miles in 40 min. ; (3) 25 miles in 45 min. Take a straight line OX. Mark off on this line OP to represent 56 minutes ; at P draw PO perpendicular to OX to represent 28 miles. Join OQ. Treat each of the other runs in the same way.

88. Draw a graph showing the wages for any number of hours up to 60 at 2 as. per hour. From it, find out how much a man gets per week of 56 hours at 2 as. per hour.

89. A train has a journey of 120 miles to perform in 3 hrs. If the train is delayed 20 minutes in starting, at what rate must it travel so as to reach the destination at the proper time ? Illustrate your answer graphically.

90. Solve the equation $3(x+5)-4(x-8)=6(x-7)+9(x+4)$ and verify that the root satisfies the equation.

91. In the formula $A=bh$, find the value b given $A=150{\cdot}5$, $h=27{\cdot}32$.

92. Draw a square the length of each side being 1·75 inches. Upon each side of the square as hypotenuse construct a right-angled isosceles triangle outside the square What figure does the whole now form ? How many times the original square is it ?

93. Make a parallelogram the sides being 4 cm. and 2·5 cm. and one angle being 45°. Construct a rhombus on the base of 4 cm. equal to the prallelogram. Measure the angles of the rhombus.

94. The Corporation of Madras assesses houses for six months at the following rates under the different items calculated on the amount of the annual rent valuation as shown below :

			RS. A. P.
House Tax (15%)	...	...	
Water Tax (8%)	...	...	
Light Tax 3%)	...	...	
		Total	

Fill up the bill assuming the annual rent valuation of a certain house to be Rs 150.

95. The terraced roof of a square Kalyánakúdam of side 20 ft. is 30 ft. above the floor. In the centre, a light is suspended by a

string passing over a pulley fixed in the middle of the roof. When the light is at a height of 5 ft. from the ground, the string can just be tied to an iron bar of a window in one of the side-walls at a point 6 ft. above the ground. Find the length of the string.

96. Motor cars are available for hire at Rs. 10 per day and each car can accommodate 4 persons excluding the driver. A railway campany charges 2 pies per mile. What is the distance in travelling which (both going and returning) the motor service will be as expensive as the railway journey? Also find what a man loses by travelling 60 miles in the motor car along with 3 others instead of journeying by rail.

97. Calicut cheque cloth costs 12 as per yard and lasts for 5 years. Long cloth costs 4 as per yard and lasts for 2 years. Which is the cheaper investment?

✓ **98.** If the cost of maintaining 5 adults and 4 children is Rs. 70, find the cost of maintaining 4 adults and 10 children, supposing that to maintain an adult it costs twice as much as to maintain a child.

99. The following are the rates of travelling from Beach to Egmore:

(1)	By railway	6 pies
(2)	,, tram	1 anna
(3)	,, rickshaw	4 annas
(4)	,, jutka	5 annas.

The rickshaw can accommodate 2 and the jutka 4. What can a party of 8 persons save by preferring the cheapest to the dearest conveyance? By how much per cent. is each of the three last modes of conveyance dearer than the first?

100. The disbursements in working the Government Press for the year 1910-11 consisted of the following :—

On account of the Central Press ...	... Rs.	3,06,987-4- 3
Penitentiary branch	... ,,	15,254-0- 8
Ootacamund ,,	... ,,	13,757-3- 3
Publication, &c.	... ,,	43,908-7- 0
Grain allowance	... ,,	20,504-0-10

Find the total expenditure and find what percentage of it is the expenditure under (1) Grain allowance (2) Central Press.

ANSWERS.

Exercise I (a). p. 6-7.

2. (*a*) (1) 2,35,495. (2) 15,06,024. (3) 2,00,78,045. (4) 15,07,00,497.

(*b*) (1) 150,349. (2) 2,045,703. (3) 1,043,207,564.

3. (*a*) (1) Twenty thousand six hundred and eighty-seven.

(2) Four lakhs, thirty-two thousand and ninety-nine.

(3) Eight lakhs, five thousand, eight hundred and seven.

(4) Twenty-one lakhs, thirty-six thousand and six hundred and thirty-nine.

(5) Two crores, six lakhs, eighty-nine thousand seven hundred and three.

(*b*) (1) Two hundred and six thousand five hundred and ninety-eight.

(2) Eight hundred and nine-thousand seven hundred and seventy-seven.

(3) Five millions, six hundred and three thousand, seven hundred and eight.

(4) Twenty-five millions, nine thousand and seventy-eight.

(5) One thousand and forty-seven millions, six hundred and eighty-seven thousand three hundred and forty-three.

Exercise I (b). p. 8.

1. (1) XXVII ; XXXIX ; XLVIII ; CLIII ; CCXCVII ; MMCCCVI.

2. 19 ; 60 ; 65 ; 79 ; 134 ; 229 ; 775 ; 1238 ; 1910.

Exercise II (a). p. 18—20.

5. 2·5" (to the nearest tenth).

6. (1) 1 1 1·1 (do.)
(2) 1 8 1·8 (do.)
(3) 1 9 1·9 (do.)
(4) 2 2 2·2 (do.)

7. (1) 2 cm. 8 mm. or 2·8 cm. (2) 4 cm. 7 mm. or 4·7 cm.
(3) 4 cm. 9 mm. or 4·9 cm. (4) 5 cm. 6 mm. or 5·6 cm.
9. 6 cm.; 5·9 cm.; 5·8 cm. 10. 2·4 in.; 2·3 in.; 2·2 in.
14. 2·6″.

Exercise II (b). p. 21-22.

5. 1·5″ and 2·5″. 6. 3 cm. and 4 cm.
7. 2·5″. 8. 1·2 cm. and 3·6 cm.

Exercise II—*Symbolic.* p. 29.

1. $a=3$. 2. $b=6$. 3. $a=9$. 4. $b=8$.
5. $a=11$. 6. $b=13$. 7. $a=4$. 8. $b=9$.
9. $a=6$; $b=7$. 10. $a=7$; $b=8$; $c=3$. 11. 7; 10; 11; 12.
12. (1) 5. (2) 17. (3) 9. (4) 13.
13. 1 and 10 or 2 and 9 or 3 and 8 or 4 and 7 or 5 and 6.
14. $a+5$. 15. $b+a$. 16. $a+b+c$.

Exercise III (a). p. 29-30.

4. AX=1 in. XB=2 in. AB=3 in.
5. AX = 2·7 cm.; XB = 5·2 cm.; AB = 7·9 cm.
6. (1) 3·8. (2) 2·6. (3) 4·4. 7. Each = 10·8.

Exercise III (b). p. 33–35.

1.	15735.	2.	13516.	3.	31963.	4.	28839.
5.	27073.	6.	27088.	7.	16006.	8.	21154.
9.	21573.	10.	14136.	11.	17169.	12.	117126.
13.	28226.	14.	75452.	15.	63834.	16.	125717.
17.	52076.	18.	45974.	19.	47026.	20.	15934.
21.	25175.	22.	47068.	23.	164128.	24.	345305.

25. 7901; 6962; 37400; 46808; 7476; 11790; 13900; 10910
7070; 30030; 32847. 28. 82655. 29. 223030.

Exercise III.—*Symbolic.* p. 36-37.

1. $d=2$. 2. $s=8$. 3. $m=15$. 4. $a=3$. 5. $b=13$.
6. $d=7$. 7. (*a*) $d=1$. (*b*) $m=12$. 8. $d=4$. 9. $d=5$.
10. (1) $a-b=7$. (2) $a-b=6$. 11. $15-a$; $a=8$.
12. $a-4$; $a=12$.

Exercise III (c). p. 38.

1. AD=3·4 nearly; AC=2; CD=1·4. 3. 1·1". 3. ·2". 4. ·2". 5. 1". 6. 3 cm.

Exercise III (d). p. 41-42.

1. 3434. 2. 2986. 3. 1078.
4. 114. 5. 64865. 6. 14834.
7. 360761. 8. 90851. 9. 12092.
10. 116082. 11. 2876659. 12. 246913.
13. 99691. 14. 3802460. 15. 692355.
16. 41125610. 17. 1033446. 18. 1432150.
19. 8888889. 20. 6294474400. 21. 3038038.
22. 8339901. 23. 31535761. 24. 16152311.
25. 4772413. 26. 7892602. 27. 93564369.
28. 86887664. 29. 430393. 30. 89710231.
31. 556355. 32. 155204. 33. 2751640.
34. The former by 6841421. 35. 751155.
36. (*a*) 3, 2, 1. (*b*) 15, 654, 2101, 89359.

Exercise III—*Symbolic*. p. 47-48.

1. $(5 + 6 - 3)$ marbles. 2. $(p - 4)$ rupees. 3. $(p - q)$ rupees. 4. $(p + q - r)$ mangoes. 5. $(a + b)$ cocoanuts. 6. $(a + 6)$ years. 7. $(b - 10)$ years. 8. $(a - b)$ rupees; $(a-b-c)$ rupees. 9. $(50 - 10+10)$ rupees; Rs. 50. 10. $(p - q + q)$ books; p books.

Exercise III.—*Graphical and Symbolic*. p. 50—52.

1. (300 — 500) rupees. 2. 28 miles to the south.
3. 98° (Fahrenheit). 4. He loses 9 annas on the whole.
5. 50 marks. 6. 1853 years.

7.

1	2	3	4	month
-5	-2	-1	8	0

8. 1 ft. 9. Rs. $(5 - 3)$; cannot be expressed as there is no loss. 10. Rs. $(b - a)$; Rs. $(a - b)$.

Exercise III (e). p. 53—59.

1. 3026. 2. 2368. 3. 2012449.
4. 2206022. 5. 78 ft. 6. (1) 9962841. (2) 16,059,513.

8. 1236 yds. 9. 82° F.
10. — 33838 ; — 559221 ; + 32098 ; + 36638.
11. 2370950. 12. 91 votes. 13. 5137 ; 13608.
14. 308. 15. 0, 84, 93, 103, 109, 137, 162, 228, 258, 276, 308, 351, 267.
16. 11, 12, 63, 24, 21, 51, 40, 46, 37, 56, 31, 10, 24, 59, 546.
17. 216534 ; 172473 ; 400 ; 37931 ; 1254 ; 94587 ; 47502 ; 74149 ; 29801 ; 163782 ; 18771 ; 428592.
18. 190946 ; 17216 ; 32614 ; 31580 ; 140704 ; 49965 ; 51170 ; 7672 ; 21832 ; 38430 ; 243991 ; 413060.
19. (1) 2709. (2) 5263. (3) 7019.
20. (1) 1087. (2) 3544. (3) 4974.
21. 3513156 ; 3630155 ; 3809250.
22. (1) 1455. (2) 3112. (3) 4314.
23. (1) 11988. (2) + 1103, — 600, — 1697, — 282, — 106. (3) — 1582.
24. Increase + 996, increase + 1073, decrease 406. Total increase 1663.
25. Tanjore 85825 ; Madura 369602 ; Trichinopoly 735340 ; Total 1190767 ; Men 905616 ; Women 178579 ; Children 106572.
26. 569403.

			Emigrants.	
27.	Ganjam ...	... ...	17857	
	Vizagapatam	... ...	166759	
	Godavari ...	... ...	28295	
	Krishna ...	... ...	234466	
	Nellore ...	... ...	105019	
	Chingleput	... ...	115629	
	Total ... 637491 ;		668025 ;	— 30534.

28. At the end of January, Rs. 23,610.
February, „ 23,524.
March, „ 23,229.
April, „ 25,549.

29. Rs. 50,277. 30. Rs. 7,601.

Exercise IV.—*Practical*. p. 67-63.

4. AC = 5″.

Exercise IV.—*Graphical and Practical.* p. 69—71.

18. 8 squares.

Exercise IV.—*Practical and Graphical.* p. 72-73.

7. 3·2 cm. nearly. 8. 5·2 cm.

Exercise IV (a). p. 75-76.

1. 17°, 13°, 8°, 37°, 21°, 30°, 20°, 39°, 39°, 15°, 16°, 85°, 37°, 143°.
2. Each 45°. 5. 15 minute divisions.
6. 150°, 120°, 210°, 180°.
7. Each = 60°. 9. Each = 60°.
10. 90°, 45°, $67\frac{1}{2}$°, $11\frac{1}{4}$°, $67\frac{1}{2}$°, $33\frac{3}{4}$°.

Exercise IV (b). p. 78-79.

1. 11°, 28°, 39°. 2. 28°, 22°, 50°. 3. 11°, 28°, 22°, ∠ABK=61°.
4. 90°, 65°, 25°. 5. 94°, 29°, 65°. 6. 117°, 65°, 25°, 27°.

Exercise IV (c).—*Graphical and Practical.* p. 80.

2. 70°. 3. 120°.
4. 13·5 miles, 17° E. of North. 5 4·75 miles, 39°.
6. 20 miles, 52° East of South. 7. 26 miles.

Exercise IV (d). p. 84—86.

2. 5·6 cm. 9. 8·2 miles. 10. 2 miles.
11. 30 feet. 12. 4·5 miles and 6·5 miles.
13. 7 cm. nearly, ∠A = ∠B = ∠C = 60° : ∠OAB = ∠OBA = 30°.
19. 10 cm

Exercise V.—*Graphical.* p. 94-95.

1. (1) 4 in. (2) 5·9 in. (3) 11 cm. (4) 15·8 cm. (5) 10·3 cm.
2. 14·6 cm.

Exercise V (a). p. 96-97.

1. 30 m. 3 dm. 6 cm 2 mm. 2. 434 m. 3 dm. 8 cm. 6 mm.
3. 162 m. 8 dm. 3 cm. 6 mm. 4. 411 m. 9 dm. 6 cm. 8 mm.
5. 90 m. 3 dm. 9 cm. 0 mm. 6. 37·44. 7. 22·474. 8. 434·5.
9 347·431. 10 3058·96068.

Exercise V.—*Graphical*, p. 98.

1. 1·4 cm. 2. ·9. 3. (*a*) 2·3 cm. (*b*) ·9 cm. (*c*) 3·8 cm. (*d*) 4·9 cm. 4. 18 miles

Exercise V (b). p. 99.

1. 9·441 m. 2. ·712 m. 3. 498·727 m. 4. 251·248 m. 5. 3·836. 6. 529·435. 7. 14 495. 8. 14·245. 9. 169·24197. 10. — 46425·07.

Exercise V (c). p. 101—103.

1. 100·35″. 2. 59·7 tons. 3. (1) Rs. 1129·22. (2) Rs. 161·19. (3) Rs. 1290·41. 4. 23 m. 6 dm 4 cm. 8 mm. 5. 6·093 tons. 6. ·99 m. 7. 23·0194 ft. 8. 121·93 gallons. 9. 5 lakhs. 10. 8·8 inches. 11. ·286 12. 2·226. 13. 1·875. 14. 7·2414. 15. 5·318. 16. $a = 3, b = 6, c = 4$. 17. 503·66 gallons. 18. $z=3·35, x=4·67, y=·65$. 19. (*a*) 16·4. (*b*) 16. 20. 3·7 yds.

Exercise VI (a). p. 111.

1. 9852, 13136, 19704, 22988. 2. 3417300, 2050380, 34173000, 20503800, 273384000, 546768000. 3. 9989508, 11654426, 13319344, 14984262, 16649180. 4. 72817269, 83219736, 97089692, 104024670, 121362115, 145634538 5. 455677650, 472554600, 540062400, 683516475.

Exercise VI (b). p. 113—115.

1. (*a*) (1) 512, (2) 1024, (3) 1280. (*b*) (1) 2500, (2) 3125.
2. (1) $2 \times 3 \times 7$, (2) $2^2 \times 3^2$, (3) 4×11, (4) $2^4 \times 5$.
3. (*a*) 3804, (*b*) 83600, (*c*) 19290, (*d*) 1672000.
4. (*a*) 20, (*b*) 24000, (*c*) 720000, (*d*) 4800000.
5. (*a*) 330652, (*b*) 525924, (*c*) 3071464, (*d*) 2373984. (*e*) 27844208, (*f*) 251671808, (*g*) 446938000, (*h*) 983263600.
6. (*a*) 199392303, (*b*) 545669684, (*c*) 49904896, (*d*) 466367496.
7. (1) (*a*) 3042, (*b*) 9120. (*c*) 127092, (*d*) 5760, (*e*) 3080. (2) (*a*) $22a$, (*b*) $8b$, (*c*) $23p$, (*d*) $17x$. 8. Rs. 725400.
9. 678559 maunds. 10. 5376 miles. 11. 17766 gallons.
12. 78232 yds.

Exercise VI (c). p. 117.

1. 875946177. 2. (*a*) 4682781675, (*b*) 4680908375.

(c) 467388350, (d) 232757525. (e) 419619200. 3. (a) 498188600, (b) 591256800, (c) 9862491900, (d) 886885200.

4. (a) 437900493982, (b) 524492481982, (c) 130325752606, (d) 494098694194, (e) 130412344594, (f 434263630486.

5. (a) 4048544, (b) 127988, (c) 14199673, (d) 9080776, (e) 1285715, (f) 79185839.

Exercise VI (d). p. 118-119.

2. $\left.\begin{array}{l} 2^2 = 4^1 = 4 \\ 2^4 = 4^2 = 16 \\ 2^6 = 4^3 = 64 \\ 2^8 = 4^4 = 256 \end{array}\right\}$ 4. (a) 648, (b) 559872, (c) 26873856, (d) 4199040000, (e) 1194393600.

6. (a) 4, (b) 10, (c) 10, (d) 2, (e) 11, (f) 1, (g) 11. (h) 3, (k) 20, (l) 8, (m) 14, (n) 0, (o) 3, (p) 12.

7. (a) 3. (b) 5, (c) 6, (d) 6, (e) 7, (f) 11, (g) 12, (h) 16, (k) 18, (l) 24.

10. 12; 72; 27648; 384. 11. 10; 14; 156; 40.

Exercise VI.—*Practical and Graphical.* p. 119—121.

8. 121, 225, 441, 1225, 1681, 10609, 41616, 93025, 1002001, 1010025.

12. 225, 361, 625, 1225, 1521, 9801, 986049.

Exercise VI (e). p. 123-124.

1. 233000.	2. 516000.	3. 728000.
4. 108490000.	5. (a) 807000, (b) 1780000, (c) 13100000.	
6. 383 lakhs.	7. 129000.	8. 89400000.
9. 695200 Km.	10. 9870000.	11. Rs. 160600.
12. 621000.	13. 24528.	14. 146097.
15. 617206.	16. 1007990.	17. 2226.
18. 157080.	19. Rs. 403565.	20. Rs. 617540.
21. Rs. 2778832500.	22. Rs. 21994.	

Exercise VII (a). p. 128-129.

3. 2312. 4. $q=13874$; $r=1$. 5. (a) 14, (b) 23.

6. (a) $q=14$; $r=5$, (b) $q=14$; $r=6$. 7. $dq=216$; $q=18$; $q=12$.

8. 256, (a) 32, (b) 16. 9. (1) $q=2741$; $r=2$, (2) $q=2081$: $r=1$.

10. (a) $q=7843$; $r=6$. (b) $p=8964$; $r=2$, (c) $q=6972$; $r=2$.

11. 337 ft.; 6 in. 12. 313 times.

32

Exercise VII (b)—p. 131.

1. 5209·3 ; 4341·2 ; 3256·0 ; 2368·0 ; 2170·8.
2. 2175·13 ; 1813·4 ; 1554·4 ; 1359·22.
3. 1125·9 ; 1028·29 ; 818·17 ; 571·36.
4. 331·39 ; 290·3 ; 265·18 ; 248·67.
5. 5042·59 ; 4051·161 ; 3781·149 ; 3545·89.

Exercise VII.—*Practical and Graphical.* p. 131-132.

1. 10 times, and 3. 3. 28 (3's) ; 5 (15's) ; 10 left over.
8. 12·7 cm. 9. 3·9 inches.

Exercise VII (c). p. 135.

1. 3628·9 ; 1464·5 ; 689·84 ; 172·377 ; 139·331.
2. 2702·12 ; 1070·44 ; 443·50 ; 93·117 ; 71·25.
3. 1489·117 ; 565·204 ; 374·378 ; 169·68.
4. 1979·132 ; 3110·365 ; 438·1661 ; 421·1938.
5. 65·1944 ; 95·1829 ; 50·49 ; 23·5787.

Exercise VII (d). p. 142.

1. (*a*) 465625 ; (*b*) 1396875 ; (*c*) 2328125 ; (*d*) 6984375.
2. (*a*) 2749·9 ; (*b*) 549·109.
3. (*a*) 8461·17 ; (*b*) 838·494 ; (*c*) 83·7739.
4. 85953·108. 5. 42977500 ; 60168500.

Exercise VII (e). p. 147-148.

2. (1) 1. (2) 20. 3. 50. 4. 3, 22 ; 0, 3.
5. 6. 6. $b=8$; $a=0$, 3, 6 or 9.
7. $b=0$, 4, 8 ; $a=1$, 6, 2, respectively. 8. 29375 ; 149
9. 76267. 10. $a=8$. 11. $a=9$.
12. (*a*) 17642676 ; (*b*) 4677673 ; (*c*) 1717848 ; (*d*) 10451456.

Exercise VII (f). p. 148—151.

1. 672. 2. 158400. 3. 7392.
4. 50544. 5. Rs. 588600. 6. 1461.
7. Rs. 6504. 8. 77082.
9. 158 times and 183.
10. 272, 68, 34. 11. 8 ft. $4\frac{4}{5}$ inches. 12. 8.
13. Ten will get 19 and one 18. 14. 24 miles per hour.

15. Rs. 5. 16. 139912.
17. 41 cart-loads and 219 bricks remaining.
18. 8 specials. 19. $15\frac{1}{2}$ sovereigns.
20. 8 min. and 20 seconds. 21. About 5 seconds nearly.
22. 29 classes. 23. 152 cars.
24. 69 miles nearly. 25. About Rs. 168.
26. 153. 27. 9 as. 5 ps.
28. 414. 29. 26 lbs.
30. 112.

Exercise VIII (a). p 163-164.

1. (a) 930 p., (b) 9554 p., (c) 114598 p. 2. (a) 10166 h. r. (b) 6168 h. r. (a) 24162 q. r. (b) 16155 q. r. (a) 29122 (2 as.) (b) 27869 (2 as.) 3. (a) 5664 as., 22656 q. as. (b) 13539 as., 54156 q. as. (c) $16696\frac{1}{2}$ as., 66786 ($\frac{1}{4}$ as.) 4. (a) 1175 sov. and 5 Rs. (b) 5496 sov. and 5 Rs. 5. 12 yds. 6. 639 oranges. 7. 854 miles. 8. 71 books.

Exercise VIII (b). p. 164.

1. 10410*d* 2. 15474 (2*d*.) 3. (a) 1699 fl. 1*s*. (b) 679 crowns 4*s*. (c) 3399 h. f. (d) 1359 h. crowns 1*s*. 6*d*.
4. £68-0-8. 5. £855-7-6. 6. £9380-14-0.

Exercise VIII (c). p. 165.

1. 12560 palams. 2. 20363 seers. 3. 120359 palams.
4. 152795 seers. 5. 42224 viss.

Exercise VIII (d). p. 166.

1. 34943 lbs. 2. 67002 lbs. 3. 212439 lbs.

Exercise VIII (e). p. 166-167.

1. 18692 ft. 2. 75720 ft. 3. 21 miles 5 fur. 203 yds.
4. 125 miles 2 fur. 191 yds. 5. 3600 seconds.
6. (a) 60. (b) 1440. 7. 519 years 330 days.
8. 35' 12". 9. 15 miles. 10. 390 yds. 1 ft.
11. 5 miles 867 yds. 1 ft. 12. 30 miles.
13. 4 miles 261 yds. 2 ft. 14. Very nearly 7 miles.

Exercise VIII (f). p. 167.

1. 5064 pints. 2. 5064 qts. 3. 818 gal. 2 qts.
4. (*a*) 808 mea. 6 oll. (*b*) 101 markals 0 mea. 6 oll.
5. 19 kalams 6 mar. 2 mea. 6. 685056 oll.
7. 400. 8. 720. 9. Rs. 5-10-0.

Exercise VIII (g). p. 167—171.

1. Rs. 1902-11-8. 2. Rs. 4127-1-1. 3. Rs. 5553-8-3.
4. Rs. 385-5-0. 5. Rs. 2303-6-9. 6. Rs. 2294-7-9.
7. Rs. 2001-6-7. 8. Rs. 4598-10-11. 9. Rs. 11583-5-0.
10. £7349-14-5. 11. £25804 7-8. 12. £6575-6-4.
13. £3944-16-7. 14. £25217-19-10. 15. £2723-2-10.
16. £2498-6-5. 17. £5345-2-9. 18. £39729-8-5.
19. Can. 589-1-6. 20. Can. 654-14-4. 21. Can. 8593-12-2.
22. Can. 190-0-7. 23. Can. 2001-18-2. 24. Can. 2722-14-7.
25. Can. 2203-6-4. 26. Can. 2719-8-0. 27. Can. 9837-8-4.
28. 908 tons 13 cwt. 3 qr. 17 lbs. 29. 3347 tons 12 cwt.
30. 1238 tons 3 cwt. 3 qr. 13 lbs. 10 oz.
31. 323 tons 15 cwt. 2 qr. 15 lbs. 15 oz.
32. 804 tons 11 cwt. 2 qr. 16 lbs. 14 oz.
33. 1496 tons 6 cwt. 1 qr. 26 lbs. 12 oz.
34. 1488 tons 1 cwt. 2 qr. 2 lbs. 11 oz.
35. 1381 tons 14 cwt. 1 qr. 24 lbs. 6 oz.
36. 5494 tons 9 cwt. 3 qr. 2 lbs. 10 oz.
37. 36 gal. 0 qt. 1 pt. 38. 76 bush. 0 pk. 0 gal.
39. 109 qr. 5 bush. 3 pk. 1 gal.
40. 20 qr. 7 bush. 0 pk. 0 g. 3 qt. 1 pt.
41. 26 qr. 1 bush. 1 pk. 1 gall. 2 qt. 0 pt.
42. 44 qr. 7 bush. 2 pk. 1 g. 3 qt. 1 pt.
43. 14 qr. 2 bush. 2 pk. 0 g. 3 qt. 1 pt.
44. 119 qr. 6 bush. 1 pk. 1 gall. 0 qt. 1 pt.
45. 1624 kal. 6 mar. 4 mea. 46. 2715 Kg. 3 Hg. 5 Dg. 1 g.
47. 68 g. 8 dg. 4 cg. 8 mg. 48. Rs. 15383 13 as. 9 p.
49. £935-7-11. 50. £2190-4-3. 51. Rs. 960 0-6.
52. 210 can. 15 md. 5 vis. 1 sr. 1 plm.
53. 98 can. 18 md. 7 vis. 0 sr. 4 plm.
54. 5 tons 6 cwt. 2 qr. 21 lbs. 15 oz.
55. 250 tons 15 cwt. 2 qr. 20 lbs. 4 oz.

56. 2 bush. 3 pk. 4 gallons.
57. 2 bush. 1 pk. 2 gal.
58. 78 Kg. 4 Hg. 2 Dg. 7 g.
59. 3 g. 0 dg. 6 cg. 9 mg.
60. Rs. 1485-14-5 ps.
61. £304-13-3.

Exercise VIII (h). p. 173.

1. Rs. 577-7-5, Rs. 742-7-3, Rs. 1237-6-9. 2. Rs. 5391-11-10. Rs. 6435-4-10, Rs. 7478-13-10. 3. Rs. 3060511-0-10. Rs. 4422606-10-1. Rs. 9870988-15-1. 4. £474719-8-0. £608783-13-6, £731859-1-6.

5. £65840937-16-0. £79288240-0-6. £10473508-18-6.

6. 214 can. 7 mds. 4 viss 35 pal.; 528 can. 0 mds. 3 viss 21 pal.; 604 can. 7 mds. 4 viss 11 pal.

7. 41928 can. 15 mds. 4 viss.; 49262 can. 6 mds. 4 viss; 65045 can. 8 mds. 0 viss.

8. 7309 tons 18 cwt. 1 qr. 11 lbs.; 10796 tons 3 cwt. 2 qr. 24 lbs.; 9784 tons 0 cwt. 3 qr. 13 lbs.

9. 583749 tons 11 cwt. 1 qr. 24 lbs.
1095633 tons 12 cwt. 3 qr. 24 lbs.
917859 tons 12 cwt. 0 qr. 0 lbs.

10. 803758 miles 1 fur. 90 yds.; 967123 miles 5 fur. 60 yds.; 4985914 miles 4 fur. 70 yds.

11. 2190 Km. 9 Hm. 7 Dm. 6 m.; 4655 Km. 8 Hm. 2 Dm. 4 m.; 6230 Km. 5 Hm. 8 Dm. 8 m.

12. 44018 m. 4 dm. 9 cm. 4 mm.; 53534 m. 1 dm. 5 cm. 6 mm.; 63939 m. 7 dm. 7 cm. 2 mm.

13. 1146 kal.; 1321 kal. 1 mar.; 748 kal. 1 mar.

14. 190 qrs.; 255 qrs. 2 bush. 2 pk.; 350 qr. 2 bush. 2 pk.

15. 928 bushels; 1863 bush. 1 pk.; 2530 bush. 1 pk.

16. 5243 Kg. 0 Hg. 1 Dg. 2 g.; 58087 Kg. 0 Hg. 5 Dg. 4 g.; 66848 Kg. 4 Hg. 0 Dg. 3 g.

17. 60767 reams 17 quires: 81023 reams 16 quires; 91147 reams 15 quires 12 sheets.

Exercise VIII (i). p. 174-175.

1. Rs. 27-12-4; r. = 7 p.; Rs. 21-9-8; r. = 4 p.; Rs. 20-13-4.

2. Rs. 251-6-3; r. = 25 p.; Rs. 107-11-10; r. = 4 p.; Rs. 82-7-9; r. = 142 p.

3. £49-13-2; r. = 6d.; £47-0-11; r. = 1d.; £42-11-3; r. = 15d.

4. £748-14-11; r. = 34d.; £464-14-9; r. = 52d.; £434-15-1; r. = 56d.

5. 8 can. 3 mds. 6 v.; r. = 58 viss ; 6 can. 12 m. 2 v. ; r. = 64 viss ; 5 can. 11 mds. 0 v. ; r. = 4 viss.
6. 2 can. 14 mds. 0 v.; r. = 599 viss.; 2 can. 3 mds. 6v.: r. = 65 viss.; 6 can. 11 mds. 2 v. ; r. = 65 viss.
7. 24 tons 9 cwt. 0 qr. 3 lbs.; r. = 21 lbs. ; 10 tons 17 cwt. 1 qr. 11 lbs. ; 6 tons 19 cwt. 2 qr. 25 lbs. ; r. = 7 lbs.
8. 634 tons 13 cwt. 1 qr. 3 lbs. ; r. = 36 lbs.
330 tons 4 cwt. 2 qr. 14 lbs. ; r. = 224 lbs.
9. 7 Kg. 9 Hg. 0 Dg. 1 g. ; r. = 20g.; 3 Kg. 7Hg. 8Dg. 6g.; r. = 15g.; 2 Kg. 3 Hg. 9 Dg. 1 g. ; r. = 27 g.
10. 44 m. 2 fur. 75 yds. ; r. = 13yds.; 34m. 5 fur. 160 yds.; r. = 8 yds.; 29 m. 6 fur. 214 yds. ; r. = 26 yds.
11. 37 qr, 2 bush. 0 pk. 1 gal. 1 qt. 1 pt. ; r. = 4 pt. ;
28 qr. 3 bush. 2 pk. 0 gal, 1 qt. 1 pt. ; r. = 17 pt. ;
23 qr. 3 bush. 3 pk. 1 gal. 3 qt. 0 pt. ; r. = 31 pt.
12. 90 qr. 0 bush. 2 pk. 1 gal. 1 qt. 1 pt. ; r. = 25 pt. ;
31 qr. 4 bush. 2 pk. 0 gal. 1 qt. 0 pt. ; r. = 292 pt. ;
14 qr. 4 bush. 3 pk. 1 gal. 2 qt. 0 pt. ; r. = 282 pt.
13. 15 kal. 10 mar. 5 mea. ; r. = 27 mea. ; 12 kal. 5 mar. 0 ; r = 23 [mea. ; 10 kal. 2 mar. 2 mea. ; r. = 59 mea.
14. 33 Km. 9Hm. 8Dm. 6m.; 16Km. 3Hm. 3Dm. 9 m.; r. = 22 m.; 10 Km. 8 Hm. 9 Dm. 2 m. ; r. = 74 m.
15. 54 Km. 7 Hm. 9 Dm. 4 m. ; r. = 6 m. ; 10 Km. 4 Hm. 8 Dm. 9 m. ; r. = 29 m.
16. 46; r. = 4. 17. 858 ; r. = 9 yds. 1 ft. 6 in. 18. 336.
19. 139—372 grs.

Exercise VIII (j). p. 175—178.

1. Rs. 327-1-6.
2. Rs. 35-10-0.
3. Rs. 162-4-6.
4. Rs. 5658-14-0.
5. Rs. 23576-6-7.
6. 16 cwt. 1 qr. 14 lbs.
7. 7924 miles 1024 yds.
8. Rs. 127-14-0 ; Rs. 50127.
9. the second ; by Rs. 75.
10. Loses £25-5*s*.
11. 276 nickel coins.
12. Rs. 7700.
13. 1 qr. 8 lb. in the buyer's favour.
14. 3 min. 56 sec.
15. 1 mile 865 yds.
16. 5610 spoons.
17. 3 miles 220 yds.
18. 87 can. 8 mds. 1 viss.
19. 4 miles 465 yds. nearly. About 125 yds. per minute.

20. 2 cwt. 0 qr. 23 lbs. 21. 104 houses.
22. Rs. 205261 12 as. 6 ps. 23. 2 carriages.
24. Rs. 250. 25. 58 yds. 2 ft.
26. 0-14 p.m. 27. £1238 10*s*. 10*d*.
28. 337 persons ; 11 as. 1 pie. 29. B, Rs. 27·8 ; A, Rs. 40·8.
30. Rs. 981. 31. Rs. 22-2-2. 32. 65 dozen.
33. 4 m. 6 fur. 104 yds. 34. 2·4 Km.
35 73 ft. nearly per second; 1466$\frac{2}{3}$ yds. per min.
36. 5682. 37. 2 min. 40 sec. past 3 p.m.
38. (1) 190 days ; (2) 327. 39. 235.
40. Tuesday, Saturday.

Exercise IX (a). p. 181—186.

1. 44 measures. 2. 670.
3 36. 4. 63668,
5. 7659866. 6. Rs. 18389001-10-8 p.
7. 780752 ; 142868. 8. 923621. 9. 73°,
10. 106, above the average. 11. 65·8613 g., 67·8613 g.
12. 16 years 9·96 mon. 13. 5 ft. 7·02 in. 14. 66360.
15. The average would be increased by Rs. 1251·6·10 nearly.
16. Rs. 450-4-5. 17. 541 nearly : 539 nearly.
18. 52 nearly. 19. 164 persons,
20. About 381 21. 4 tons 5 cwt. 3 qrs. 27·6 lbs.
22. Rs. 205-8-0. 23. 39 ; 35 ; 25 ; 50 ; 40,
24. Rs. 693·3 4 nearly.

Exercise IX (b). p. 187.

1. Re. 0-3-2 ; Rs. 3. 2. Rs. 18-12-0 ; Rs. 312-8-0.
3. 4 as. 9$\frac{3}{5}$ ps. Re. 1-12-9$\frac{3}{5}$ p. ; Rs. 3-3-4,
4. Rs. 55-0-5 ; Rs. 1572-3-7 pies nearly.
5. 25 yds. 6. Rs. 1206-6 as.
7. Rs. 76-14-0. 8. Rs. 29-12-0.
9. 45 days. 10. £36-17-4.
11. 3 hrs 45 min. 12. 21 hrs, 53 min. nearly.
13. Rs. 75-6-0. 14. 1 quarter.
15. Rs. 5-5-4. 16. 1018 Kg. nearly.
17. 17 hrs. 20 min. 18. £18333-6*s*. 8*d*.
19. 97$\frac{1}{4}$ lbs. nearly. 20. 4 as. 10 ps.
21. 1814 tons. 22. 2648$\frac{3}{4}$ lbs.
23. 151·2 miles. 24. Rs. 3907.

25. 46 tons. 26. Rs. 5-9-10 nearly.
27. By 2 minutes, (1) 15 weeks, (2) 30 weeks, (3) 45 weeks.
17 candles. 29. 1 qt. nearly.
30. Rs. 97161856 nearly.

Exercise IX (c). p. 191—195.

1. 492. 2. 784135.
45 yrs. 31 yrs. 23 yrs. 15 yrs. 6 yrs.
5. 170. 6. 5004.
7. 6059. 8. 209.
92900, 94425. 10. (*a*) 84; (*b*) 547; (*c*) 1238 by 2134.
11. 15 seconds, 5 first, 600 third classes.
12. C 66, B 134, A 87. 13. 44 yrs. 24 yrs. 14. Rs. 240, Rs. 10.
15. Value of a cow = Rs. 20 ; of a sheep = Rs. 5.
16. 100 mangoes ; 200 mangoes.
17. 1 anna each pencil. 18. 91 of each set.
19. 6 men, 12 women and 18 boys. 20. 10 sovereigns, 30 cr. 70 fl.
21. 50 two annas 400 ($\frac{1}{4}$ as.) ; 200 ($\frac{1}{2}$ as.)
22. £12-13-0 loss. 23. Gain Rs. 15-3-0.
24. Rs. 3-8-10 pies nearly. 25. 8448 revolutions.
26. 20 miles per hour. 27. 13 ft. 4 inches.
28. B = £1046, A = £1121, C = £1155.
30. 9 annas. 31. Child Rs. 10, woman Rs. 30, man Rs. 60.
32. 12 shillings ; £903.
33. Rs. 122435937-8 annas. 34. Rs. 42-9-10.
35. £8-8-9. 36. Rs. 535-0-7.
37. The purchaser should pay £11-16-7 more. 38. Rs. 2-8-0.
39. A must pay B Rs. 4000. 40. Rs. 102-2-6.
41. Rs. 91-8-2 to the nearest pie. 42. 23 times.
43. A must pay C Rs. 11-1-9$\frac{1}{3}$; B must pay C Rs. 5-13-5$\frac{1}{3}$.
44. 3840. 45. 8*s*. 11*d*. per yard nearly.

Revision Papers. I Series. p. 196—204.

1

2. 684 rem. 82.
3. 14·8 cm. ; 41·6 cm.
4. N = 50.

2

1. 2685.
2. 552000 ; 55752000.
3. 2593.

1—contd.

5. 3080*d*.
6. 1026·363.
7. $4(a - b)$ rupees.
8. Rs. 30240.

2—contd.

5. $a=60$.
6. 85*d*.
7. 3 m. 5 cm. 6 mm.
   2 m. 2 cm. 2 mm.
8. Rs. 529-2-8.

3

2. 11 hrs. 2 min. ; 11-59 a.m.
   11 hrs. 6 min. ; 12 p.m.
   11 hrs. 8 min. ; 12 p.m.
   11 hrs. 16 min.; 12 p.m.
   11 hrs. 26 min.; 12 p.m.
   11 hrs. 30 min.; 12 p.m.
   11 hrs. 38 min.; 12 p.m
3. 15·6 cm. nearly.
4. 6 annas.
5. (*a*) 99, 224, 255.
   (*b*) 8001, 15626.
6. 100 revolutions.
7. Rs. 46.
8. 1 minute to 1 p.m. or 0·59 p.m.

4

1. 44 ft. per second.
2. 30415095690.
4. 200 yds. 37° S. of East.
5. $(16\,ax + by)$ annas.
6. 2 Kg. 3 Hg. 6 Dg. 2 g. 1 dg.
7. Rs. 5-5-6.
8. 23 and $\frac{1}{40}$ marks.

5

1. $12 \times 6 \times 10 - 38 + 25 - 47 + 32 = 692$.
2. 2116.
3. 20 ft.
4. $b = 7$, $a = 9728$.
5. 21 times.
6. (1) $n + 1$. (2) $n - 1$; 110, 109, 111.
7. Rs. 12-3-2.
8. Loses 1 anna.

6

2. 40 remainder 3.
4. (*a*) 7836×10^3.
   (*b*) 64389×10^4.
   (*c*) 18×10^7.
5. Rs. 155.
6. No ; 12 miles an hour.
7. Gains Rs. 199.
8. £60, 120 crowns, 300*s*.

7

1. Rs. 19.
2. 75 ; 438.
3. ·7 cm.
4. 280 ; 420.
5. 288 knives.
6. A and C must pay B 11*s*. 6*d*. and 2*s*. 4*d*. respectively.
7. 528 revolutions.
8. 71°.

8

1. $3^{8} \times 11 \times 37$.
2. 3408000.
6. $100a + 10b + c$.
7. 808.
8. 10 as. 11 pies.

9

2. Rs. 4740
3. 123, 91389.
5. 117°.
6. Rs. 198-10-9½.
7. 44 minutes.
8. £2 12*s*. 8*d*.

10

1. 30766 rem. 171.
2. Rs. 158-4-0.
4. 168 revolutions; 14 revlns.
5. Walking diagonally; 2 minutes.
6. 80 litres.
7. $(2x+3y)$ miles.
8. 39; 79; 130.

Exercise X (a). p. 208.

1. 50 g. 2 dg. 7 cg. 4 mg.; 58 g. 6 dg. 5 cg. 3 mg.; 67 g. 3 cg. 2 mg.; 75 g. 4 dg. 1 cg. 1 mg.

2. 31 m. 5 dm. 6 cm.; 35 m. 5 dm. 5 mm.; 7 m. 8 dm. 9 cm. 0 mm.; 11 m. 8 dm. 3 cm. 5 mm.

3. 25 fur. 9 ch. 14 lks.; 43 fur. 1 ch. 90 lks.; 60 fur. 4 ch. 66 lks.; 69 fur. 1 ch. 4 lks.

4. 604·492; 5440·428; 6217·632; 5526·784; 4317·8; 5181·36; 60449·2; 69084·8.

5. 2·595; 7·785; 19 778·5.
6. 151·2 cm.
7. 100·8 in.
8. Rs. 1652.
9. 13600 g.
10. 57·06 km.
11. $892{\cdot}8 \times 10^5$ miles.

Exercise X (b). p. 213.

I. (*a*) 284·16; 303·104; 304·5248. (*b*) 4·9592; 1324·1064.
(*c*) 112032; 1125·9216; 11·259216.
(*d*) 55·156656; 1614·8372. (*e*) ·00001739184.

II. (*a*) ·01406594. (*b*) ·1226736.
(*c*) 13·63599.

III. (*a*) ·000499905. (*b*) ·401956; 61023·377953.

Exercise X (c). p. 214.

1. 10·528; 9·024; 7·896; 5·264; 3·948.
2. 36·4377375; 32·3891.
3. ·00329625; ·00293.
4. ·052636; ·029242.

Exercise X (d). p. 215.

1. 13·334-R. ·031; 11·474-R. ·007; 6·758-R. ·055; 5·543-R. ·062.

2. 77·5525-R. ·0062; 67·9336-R. ·0043; 58·8150-R. ·0037; 50·6557-R. ·0026.

3. 7·1469, R. ·0084; 5·4461, R. ·0335; 4·1653, R. ·0432; 3·5362, R. ·0237.

4. 12·80008. R. 00871; 10·90056, R. ·00159: 3·68716, R. ·01527; 2·62704-R. ·00183.

Exercise X (e). p. 215.

1. (1) ·8071, R. ·0091; (2) ·6715, R. ·01; (3) ·5829, R. ·0099; (4) ·4996, R. ·0147; (5) ·4372. R. ·0051.

2. (1) ·011078, R. ·0075; (2) ·12925, R. ·0027; (3) ·009694, R. ·0003; (4) ·040462, R. ·0001.

3. (1) ·004477589, R. ·0000103; (2) ·002368311, R. ·0000363; (3) ·00020468980, R. ·0000007; (4) 0·001326693, R ·0000135.

4. (1) ·00593, R. ·00049; (2) ·00162, R. 00241; (3) ·00137-R ·00565; (4) ·00100-R. ·00425.

5. (1) ·00001304, R. (2) ·000001159, R. ·000008; (3) ·00000096, R. ·0000512; (4) ·0000077. R. ·0000296.

Exercise X (f). p. 217.

1. 3·605. 2. 1·53. 3. ·106. 4. 1·65.
5 ·093. 6. 2·0017. 7. 7096·4 8. ·048206.
9. 10·3. 10. ·4506. 11. ·54518.
12. 743·7714...... 13. ·0104...... 14. 41875·06.
15. 14 and 34·37 cm. 16. 429 times. 17. 6·508 oz.
18. 117932·56 c.c. 19. 106 metres.
20. 258 times; 7·36 pints. 21. 5250.
22. 29·92739......; (*a*) 299·2739; (*b*) 29·92739.
23. 12·1326; (1) 121·326; (2) ·0121326.
24. 123·27 grains; £3-17-10$\frac{1}{2}$*d*. 25. £1749016·2159.
26. 3·72 dm. 27. Rs. 28 12 as. 11 ps. nearly.
28. Rs. 91-8 as. 5 ps. nearly. 29. 60 Km.
30. $2{\cdot}746532 \times 10^{13}$. 31. 50. 32. Rs. 222-5-5.
33. ·0355 m. 34. 35 miles. 35. ·00489 in.
36. 37·3 gallons nearly. 37. 7·28 in.
38. 12 times. 39. 1·340625 in. 40. ·000144 ft.

Exercise XI (a).—*Practical*. p. 228.

1. $\angle 2 = \angle 3 = \angle 6 = \angle 7 = 148^\circ$ and $\angle 1 = \angle 4 = \angle 5 = \angle 8 = 32^\circ$.

2. $\angle 2 = 120^\circ$. 6. 7·8 cm. and 6·2 cm. and 46°, 134° and 46°.

7. 9·8 cm. and 3·4 cm. and 104°, 76°, 104°.
10. 6·3 cm. and 3·1 cm. 12. AD=2·3 inches; ∠BAD=50°.
13. AD=4·35 in. 14. each diagonal=4·3″.
15. 56°, 124°, 124°, diagonals=3·9 inches and 2·2 inches and 90°.
16. 12·1 cm. and 6·4 cm. 90°.

Exercise XI (c). p. 240.

5. (1) A=B=C=60°. (2) 51°, 51°, 78°. (3) No triangle.
(4) No triangle. (5) 41°, 41°, 100°. (6) No triangle.
(7) 21°, 16°, 143°. (8) 76°, 30°, 74°.

Exercise XI (d). p. 242.

1. a=10·2 cm. C=33°. B=40°. 2. a=11·9, 42° each.
3. (1) 1·3″, 42°, 98°; (2) 10·8 cm 28°, 44°; (3) 8 cm. 43°, 52°; (4) 7·8 cm. 35°, 28°. 4. 51°, 39°. 5. 28°, 22°.

Exercise XI (e). p. 243.

1. 35°. 2. 74°. 3. 2·1° each. 4. 78°, 4·9 cm., 6·4 cm.

Exercise XI (f). p. 245.

. c=5 cm., C=125°, A 15°. 2. a=7·3 cm. 68°, 32°.
3. 9·6 cm. 55°, 51°. 4. No △. 5. 1·2″, 20°, 42°.

Exercise XI (g). p. 245-246.

1. 90°, 38°, 52°. 5. 10 miles from P; after 3 hours from starting. 11. 4 solutions. 12. 12 ft., 42°.

Exercise XII (a). p. 248-249.

1. $(ap+bq)$ annas; Rs. 9 12 as. 2. $am+bn$; 695.
3. (a) 20; (b) 6; (c) 104; (d) 20; (e) 92; (f) 2; (g) $26\frac{2}{3}$; (h) 64; (i) 512; (j) 4096; (k) 40960; (l) 131072.
4. (a)−31; (b)−5; (c) 141; (d) 218. 5. 2, 1, 6, 17, 57, 121.
6. 1, 105; 156, 369. 7. $5280x$ ft. 8. $(12x+6y)$ pence.
9. $a+b$, $a+2b$, $x=a+2b$. 10. (a) $x+3$, $x+4$, $x+5$. (b) x, $x-1$, $x-2$. 11. (a) $2n+1$, $2n+3$, $2n+5$; (b) $2n+1$, $2n-1$, $2n-3$.
12. $x=4$. 13. $x=4$. 14. (a) ax pence; (b) $(ax+by)$ pence.
15. $7x$ miles. 16. vt ft., $s=vt$.
17. $\frac{28}{x}$ miles per hour, 4 miles. 18. $5x=6y$.
19. (1) 1056; (2) 88; (3) $t=5$.
20. (1) 16 ft., 64 ft., 144 ft., 256 ft., 400 ft. (2) $t=1, 2, 3$.

Exercise XII (b). p. 251.

1. (*a*) $x = 4$, $5x = 20$, (*b*) $x = 10$, $6x = 60$.
2. (*a*) $x = 8$, (*b*) $x = 12$, (*c*) $x = 5$, (*d*) $x = 8$. 4. $x = a - b$.
5. $x = 4a + 3b + c$, $x = 5$.
$tr.$, $x = 48$ miles. 7. $x = ab + cd$, $x = 115$ miles.
8. $x = ap + bq + cr$, $x = 220$ rupees.
9. $hx = r$, $h = 65$. 10. $\dfrac{ab + cd}{a + c}$ rupees.

Exercise XII (c). p. 253.

1. $16a$. 2. $21a^2b$. 3. $10a^2b^2$. 4. $-5p$.
5. $-14xy$. 6. $8pq^2$. 7. $13c^2$. 8. $11xyz$.
9. 8, 40. 10. 7. 11. 6, 54. 12. 60, 5.
13. C, £20 ; A, £100 ; B, £120. 14. 40. 15. 12 yrs. 48 yrs.
16. B, Rs. 705 ; A, Rs. 6345, C = Rs. 2115.

Exercise XII (d). p. 254.

1. $4a - 2b + 9c + d$. 2. $7y + 7z$.
3. $6p - 5q + 17r$. 4. $15ab - 8ac + 3bc$.
5. $24xy - 15xz + 13yz - 15xl$. 6. $2a + 3b + 3d$.
7. $15xy + yz - xz$.
8. $5a^2b - 3a^2c - 20b^2c - 18bc^2 + 4a^3b + 6ac^2 + 4ab^3$.
9. $4 + 10a - 8b - 12c - 10ab - 7ac - 10bc$.

Exercise XII (e). p. 257.

1. $5a - 13b$. 2. $5x^2 + 2y^2 - 3z^2$.
3. $xy + 9xz + yz - 8ab$. 4. $a^3 + a + 1$.
5. $x^4 - x^3 + 3x^2 - x - 1$. 6. $3p^3 + 5q^3 - 10r^3 - 15pqr$.
7. $x^4 + y^4 - 14x^2y^2 - 5x^3y - xy^3$.
8. $a^4 + a^3 + 5a^2 + 9$. 9. $12x^3 - 17x^2y + 26xy^2 - 15y^3$.
10. $4b$. 11. $a - 7b + 3c$.
12. $-3a^2 + 2ab - 10ac - 8bc - 12c^2 - 10b^2$; -225.
13. $a + b - c - d$.
14. $a + b - p - q$. 15. $-8b$ miles.

Exercise XII (f). p. 259.

1. $x=2$. 2. $x=5$. 3. $x=7$. 4. $y=9$. 5. $x=9$.
6. $y=9$. 7. $x=8$. 8. $x=2$. 9. $x=10$. 10. $x=5$.

Exercise XII. (g). p. 269.

1. 13 and 21. 2. 52°, 32°, 96°. 3. 8.
4. 22. 5. 7, 8, 9, 10.
6. C, Rs. 100; A, Rs. 200; B, Rs. 300. 7. 21, 23, 25.
8. Rs. 15 and 5 quarter rupees. 9. A, 16 years; B, 8 years.
10. A = B = 55 sheep.

Exercise XIII—*Practical*. p. 261.

1. (*a*) 8 sq. in.; (*b*) 21 sq. cm.; (*c*) 15 sq. in.; (*d*) 32 sq. cm.
2. (*a*) 20 sq. in.; (*b*) 32 sq. cm.; (*c*) 11·2 sq. cm.
3. (*a*) 12 sq. in.; (*b*) 30 sq. cm.; (*c*) 48 sq. in.; (*d*) 80 sq. cm.

Exercise XIII (a). p. 264-265.

1. 234 sq. ft.; 12 ft.; 20 ft.; 7 sq. yds.; 5 ft.; 14 ft. 7 in.
2. 112 sq. yds.; 3 sq. ft. 105 sq. in.
3. 34 ac. 657 sq. yds.; 5 sq. ft. 4. 213 sq. ft. 48 sq. in.
5. 486 sq. in. 6. 2050 sq. yds.
7. 162 sq. ft. 8. 518 sq. ft.; 90. sq. in.
9. 564 sq. in. 10. 98·5 sq. in.
11. 44 sq. yds.; 4 sq. ft.; 25 ft. 12. 88 cm.; 448 sq. cm.
13. 27500 sq. m.; 2·75 hectares. 14. (i) 180 sq. mm; (ii) 1·8 sq. cm.
15. 15 dm. 16. 500 cm. 17. 833 tickets 18. 1,944,000 lbs.

Exercise XIII (b). p. 266-267.

2. 330 ft, in. both cases. 3. 132 ft.; 90 sq. ft.
4. 72 ft. 5. 160 slabs.
6. 1080 tiles. 7. 1248 tiles.
8. 280 ft. 9. 28 benches each 11 ft. long
10. 432 sq. ft. 11. 700 sq. ft. Re. 1-5 as.
12. 10 sq. ft. 13. 21 sq. ft. Re. 1-5 as. 14. 497 sq. ft.

Exercise XIII (c). p. 270—273.

1. 198·66 sq. ft. 2. 4·03 sq. ft.
3. 4·89 ft. nearly.
4. (1) 156¼ sq. ft. (2) 435600 sq. yds.
5. 28·288 acres. 6. 1170·8928 acres. 7. 110 20 ft.
8. (1) 840 sq. ft. (2) 1150·92 sq. ft. (3) 920 sq. ft.
9. (1) 4 16 ft. (2) 6·42. 10. (1) L = 20 ft. (2) 24·4125 ft.
11. (1) 181·75 ft. (2) 6·525 ft. 12. £264-12s

13. Rs. 12-15-11 ps. nearly.
14. Rs. 17-13-9 nearly.
15. Rs. 28-3-5.
16. (1) 120 yds.; (2) 261·3 yds.
17. (1) Rs. 15; (2) Rs. 32 10 as. 7 p.
18. 56 rolls.
19. B = 10 ft.; L = 30 ft.
20. 15 ft. and 450 sq. ft.
21. Rs. 7-10-9.
22. 12 ft.
23. B = 16 ft.; H = 12 ft.
24. L = 32 yds.
25. 13 yds. 0 ft. 2 in.
26. 1980 meshes.
27. 2904 sq. yds., 508 yds.
28. 117 sq. in.
29. 344 sq. ft.
30. 12 ft.
31. $62\frac{1}{2}$ sq. ft.
32. 25 big sheets; nothing will remain.
33. 8 sq. ft. 18 sq. inches; 203 sq. ft. 18 sq. in.
34. 3200 acres.
35. The first is cheaper by as. $3\text{-}4\frac{1}{5}$ ps.

Examples. p. 273.

Ex. 1. about 1·3 sq. in.; 1·27 sq. in.; 1·12 sq. in.; 1·63 sq. in. ·26 sq. miles.

Exercise XIII (d), p. 278.

1. 6 cubic in. $3'' \times 2'' \times 1''$.
2. (*a*) 8 cubic in. $4'' \times 2'' \times 1''$ (*b*) 12 cubic in. $3'' \times 4'' \times 1''$ (*c*) 20 cubic in. $5'' \times 4'' \times 1''$.
3. 3″, 2″, 2″; 4″, 2″, 2″; 3″, 4″, 2″; 5″, 4″, 2″. Each volume is doubled.
4. (*a*) 18 cub. in. (*b*) 472·5 cub. in. (*c*) 1043·04 cub. in. (*d*) 3600 c. c. (*e*) xyz c. c. (*f*) pqr c. in.

Exercise XIII (e). p. 280—283.

1. 426 cub. ft. 1152 cub. in.
2. 6840 cub. ft.
3. 33 cub. ft. 351 cub. in.
4. 84000 Kg.
5. (*a*) 54872 cub. in. (*b*) 117·649 c. c.
6. (*a*) 2 cub. ft. 1095 cub. in. 22500 cub. ft. (*b*) L = 20 ft. L = 34 ft. (*c*) B = 4 cm. B = 3 ft. 8 in. (*d*) H = 10 m. H = 2 ft. 6 in.
7. 4·24 in.
8. 18·7 in.
9. 63·15 cub. ft.
10. Rs. 181-4.
11. Rs. 56.
12. 52·083 lbs.
13. Rs. 5-3-10 nearly.
14. 1000 c. ft.; 960 c. ft.; 90 c. ft.
15. 24030 cub. in.; £1-11-3·5*d*. nearly.
16. 1·7664 metres.
17. 12·75 in.
18. 284·5 c. ft.
19. 12600 bricks.

20. 2667 c. ft., 144 c. in.; 12001 c. ft., 1512 c. in.
21. 5 in.
22. 2616000 leaves.
23. ·055 sq. in.
24. ·44 in.
1266·6 oz.; 6188 oz.
26. 19144 c. in.; £2-2-5·1d.
27. $1189\frac{7}{12}$ oz.
28. 4152 c. c.; 7·098 litre.
29. 3 tons 11 cwt. 3 qrs. 13 lbs.
30. 36,000 sheets.
31. Rs. 43-12.
32. $1\cdot\dot{3}$ c. c.; 49·38 cm.
33. $266\cdot\dot{6}$ cub. ft.
34. 10 tons.
35. ·05 sq. in.
36. 25 metres.
37. 4400 c. ft. of water.
38. ·15 in.
39. No. only 24.
40. $11\cdot\dot{3}$ cub. ft.; Rs. 58-4 as.

Exercise XIII (f). p. 283.

1. 66·1 acre.
2. 129·6 sq. ft.
3. Rs. 9-1-1 nearly.
4. 272 tons.
5. 1074 lbs., 3·5 oz.
6. 1243·5 acres.
7. Rs. 150, Rs. 20.
8. 415·8 gallons.
9. 1536 stones.
10. (1) 48 m. by 38 m. 1824 sq. m. (2) 20 m. by 10 m. 200 sq. m.
(3) 30 m. by 20 m. 600 sq. m.
11. 11 cub. in.; 27 cub. ft.
12. 10638; 1063 oz.
13. 252 millions of gallons.
14. 2433·375 cub. in.
15. (*a*) 56 sq. ft. (*b*) 96 sq. ft.
(*c*) 175 sq. ft. (*d*) 78 sq. ft. (*e*) 128 sq. ft. (*f*) 80 sq. ft.
(*g*) 112 sq. ft. (*h*) 96 sq. ft. (*i*) 132 sq. ft.

Exercise XIV.—*Practical.* p. 297.

7. 45°.

Exercise—*Practical.* p. 300.

1. 2·5 cm., 37°.
2. 8·5 cm., 96°.

Exercise XV (a). p. 308-309.

1. 60.
2. 52.
3. 4.
4. 33.
5. 432.
6. 108.
7. 1356.
8. $1111\frac{1}{9}$.
9. 288.
10. 87696.
11. 268320.
12. 8712.
13. 115920.
14. 818125.
15. 1194480.
16. 47520.
18. 2016 (least number).
19. 240 and 300.

17. $146\frac{1}{4}\,^0/_0$; he gets Rs. 15 more. 18. Rs. 9336.
19. 1 ft. 5·3 in. 20. 55. 21. 11·2 in., 5·6 in., 2·4 in.
22. 113·04 gr. nearly. 23. 7·76.
24. (1) 1·094 nearly. (2) ·914 nearly. 25. 12 turns nearly.
26. ·462 nearly. 27. 54 times ; ·093 of a gallon.
28. $\left\{\frac{c^2}{36} - \frac{1}{6}c(a+b) + ab\right\}$ sq. ft. $312\frac{17}{18}$ sq. ft.
29. Rs. $2g\left(\frac{2g+l+w}{27 \times 12}\right)d \times \frac{c}{16}$; Rs 4-0-2 nearly.
30. 80894 lbs. $8\frac{1}{2}$ oz. 31. 7·1 inches nearly.
32. 7296 × 114. 33. $2275 \times 5 \times 7^2 \times 13^2$.
34. 7, 12, 17, 22, 27, 32, 37, 42, 47............292, 297.
15, 23, 31, 39, 47 ....87, 95.
47, 87. 127, 167, 207. ....
35 Rs. 5-12-2 nearly. 36. Rs. 271980 and Rs. 283500.
37. 26″ 1 ; 26″·0, 19″·9. 38. 65° and 63°.
39. 22 sq. in 40. $(-\frac{3}{2}, 0)$; (0, 3).
41. 30 days. 42. Rs 36455-8-6.
43. 10 ft, 44. 7309·25 sq. ft.
45. 8·3 ft. 47. 51·4 ft.; 11·7 ft.
49. $572\frac{1}{2}$ lbs. 50. $1631\frac{1}{4}$ yds.
51. $2·97\,^0/_0$ nearly. 53. Rs. 28-15-9 nearly.
54. Rs. 5. 55 $14\frac{2}{7}\,^0/_0$. 56. 12 as. 4 ps. nearly.
57. 2·8″; ·8″. 2·24 sq. in , $152·1\dot{3}$ lbs.
58. A = Rs. $387\frac{1}{2}$; B = Rs. $516\frac{2}{3}$; C = Rs. $645\frac{5}{6}$.
59. 185·85 yds. 60. Rs. 7-14-8.
61. 12·45 miles from the starting point in a direction 60° N. of E.
62. $5\frac{5}{16}$ hrs. 63. 8 months.
64. 385 men. 65. $\frac{1}{10}, \frac{7}{30}, -\frac{1}{6}$; 12 minutes.
66. $14\frac{2}{5}$ hours. 67. After 60 full days (12-18 ; 11-48) or 1380 days ; 6·54.
68. 360 days. 69. $7\frac{1}{5}$ days, $7\frac{3}{7}$ days.
70. Rs. $\left(\frac{1}{8} + \frac{x-1}{16}\right)$, 6 as. 10 as.; 9 miles.
71. 75. 72. 189 miles. 73. $40\frac{1}{3}$ miles.
74. Loss $3\frac{4}{5}\,^0/_0$. 75. $29\frac{21}{125}\,^0/_0$ gain.
76. $7·8\,^0/_0$. 77. Rs. 4 6 as.
78. 43·25 ; no difference in this case. 79. ·87 acre.
80. To the nearest integer. 81. 6780 sq. miles.

82. 766 ft. 8 in ; 300 ft. 6 in.; ·65 of an acre.

83. (1) $px+qy$; (2) $\dfrac{px+qy}{x+y}$.

84. 22 nearly.

85. $\dfrac{px-qy}{x-y}$.

86. Rs. 611-10-3 nearly.

87. 25 miles in 45 minutes.

88. Rs. 7.

89. 45 miles per hour.

90. $x = -1\frac{15}{16}$.

91. $b=5{\cdot}51$ nearly.

92. A square ; Twice.

93. 26° and 154°.

94.

Rs.	22	8	0
	12	0	0
	4	8	0
	39	0	0

95. 51 ft.

96. 120 miles, Re. 1 14 as.

97. The latter.

98. Rs. 90.

99. 6 as., 100 %, 300 %, 150 %.

100. Rs. 400, 410-13-0, 5·12 %; 76·67 %.

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20. 97053 (less than 100000). 21. (a) $5a^2bc$. (b) xy. (c) $10xy$.
22. (a) 360 $a^3b^3cd^2$. (b) 798 a^2cbd^3. (c) 600 $a^3cbx^2y^2z^2$.

Exercise XV (b). p. 312-313.

1. 52. 2. 89. 3. 428571. 4. 11.
5. 1836. 6. 14. 7. 16. 8. 13596.
9. 8 inches; 3 windows. 10. Re. 1-4-0.
11. 9^3 c. in., 4522 packets. 12. 10 palams.
13. 167, 3 and 6. 14. 167.
15. 298. 16. 324. 17. 216, 12 and 17 deep.

Exercise XV.—*Graphical.* p. 314.

1. 288 ft. 2. 378 ft. 3. Every fifth on the former and every fourth on the latter are opposite to one another.
4. $\frac{1}{2}$ inch. 5. 409·5.

Exercise XV (c). p. 316.

1. 9360. 2. 81627. 3. 94080. 4. 780390.
5. 5220. 6. 355300. 7. 2359188. 8. 216216.

Exercise XV (d). p. 317-318.

1. 1872. 2. 5040. 3. 1680 sheets. 4. 729.
5. 498. 6. 1090. 7. Rs. 651. 8. 506.
9. Rs. 18-6; 2 as. 4 p. 10. 4 as.
11. 10 shillings. 12. 57600 km. = 35489 miles.
13. 25 lbs. 14. 40 ft. 15. 2 hours.
16. 120,090,547$\frac{1}{2}$ years. 17. 1$\frac{1}{2}$ hours.
18. 864 and 756.

Exercise XVI (a). p. 322.

1. Rs. 7. 2. £104-4-4. 3. 180 lbs.
4. 5 yds. 2 ft. 5$\frac{41}{54}$ in. 5. 4 dm. 2 cm. 5 mm.
6 Rs. 110-1-0. 7. 2 tons 10 cwt. 1 qr. 6 lbs.

Exercise XVI.—*Graphical.* p. 323.

1. 7 : 20. 2. (i) 1 : 5. (ii) 1 : 5. (iii) 1 : 4. (iv) 1 : 4. (v) 3 : 4. (vi) 3 : 16. (vii) 25 : 8. (viii) 5 : 8.

Exercise XVI (b). p. 325-326.

1. (a) 9$\frac{1}{3}$; (b) 40$\frac{5}{12}$; (c) 43$\frac{5}{13}$; (d) 29$\frac{17}{25}$; (e) 26$\frac{1}{31}$; (f) 1$\frac{459}{628}$.
2. Rs. 5796. 3. Rs. 17250-2-6.

4. 2 can. 8 m. 0 viss 15 pal. 5. $16\frac{1}{9}$ days.
6. Rs. 270-2-$6\frac{5}{12}$.

Exercise XVI. (c) p. 327-328.

1. (a) 3·1 in.; (b) 4·2 cm.; (c) 6·4 in.; (d) 8·5 cm.
2. 5·9 cm. 3. 6 cm.
4. (a) 2·8 cm.; (b) 17·9 in.; (c) 42 yds.; (d) 67·2 yds.
5. 14·9 cm. 6. (1) 8·6 ft.; (2) 26·1 ft.; (3) 15 ft.
7. 98. 8. (1) 30·5 ft. nearly; (2) 25 ft.

Exercise XVI (d). p. 335.

1. (a) $\frac{10}{11}$; (b) $\frac{12}{13}$; (c) $\frac{24}{25}$; (d) $\frac{13}{17}$. 4/3
2. (a) 7 : 6; (b) 27 : 35; (c) 4 : 3; (d) 26 : 31.
3. $(\frac{8}{24}, \frac{24}{72}, \frac{32}{96}, \frac{120}{360})$; $(\frac{7}{15}, \frac{25}{75}, \frac{105}{225})$; $(\frac{6}{23}, \frac{72}{276}, \frac{78}{299})$. 4. (1) $\frac{3}{5}, \frac{5}{8}, \frac{5}{6}$.
6. (a) x = (1) 81; (2) 108; (3) 135; (4) 72.
(b) x = (1) 120; (2) 90; (3) 72; (4) 135.
(c) x = (1) 126; (2) 168; (3) 210; (4) 112.
(d) x = (1) 200; (2) 150; (3) 120; (4) 225.
(e) x = (1) 280; (2) 210; (3) 168; (4) 315.
(f) x = (1) 144; (2) 192; (3) 240; (4) 128.
7. (a) 5 can. 10 mds.; (b) 13 can. 15 mds.; (c) 132 candies.
8. 60 marks. 9. Rs. 156 4 as.

Exercise XVI (e). p. 337.

1. $\frac{20}{60}, \frac{15}{60}, \frac{12}{60}, \frac{10}{60}, \frac{1}{3} > \frac{1}{4} > \frac{1}{5} > \frac{1}{6}$.
2. $\frac{990}{3465}, \frac{2079}{3465}, \frac{1540}{3465}, \frac{1575}{3465}, \frac{3}{5} > \frac{5}{11} > \frac{4}{9} > \frac{2}{7}$.
3. $\frac{196}{420}, \frac{126}{420}, \frac{112}{420}, \frac{108}{420}, \frac{7}{15} > \frac{6}{20} > \frac{8}{30} > \frac{9}{35}$.
4. $\frac{1008}{2016}, \frac{1872}{2016}, \frac{1512}{2016}, \frac{1064}{2016}, \frac{13}{14} > \frac{18}{24} > \frac{19}{36} > \frac{8}{16}$.

Exercise XVI (f). p. 339-340.

1. 11 as. $2\frac{1}{5}$ p. 2. 8 as. $11\frac{3}{7}$ p. 3. £2-2-10.
4. 6 as. $10\frac{9}{39}$ p. 5. $\frac{87}{120}$. 6. $1\frac{4987}{6552}$.
7. $2\frac{123}{160}$. 8. $4\frac{3671}{10080}$. 9. $\frac{667}{1716}$.
10 $\frac{43}{1200}$. 11. $3\frac{19}{45}$. 12. $\frac{193}{200}$. 13. $9\frac{1}{3}$.
14. $125\frac{3467}{5040}$. 15. $12\frac{1}{4}$. 16. $15\frac{7}{8}$.
17. $124\frac{117}{272}$. 18. $44\frac{3799}{4800}$. 19. $2\frac{23573}{45600}$. 20. $7\frac{883}{1938}$.
21. $22\frac{135}{476}$. 22. (a) Rs. $6\frac{4}{4}$ (b) £$104\frac{683}{840}$. 23. (a) $\frac{114}{151}$.
(b) $\frac{43}{144}$. 24. $\frac{5639}{12768}$. 25. (a) Rs. 8-10-0.
(b) 13 tons 7 cwt. 3 qrs. 25 lbs $10\frac{5}{12}$ oz. (c) 1651471 cg. 6·25 mg.

26. $\frac{118}{160}$. 27. $\frac{7}{12}$ cis. 28. 5 : 12.
29. $\frac{28}{60}$; 60. 30. 1200.

Exercise XVI—*Symbolical*. p. 340.

1. $\frac{ps+rq}{qs}$.
2. $\frac{adf+bcf+ebd}{bdf}$.
3. $\frac{ps-qr}{qs}$.
4. $\frac{aqd+pbd-cbq}{bqd}$.
5. $\frac{aqd-pbd-cbq}{bqd}$.
6. $\frac{xbq+ayq-pqb}{ybq}$.

Exercise XVI (g). p. 342.

1. ·8$\dot{3}$. 2. ·875. 3. ·9$\dot{2}$. 4. 1·4
5. ·4$\dot{1}4285\dot{7}$. 6. ·0475. 7. ·8125. 8. ·296875.
9. ·0006988120. 10. ·00561. 11. ·000513......
12. ·00025. 13. 3·045...... 14. 8·039. 15. 10·023.
16. ·4545. 17. ·9412. 18. ·4444.

Exercise XVI (h). p. 348.

1. $\frac{75}{176}$; $\frac{45}{154}$; $\frac{15}{26}$
2. $1\frac{1}{7}$; $1\frac{17}{35}$; $7\frac{227}{815}$.
3. $5\frac{41}{75}$; $5\frac{17}{65}$; $5\frac{1}{35}$.
4. $618\frac{14}{27}$; $1224\frac{2}{3}$; $1143\frac{172}{225}$
5. $341\frac{2}{3}$.
6. $4329\frac{13}{45}$.

Exercise XVI (i). p. 349.

1. $6\frac{2}{5}$. 2. $1\frac{13}{30}$. 3 $563\frac{3}{4}$. 4. $474\frac{301}{576}$.
5. $381\frac{597}{935}$. 6. Rs. 4-12-0$\frac{2}{3}$. 7. £19-1-7. 8. $-$£8-3-3.
9. 13 mds. 6 v. 1 p. 0$\frac{7}{40}$ tola. 10. 3 cwt. 2 q. 6$\frac{31}{60}$ lbs.
11. Rs. 1989-3-7. 12. 4 tons 0 cwt. 3 qrs. 8·54 lbs.

Exercise XVI (j). p. 351.

1. $\frac{18}{125}$; $\frac{18}{175}$; $\frac{9}{100}$; $\frac{2}{25}$.
2. $\frac{13}{294}$; $\frac{13}{126}$; $\frac{13}{630}$; $\frac{13}{840}$.
3. $\frac{161}{17620}$, $\frac{161}{22400}$, $\frac{161}{26880}$.
4. $5\frac{1}{3}$; $2\frac{1}{2}$; $2\frac{1}{3}$; $1\frac{8}{4}$.
5. $5\frac{3}{5}$; $5\frac{13}{15}$; $13\frac{11}{25}$; 10.
6. $\frac{20}{169}$; $\frac{4}{13}$.
7. $\frac{25}{27}$.
8. (*a*) 89$\frac{3}{5}$ lbs. (*b*) 37$\frac{1}{3}$ lbs. ; 2$\frac{2}{5}$.
9. $7\frac{1}{2}$.
10. $2\frac{2}{7}$ times.
11. 42 times $\frac{1}{6}$ qr.

Exercise XVI (k). p. 351-352.

1. (*a*) £106-17-5$\frac{5}{11}$. (*b*) 9 m. 4 fur. 180 yds. (*c*) Rs. 13-0-6$\frac{151}{322}$.
(*d*) 11 tons 10 cwts. 2 qrs. 24$\frac{20}{27}$ lbs.
2. Rs. 5-15-2$\frac{6}{7}$.
3. Rs. 516-10-4$\frac{2}{5}$.
4. 31$\frac{3}{7}$ miles per hour.
5. $\frac{17}{47}$; ·3617002.
6. $\frac{292}{321}$ part. ·9364...

7. $\frac{1}{11664}$. 8. $1\frac{54029}{63099}$.
9. $25\frac{81}{128}$. 10. $\frac{403}{3350}$.
11. Rs. $1\frac{13}{28}$.

Exercise XVI (l). p. 353-354.

1. $1\frac{7}{44}$. 2. $\frac{5}{8}$. 3. $\frac{551}{3124}$. 4. $\frac{32}{45}$.
5. 1. 6. $3\frac{23}{35}$. 7. $23\frac{1}{3}$. 8. $2\frac{47}{291}$.
9. $1\frac{4}{31}$. 10. $12\frac{1}{3}$. 11. $1\frac{1}{15}$. 12. $1\frac{2713}{7800}$.
13. 6. 14. $7\frac{41}{47}$. 15. $\frac{3373}{5124}$ 16. $3\frac{486}{925}$.

Exercise XVI (m). p. 354-355.

1. $\frac{2}{5}$; Rs 5; Rs. 3. 2. $\frac{3}{4}$. 3 $21\frac{1}{4}$ yds.
4. 6 miles. 5. $\frac{1}{5}$ day; $\frac{1}{6}$ day, $\frac{11}{30}$ d.
6. $1\frac{23}{25}$ as.; 3 as. 7. 13 pieces; $5\frac{1}{4}$ ft. will be left behind.
8. Rs. 10,000; Rs. 2530 each. 9. $\frac{6}{25}$.
10. 18 : 25. 11. 10 : 9. 12. 1 : 19.
15. ·5, ·25, ·125, ·2, ·04, ·608, ·025, ·0125, ·002, ·00125.
16. (*a*) $\frac{7}{8}$. (*b*) $\frac{3}{8}$. (*c*) $\frac{9}{16}$ (*d*) $\frac{11}{16}$.

Exercise XVII (a). p. 357.

1. 6. 2. (*a*) $\frac{201}{400}$, ·5025. (*b*) 1. (*c*) $1\frac{1}{4}$, 1·25. (*d*) $\frac{1}{200}$, 1·005.
(*e*) $\frac{273}{400}$, 68·25 (*f*) $\frac{499}{500}$, ·998. (*g*) $\frac{1967}{2000}$, ·9835. (*h*) $2\frac{1}{2}$, 2·5.
3. (*a*) ·520825 p. c. (*b*) $2\cdot\dot{2}$ p. c. (*c*) $101\cdot\dot{5}\dot{1}$ (*d*) $\frac{100}{x}$p. c.
4. 5 m., 4. m., ·25 m., ·85 m.

Exercise XVII (b). p. 358-359.

3. (*a*) $2\frac{3}{16}$ % (*b*) $6\frac{1}{9}$ %. (*c*) $16\frac{7}{30}$ % (*d*) ·077 %.
(*e*) $1\frac{8}{22}$ %. (*f*) $4\frac{1}{8}$ %. 4. (*a*) $6\frac{2}{3}$ %. (*b*) ·25%.
(*c*) $17\frac{1}{2}$ %. (*d*) 10·71875 %. 5. 6%.
6. 27·7 % nearly. 7. 3·2 % nearly.
8. 91·6; 93·1. 9. 62 %. 10. 300; 99; 105.
11. Brahmins 56·5 %, 35 %, 8·5 %. 12. (*a*) Rs. 80.
(*b*) £285-10-0. 13. (*a*) 18 miles – $0\frac{4}{9}$ furlong.
(*b*) 38·2382. 14. (*a*) 1 ton 1 cwt. 12 lbs.
(*b*) ·$120\frac{2}{3}$ mds. 15. $106\frac{2}{3}$. 16. 8100.
17. 150, 360, 75, 15.
18. 60 boys, 66, 24.
19. 377·7 cwt of air, 116·65 cwt. of oxygen, 5·65 cwt. carbonic [acid.
20. Rs. 528-2-0

Exercise XVII (c). p. 362.

1. Rs. 679 4 as. ; Rs. 977-11-4.
2. £3659 5*s*. ; £5610 17*s*.
3. Rs. 53893-5-4 ; Rs. 199125-6-8. £287547-7-0.
4. £120838-2-6 ;
5. Rs. 821-11-3$\frac{3}{4}$.

Exercise XVII (d). p. 364.

1. Rs. 43. 2. Rs. 240-0-0. 3. Rs. 2158-12-0.
4. Rs. 621-2-0. 5. £136-18-0. 6. Rs. 337-13-0.

Exercise XVII (e). p. 365—368.

1. Rs. 10-0-5$\frac{1}{3}$; 2 as. 10 ps.
2. Rs. 36-4-5.
3. 21·6 km. 4. $3334\cdot\dot{5}\dot{4}$ Kg. 5. $\frac{5}{12}$. Rs. 24.
6. $\frac{59}{120}$. 7. Rs. 28000. 8. $\frac{1}{225}$. 9. $36\frac{7}{8}$ gallons.
10. 2 days more. 11 $\frac{17}{18}$. 12. 293 steps ; 2346.
13. $\frac{19}{96}$. 14. Re. $\frac{2}{3}$; Rs 6$\frac{2}{5}$. 15. £1899-2-3$\frac{3}{4}$.
16. Rs. 183-8-3. 17. Rs. 11880-1-9 nearly.
18. Rs. 908-3-1 ; Rs. 1453-5-4. 19. Rs. 101-7-7.
20. Rs. 2-12-1·2. 21. Rs. 362-9-6 nearly.
22. Rs 961187-8-0. 23. Rs. 10433-2-0.
24. Rs. 38-3-4. 25. Rs. 3344-1-0. 26. 3071687$\frac{1}{2}$ tons.
27. 3 hrs. 28. 1$\frac{1}{5}$ tons ; $\frac{3}{10}$; $\frac{1}{3}$ ton.
29. Rs. 81-5-7 nearly. 30. 384. 31. 1694.
32. 180, 300. 33. 765.
34. 5676860. 35. B Rs. 7-6-2 nearly ; A Rs. 18-7-4 nearly.
36. C Rs. 3375, A = Rs. 281-4-0, B = Rs. 843-12-0.
37. 400 runs nearly. 38. 108 voters 58, 38.
39. $-5\frac{15}{53}$. 40. 10560 persons.

REVISION PAPERS—II Series.

1.

1. Rs. 12 2 as. 7 pies nearly. 2. 1$\frac{1}{2}$ viss. 3. $\frac{677}{4004}$.
6. 8472·89 lakhs. 7. 11·9 hectolitres. 8. ·0036.

2.

1. 1·43. 2. Re. 1-2-4. 3. 2 years 6 months 12 days.
4. 105 lbs. ; 302 bars. 5. A rhombus.
6. H. C. F. = $5^2 \times 13 \times 3$ or 975.
   L. C. M. = $5^2 \times 3^3 \times 2 \times 13 \times 37$ or 649350.
7. (1) $\frac{881}{13200}$. (2) ·02507. 8. $xy+pq$ miles.

3.

1. ·082. 2. Rs. 6-1-9 nearly. 3. $5\frac{21}{43}$. 4. $\frac{31}{240}$.
5. 110 ft. 40°, 20°. 6. − 2. 7. 7 as. 4 pies ; 45·83%.
8. Rs. 404-10-0 ; the former.

4.

1. (1) 64 can. 4 md. 1 v. ; (2) Rs. 96487-6-0 ; (3) Rs. 12-8 4.
2. 76·60 acres. 3. (*a*) £2 18*s*. 6*d*. nearly ; (*b*) $\frac{64}{105}$.
4. 84 ft. 5. 2. 6. C $34^\circ\frac{26}{31}$. B $=87^\circ\frac{3}{31}$, A $=58^\circ\frac{2}{31}$.
7. Rs. 70. 8. (i) 8 sq. ft. (ii) 7 sq. ft. ; (iii) $3\frac{1}{2}$ sq. ft. ; (iv) 37 sq. ft.

5.

2. Rs. 2-13-0. 3. 90°, 56°, 34°, 4·8. 4. 30*mn* gallons.
5. ·001, about 77. 6. $4\frac{1}{3}$ miles, $6\frac{1}{3}$ miles 7. 13 ac. 84·5 cents.
8. 86552.

6.

1. $\frac{600457}{1036840}$. 2. 15408 ; ·0494...... 3. Rs. 162.
4. 9·2 ft. 5. $180\frac{15}{29}$ oz. ; $144\frac{12}{29}$ oz. ; $722\frac{2}{29}$ oz.
6. $\frac{a}{b}=\frac{c}{d}$. 7. 12·1923 sq. inches and 78·5712 sq. cm. 6·44 sq. cm.
8. 527500888·92 days.

7.

1. 12 h. 4 m ; 11 h. $32\frac{1}{4}$ m.
2. $\frac{8}{13}$ is the greatest and $\frac{48}{88}$ is the least. 5. 10·4 inches.
6. 30, 31, 32, 33. 7. Re. 1-10-$7\frac{3}{8}$.
8. (1) $7\frac{1}{3}$ sq. ft. ; (2) 5 cub ft.

8.

1. £6875. 2. 19824 revolutions ; 79296.
3. 3135 packets ; 9^3 cub in. 4. C Rs. $2607\frac{9}{13}$; B Rs. $3911\frac{7}{13}$; A Rs. $1955\frac{10}{13}$. 5. 10·9m. ; 87°. 6. $d = tr$.
7. 6·366 miles. 8. Rs. 9-12-9.

9.

1. 1614 m. 2. ·5859375 of a ton. 3. Rs. 9734-12-1 nearly.
4. Rs. 58. 5. 2 ft. 6. 12·875. 7. 1480 sq. ft.

10.

1. 7 tons 10 cwts 1qr. 11 lbs nearly. 2. 46835 bricks.
3. $22\frac{5}{24}$ sq. ft. ; $29\frac{1}{3}$ sq. ft. 4. 46 ft. ; 38 ft.

5. $ab - 8 = cd + 8$.
6. $x = -48$.
7. Rs. 1463 9 as. 2 ps.
8. 399·645 francs.

Exercise XVIII (a). p. 383-384.

1. Between 3 cm. and 15 cm.

Exercise XVIII (b). p. 386-387.

4. 60°.

Exercise XVIII (d). p. 391-392

7. 60 knots.
8. 90 yds.
9. C's rate 2·2 miles.
10. 3·5 furl. nearly.
11. 4·4 miles, 2·9 miles.
12. AC = 2 miles, BC = 6·74.
13. AC = 3 miles, BC = 3·43 miles.

Exercise XIX. p. 399-400.

3. (a) 5·4 ; (b) 9·9 ; (c) 7·8 ; (d) 18·4 ; (e) 5·4 ; (f) 11·2.
4. (a) 6·7 ; (b) 8·5 ; (c) 9·8 ; (d) 7·8 ; (e) 12·8 ; (f) 7·9.
6. (1) (–3·9, –3·5) ; (2) 1, 1·3.
7. 3·3.
11. 0, 0 ; 0, 0.
12. (a) 5 ; (b) 13.

Exercise XIX (b). p. 406—409.

1. (i) $116\frac{1}{2}$ lbs. (ii) $135\frac{1}{2}$ lbs.
2. 10·25 ft. ; about 1·12 seconds.
4. (1) 10·5 in. (2) 8·5 in.
6. 66 in., $68\frac{3}{4}$ in.
7. Rs. 23.
9. (i) 5·6 ft. (2) 28 ft.
10. 33·18 sq. ft.
11. 32 tons ; 45 in.
12. Rs. 21 and Rs. 38.
13. 69 lbs. and 79 lbs.

Exercise XX (a). p. 415.

1. (a) 36 : 50. (b) 16 : 21. (c) 1495 : 1232.
2. (a) $x = \frac{48}{13}$. (b) $x = \frac{117}{8}$. (c) 28.
   (d) $x = \frac{1}{45}$. (e) $x = \frac{ab}{e}$.
3. (a) $12\frac{1}{3}$. (b) $12\frac{4}{5}$. (c) $13\frac{1}{11}$.
4. (a) $16\frac{1}{5}$. (b) $91\frac{1}{7}$. (c) 25.
5. (a) 9. (b) 12. (c) $\frac{1}{3}$. (d) $\frac{2}{15}$.
6. (a) 9 : 8.
   (b) 4 : 9.
   (c) 1 : 1.
7. (1) $7\frac{1}{5}$.
   (2) $8\frac{2}{5}$.
   (3) $13\frac{8}{19}$.
8. $\frac{2}{19}$, $\frac{8}{21}$, $\frac{4}{7}$, $\frac{9}{87}$, $\frac{3}{x}$, $\frac{y}{a}$.

Exercise XX (b). p. 421—423.

1. Rs. 812-8-0.
2. Re. 0-12-0.
3. 17203200 cub. ft.
4. Rs. 24275 13 as. 10 ps. nearly.
5. 99·9 sq. ft.
6. 8·998 lbs.
7. 4·3 miles.
8. Saidapet 8-36 A.M.; St, Thomas Mount 9-30 A.M.; Chingleput 11-36 A.M.; Conjeevaram 2 P.M.
9. About 2·2 pies per mile; 14 as. 9 ps.
10. 336 tons.
11. 360 working days.
14. 10 in., $2\frac{1}{2}$ in.; $12\frac{1}{2}$ in. 12 in., 14 in.
15. 1680 lbs.
16. $87\frac{1}{2}$ lbs. $4\frac{1}{2}$ mds.
17. £1 13*s*. 4*d*.
18. XU = $15\frac{5}{9}$ miles; XZ =about 22 miles; YZ = 40 miles.
21. $17\frac{1}{2}$ lbs.
22. pr.
23. $\frac{c}{b}$ men.
24. 11·20 A.M. or 11·20 P.M.
25. 7-50 A.M.

Exercise XX (c). p. 425-426.

1. 10 hrs.
5. $90\frac{4}{17}\frac{0}{7}$ yds.
7. 240 rails; 176 rails.
8. 75 cm.
9. 34 days.
10. 12 pairs.
11. 100 days.
12. $\frac{15}{256}$ inch.
13. By $\frac{1}{5}$ gallon.
14. $\frac{xy}{p}$ pies.
15. $\frac{ab}{c}$ rooms; $\left(a - \frac{ab}{c}\right)$ rooms.

Exercise XXI (a). p. 431.

1. £27.
2. £155 10*s*. 5*d*.
3. £63.
4. £43 15s.
5. Rs. 37 8 as.
6. Rs. 32 8 as. 1 p.
7. Rs. 99.
8. £2 2*s*. 8*d*.
9. Rs. 4 7 as 4 p.
10. Rs. 5 1 a. 2 p.
11. $2\frac{6}{7}$ yrs.
12. 4 yrs.
13. 20 yrs.
14. 5 yrs.
15. 5 yrs.
16. $5\frac{2}{3}$ yrs.
17. $2\frac{1}{2}$%.
18. 5%.
19. $5\frac{1}{3}$%.
20. $4\frac{1}{3}$%.
21. Rs. 400.
22. Rs. 750.
23. £400.
24. £250.
25. £75.
26. Rs. 85.

Exercise XXI (b). p. 434.

1. (i) 8 as. 4 ps. (ii) 4 as.
2. 7 as. 10 ps.
3. Rs. 186-10-8.
4. Rs. 720-0-0.
5. £518-8-0.
6. Rs. 576.
7. Rs. 500.
8. $4\frac{3}{8}$%.
9. 5 yrs.
10. 6 p c.

11. £529-14-4½.
12. Rs. 3203-2-0.
13. Rs. 315.
14. 5·6 per cent. nearly.
15. Re. 1-2-0.
16. 37½ % ; Rs. 6.
17. £ $\frac{\bullet}{240}$; 8s. nearly.
18. Rs. $\frac{9p.}{8}$; Rs. 216.

Exercise XXII. *Practical*. p. 437-438.

1. 6 sq. in. 2. 36°, 54° ; 24 sq. cm. 3. 10". 4. 30 sq. mm.

Exercise XXII (a). p. 439-440.

1. 3·68 sq. in.; 3·60 sq. in.; 3·64 sq. in. 2. 4·32 sq. in.
3. 2·1 sq. in.
4. 3·1 in.
5. 42°.
6. 1·4".

Exercise XXII (b). p. 441-442.

1. 5·2 sq. cm. ; 5·1 sq. cm. ; 5·07 sq. cm. ; 5·12 sq. cm.
2. 8·8 in.
3. 11·73 sq ft.
4. 155·4 sq. ft.
5. 3·9 sq. in.
6. $\frac{d_1 d_2}{2}$, where d_1 and d_2 are the diagonals.
7. $\frac{D^2}{2}$, where D is the diagonal. 8. 45 yards ; 1008 sq. yds.
9. 2·6 sq. in.
10. $\frac{pr}{q}$.
11. 40 ft.
12. 16 sq. cm.

Exercise XXII (c). p. 443.

5. Four solutions.
6. Four solutions.

Exercise XXII (d). p. 446-447.

1. $c^2=a^2+b^2$.
2. 15 miles.
3. 15.
4. 3·3 nearly.
5. 3·46".
6. 10 ft.
7. 5·2 inches nearly.
8. 13 miles.
9. 5 miles an hour.
10. 6 miles an hour.

Exercise XXII (e). p. 450-451.

1. 330 ft. 2. 37 yds. 3. 58°.
4. 16°. 5. 104 ft. 6. 11°, 150 ft and 200 ft.
7. 25 ft. ; 36 ft. 8. 245 ft. 9. 35 yds.
10. 40 ft. 11. 30 ft. 12. $1\frac{3}{32}$ miles per hour.
13. 250 yds. 14. 127°. 15. ·5 of a mile.

16. (i) 18 squares ; (ii) 24·5 squares ; (iii) 8 squares.
17. 36 cm. 18. 25°, 155°. 19. 3·6 sq. in.
20. 3·5 sq. in. 21. 4 sq. units. 24. 27 ft.
26. 25 ft. 28. $\sqrt{x^2 - y^2}$ in.
29. (a) 50 ; (b) 10 ; (c) $(x_1 - x_2)^2 + (y_1 - y_2)^2$.
30. 20, 65, 89.

Exercise XXIII (a). p. 455-456.

1. 159; 212 ; 318. 2. $453\frac{1}{3}$, $226\frac{2}{3}$, 170.
3. A, 15 as. ; B, Re. 1 4 as. ; C, Re. 1 9 as.
4. Rs. 442, Rs 936, Rs. 195. 5. Rs. 133-3-2 p.
6. Rs. 1800 ; Rs. 750
7. 1 child gets Rs. $82\frac{2}{3}$; 1 woman, Rs. $165\frac{1}{3}$; 1 man, Rs. $330\frac{2}{3}$.
8. Third class, Rs. 2-1-4.
Second class, Rs. 6-4-0. } 6 p. per mile.
First class, Rs. 12-8-0.
9. B, Rs. 1500 ; C, Rs. 3000 ; A, Rs. 2500.
10. B, 91 mks. ; A. 46 mks.
11. Youngest, Rs. $1073\frac{5}{11}$; Eldest, Rs. $3220\frac{4}{11}$; Second, Rs. $1610\frac{2}{11}$.
12. 20 men, 40 women, 60 boys.

Exercise XXIII (b). p. 458-460.

1. In 9 days more. 2. In $22\frac{1}{2}$ days. 3. $3\frac{9}{13}$ days.
4. C in 24 days, A in $18\frac{2}{3}$ days, B in $33\frac{3}{5}$ days.
5. (i) $5\frac{5}{47}$ days. (ii) C in $34\frac{2}{7}$ days ; A, $14\frac{2}{17}$ days, B, $10\frac{10}{23}$ days.
6. $78\frac{1}{3}$ days. 7. A, $10\frac{2}{3}$ days, B in 32 days.
8. A gets Rs. $5\frac{1}{3}$, B gets Rs. $4\frac{2}{3}$.
9. $11\frac{3}{7}$ days. 10. 86 men. 11. $58\frac{56}{193}$ hours.
12. 3000 gallons. 13. $3\frac{8}{5}$ hours. 14. 24 hours.
15. $\frac{7}{19}$ cistern. 16. 40 hours. 17. 30 minutes.
18. $9\frac{5}{19}$ hours.

Exercise XXIII (c). p. 461-462.

1. 10 hours—11 miles. 2. 2 hours.
3. 2 hours later. 4. 60 miles.
5 In 12 days. 6. $3\frac{1}{3}$ hours, $33\frac{1}{3}$ miles from the starting point.
7. $3\frac{3}{7}$ miles an hour. 8. $16\frac{2}{13}$ miles. 9. $6\frac{8}{22}$ seconds.
10. $38\frac{19}{22}$ seconds. 11. $1\frac{2}{3}$ miles per hour.
12. 1 mile per hour. 13. Still water, 6 miles per hour, stream, 2 miles per hour.
14. 4-14 P.M. 15. 1-29 P.M.
16. $3\frac{45}{76}$ miles per hour.

Exercise XXIII (d). p. 465.

1. Loses Rs. 7-13-0 ; $7\frac{13}{16}$ %. 2. 25 %; $\frac{100\,(q-p)}{p}$ p. c.
3. Rs. 6-5-3 nearly. 4. £378 ; 37·8 %.
5. 400. 6. $4\frac{1}{6}$ p. c. 7. $7\frac{1}{37}$ %.
8. 20 % increase. 9. $62\frac{1}{2}$ %, 25 %, $6\frac{1}{4}$ %, $6\frac{1}{4}$ %.
10. $11\frac{1}{9}$ %. 11. Rs. 12-14-8. 12. Rs. 27-9-7 nearly.
13. $17\frac{1}{2}$ %. 14. $\frac{19}{16}x$. 15. 1s. $10\frac{1}{2}d.$, 2s. $0\frac{3}{16}d.$; 1s $11\frac{5}{8}d.$

Exercise XXIII (e). p. 467-468.

1. £2925. 2. Rs. 122-6-4. 3. Rs. 153-3-0.
4. 6 ps. 5. £1050. 6. Rs. 220.
7. £1687-10-0. 8. Rs. 271-8-0.

Exercise XXIII (f). p. 468-469.

1. Rs. 1440. 2. 12s in the £. 3. Rs. 8400, Rs. 5600.
4. £3 17s. 2d. nearly. 5. By 10 ps. nearly.
6. Rs. 335-15 ; $\frac{48}{64}$. 7. 12 as. in the rupee. 8. Loss $56\frac{1}{4}$ %.

REVISION PAPERS—III Series.

1. (a) $\cdot 1458\dot{3}$. (b) 10s. 9d. 2. 5500 sq. yds. 3. £636.
4. $x = 4$. 5. 10·4 inches nearly. (b) 62·3 sq. in.
6. 1·9 lbs. nearly. 7. C is the fastest. 8. 195 yds.

2.

1. (a) ·7625. (b) Rs. 667-8 as. 2. 128 : 105. 3. $4\frac{1}{2}$ %.
4. 1 to 12. 5. 80. 6. $58\frac{2}{3}$ ft. per second.
7. 155700 sq. yds., (taking $\sqrt{3} = 1{\cdot}73$) ; £2412-14-2 nearly.
8. A, Rs. 75, B, Rs. 195, C, Rs. 270.

3.

1. Rs. 1708-5-4. 2. Rs. 203-2-0. 3. 16s. $11\frac{3}{7}d.$ a gallon.
4. 23′ past 1, 48′ past 1, 53′ past 1. 5. $18\frac{3}{4}$ %, $5\frac{1}{3}$ yrs.
6. 1176. 7. ·001729734 of an acre. 8. 34 per 1000 nearly.

4.

1. $\frac{1}{791}$. 2. 16 cawnies.
3. y = 72, 48, 36, 24, 18, 14·4, 13·1, 12, 10·3, 9, 8, 6, 4, 2.
4. (8 5, 5), (5, 8·5), (−3, 9·5). 5. $33\frac{1}{3}$ days.
6. 25 + 5 is the cheapest. 7. Rs. 180, 50 %. 8. About 94 yds.

5.

1. $\frac{3}{4}$ full. 2. 30 %. 3. Rs 251 10 as. 4. Rs. 10.
5. $\frac{1}{3}$. 6. 3 ft., $5\frac{1}{3}$ ft.; $6\frac{2}{3}$ ft.
7. 20 miles, 4-57 A.M. next day. 8. 9·75 cub. ft.

6.

1. 35·3 nearly.
2. 50 %; 16 %; 13 %.
3. Rs. 473 10 as. 11 ps. nearly.
4. Rs. 24-10-6 nearly.
5. 8*s*.
7. 31·14 sq. inches.
8. £159-12 copper, £36 spelter

7.

1. 103680 bricks ; Rs. 440.
2. $27\frac{3}{11}$ days.
3. A = Rs. $685714\frac{2}{7}$, B = Rs. $1028571\frac{3}{7}$, C = Rs. $1285714\frac{2}{7}$.
4. Y = 12, 18, 24.
5. $-1\frac{29}{34}$.
6. 99 ft.
7. 52′.
8. $58\frac{1}{3}$ cwt., $97\frac{1}{2}$ cwt.

8.

1. 298, 2634320.
2. 13 as. 9 p.
3. ·35 of an acre.
4. Rs. 1,300.
5. 17 ft.
6. 1·9″ by 2″, 3·8 sq. in.
7. 41.

9.

1. 12 yds. ; 9 sq. yds.
2. $2\frac{2}{7}$ yrs. 75 %.
3. 14·6 sq. cm.
4. 5*s*. $\frac{1}{2}$*d*. nearly.
5. £6 6*s*.
6. 4 days earlier.
7. 17 units.
8. D = Rs. $311\frac{1}{9}$, S = Rs. $933\frac{1}{3}$.

10.

2. Rs. 104.
3. 3 hours 12 minutes.
4. (*a*) $p^2 + q^2$ (*b*) 10.
5. ·283 of an acre.
6. Reaches completion on the 4th day.
7. £7, £273.
8. $\frac{27\,abc}{4}$; 7000 bricks.

Chapter XXIII. Miscellaneous Examples. p. 478—489.

1. 1, 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79.
   $3^3 \times 11 \times 13 \times 7 \times 37$.
2. Re. 1 6 as. 5 p. nearly.
3. (1) $3^4 \times 4^3 \times 19 \times 41$ (2) $3^3 \times 4^2$.
4. Re. 0-1-4 in the rupee.
5. $p = 5$.
6. 1·6 cm. ; 5·6 cm.
7. 165°.
6. 8 as. a viss ; 9 as., 10 as., 11 as., 12 as., Re. 1 4 as. 0 p., Rs. 5.
10. 40 measures.
11. 71·24 cwt.
12. 19·589 miles.
13. Rs. 96
14. 1 foot towards the 6 ft. encroachment.
15. 6 miles 76 yds.
16. 1000944.